Question 0 **2025 Mu Alpha Theta National Convention Mu Bowl**

This is a practice question and will not count your overall score in the Mu Bowl.

$\sim\sim\sim\sim\sim\sim\sim\sim$ *Good luck, and have fun!* $\sim\sim\sim\sim\sim\sim\sim\sim$

A. Let $A = \displaystyle\sum_{n=0}^{\infty} \frac{1}{2^n}$.

B. Let $B = \displaystyle\sum_{n=0}^{\infty} \frac{3^n}{n!}$.

C. Let $C = \displaystyle\sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{1}{5^m 7^n}$.

D. Let $D = \displaystyle\sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{11^m \cdot 13^n}{m! \cdot n!}$.

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Question 1 **2025 Mu Alpha Theta National Convention Mu Bowl**

Let $f(x) = (x-1)^7(x-4)^2$.

A. Let $A = a$, where $(a, f(a))$ is a local maximum of $f(x)$.

B. Let $B = b$ where $(b, f(b))$ is a local minimum of $f(x)$.

C. Let $C = f''(2)$.

D. Let $D = \lim_{x \to \infty} \dfrac{f(2x)}{f(x)}$.

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Let $f(x) = 4 - x^2$, and let $\mathcal{R}$ be the region bounded by $f(x)$ and the x-axis.

A. Tangent lines to $f(x)$ at $x = \pm a$ are drawn. The region bounded by these tangent lines and the x-axis is an equilateral triangle. Let A equal the area of this triangle.

B. Let B equal the area of $\mathcal{R}$.

C. A solid has a base bounded by $\mathcal{R}$, and cross-sections of the solid perpendicular to the x-axis are semicircles. Let C equal the solid's volume.

D. $\mathcal{R}$ is rotated over the line $x = 6$. Let D equal the volume of the solid formed.

Question 2 **2025 Mu Alpha Theta National Convention Mu Bowl**

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Question 3 **2025 Mu Alpha Theta National Convention Mu Bowl**

You may wish to use patterns or Maclaurin series to find the following derivatives.

A. Let $A = \dfrac{d^6}{dx^6}\left[\sin(x^2)\right]_{x=0}$.

B. If $\dfrac{d^{20}}{dx^{20}}\left[\dfrac{1}{(1-x)^3}\right]_{x=0} = b_1 \cdot b_2!$ where b_1 and b_2 are positive integers and b_1 is minimized, let $B = b_1 + b_2$.

C. Let $C = \dfrac{d^8}{dx^8}\left[e^{-x^2}\right]_{x=0}$.

D. Let $D = \dfrac{d^4}{dx^4}\left[\dfrac{6x-16}{x^2-6x+8}\right]_{x=0}$.

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Question 4 **2025 Mu Alpha Theta National Convention Mu Bowl**

Evaluate the following integrals.

A. Let $A = \displaystyle\int_0^4 \frac{x}{x^2+16}\,dx$.

B. Let $B = \displaystyle\int_0^4 \frac{x^2}{x^2+16}\,dx$.

C. If $\displaystyle\int_0^4 \frac{x}{x+16}\,dx = Q + R\ln(P_1) + S\ln(P_2)$ for integers Q, R, and S and primes P_1 and P_2, let $C = P_1 + P_2 + Q + R + S$.

D. If $\displaystyle\int_0^4 \frac{x^2}{x+16}\,dx = Q + R\ln(P_1) + S\ln(P_2)$ for integers Q, R, and S and primes P_1 and P_2, let $D = P_1 + P_2 + Q + R + S$.

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Question 5 **2025 Mu Alpha Theta National Convention Mu Bowl**

Croix is chasing her girlfriend Ursula in the air above Luna Nova Magical Academy. At the start of the chase, Croix is 330 feet south of Ursula, and Ursula begins flying on her broom east at 50 feet per second. Croix's flying Roomba travels at 60 feet per second.

A. Croix knows the speed of both her Roomba and Ursula's broom, and flies in a straight line towards the point she knows she will catch Ursula. Let A equal the number of seconds equal to the duration of the chase.

B. Croix thinks Ursula doesn't notice her sneaking up on her, and flies in a straight line towards Ursula's initial position. Let B equal the number of seconds until Ursula will be increasing the distance between her and Croix.

C. It's a stormy night, so Croix can only see Ursula when lightning flashes, every 2 seconds. There is a flash of lightning at the start of the chase. During a lightning flash, Croix changes direction to fly towards where she sees Ursula. Let $\vec{v_0}$ be the vector representing Ursula's movement after the $t = 0$ lightning flash, and let $\vec{v_2}$ be the vector representing Ursula's movement after the $t = 2$ lightning flash. Let θ be the obtuse angle between $\vec{v_0}$ and $\vec{v_2}$. Let $C = \cos^2 \theta$.

D. Croix flies east until she can turn north and crash into Ursula to give her a kiss. Let D equal the total distance Croix travels, in feet.

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Question 6 **2025 Mu Alpha Theta National Convention Mu Bowl**

Consider the graph $\mathcal{G}$ of $x^3 + y^3 = 2xy$ in the first quadrant. The shape of this graph is a closed loop.

A. $\mathcal{G}$ passes through the point $(1,1)$. Let $A = \dfrac{dy}{dx}$ at this point.

B. There are two points on $\mathcal{G}$ such that the tangent line to $\mathcal{G}$ at those points has slope 1. Let B be the area of the circle passing through those points and the origin.

C. The line $y = c$ intersects $\mathcal{G}$. If the maximum possible value of c is $\frac{c_1 \sqrt[3]{c_2}}{c_3}$ in simplest form, let $C = c_1 + c_2 + c_3$.

D. Let D equal the area of $\mathcal{G}$.

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Question 7 **2025 Mu Alpha Theta National Convention Mu Bowl**

The matrix exponential e^X of a square matrix X can be represented as the power series $e^X = \sum_{k=0}^{\infty} \frac{1}{k!} X^k$, where X^0 is the identity matrix.

A. Let A equal the product of the nonzero entries of e^{X_a}, where $X_a = \begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix}$.

B. Let B equal the product of the nonzero entries of e^{X_b}, where $X_b = \begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix}$.

C. Let C equal the sum of the entries of e^{X_c}, where $X_c = \begin{bmatrix} 1 & 2 \\ 4 & 8 \end{bmatrix}$.

D. Let D equal the sum of the entries of e^{X_d}, where $X_d = \begin{bmatrix} 5 & 3 \\ 4 & 6 \end{bmatrix} = -\frac{1}{7} \begin{bmatrix} -1 & 3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} 4 & -3 \\ -1 & -1 \end{bmatrix}$ and $\lambda_1 < \lambda_2$.

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Question 8 **2025 Mu Alpha Theta National Convention Mu Bowl**

Solve the following initial-value differential equation problems.

A. Let $A = y(4)$ if $y' = 6x\sqrt{x^2+9}$ and $y(0) = 19$.

B. If $y' = \frac{(x^2+4x+1)(y^2+4y+1)}{(x+2)(y+2)}$ and $y(0) = 1$, then $y(-1) = b_1 + \sqrt{b_2 + b_3 e^{b_4}}$ for integers b_1, b_2, b_3, and b_4. Let $B = b_1 + b_2 + b_3 + b_4$

C. Let $C = y(2\ln 2)$ if $y' = \frac{e^x - y^3}{3y^2}$ and $y(0) = 0$.

D. Let $D = y(4)$ if $x^2y'' + 6xy' + 6y = 0$, $y(1) = 4$, and $y(2) = -\frac{1}{2}$.

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Question 9 **2025 Mu Alpha Theta National Convention Mu Bowl**

Each of the following parts describes a solid of maximum possible volume that can be inscribed in a sphere of radius 1.

A. The cube would have volume A.

B. One of two identical non-intersecting spheres would have volume B.

C. The cylinder would have volume C.

D. The cone would have volume D.

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D. The cone would have volume D.

The line $y = 20x + 25$ is tangent to all of the following graphs.

A. $f(x) = \ln x + a$. Let $A = e^a$.

B. $g(x) = x^2 + 4bx + 61$. Let B equal the largest possible value of b.

C. $h(x) = cx^2 + 4x + 61$. Let $C = c$.

D. $j(x) = \frac{d}{x}$. Let $D = d$.

Question 10 **2025 Mu Alpha Theta National Convention Mu Bowl**

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Question 11 **2025 Mu Alpha Theta National Convention Mu Bowl**

Particle P_1 moves along the parametrized curve $\vec{r_1}(t) = \left\langle \frac{t^3}{3} - t, t^2 - 1 \right\rangle$, and particle P_2 moves along a curve with parametrized velocity $\vec{v_2}(t) = \left\langle \frac{t^3}{3} + 4t, t^2 + 4 \right\rangle$.

A. Let A equal the product of the magnitudes of the acceleration of P_1 and P_2 at $t = 2$.

B. Let B equal the product of the speeds of P_1 and P_2 at $t = 3$.

C. Let C equal the arc length of the curve traveled by P_1 between $t = 4$ and $t = 5$.

D. Let D equal the unique value of t such that the slope of the curve traveled by P_2 is $\frac{1}{2}$.

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Question 12 **2025 Mu Alpha Theta National Convention Mu Bowl**

This question is relay-style, where each part's answer depends on another part.

A. Ryuko is collapsing Mako's spherical beach ball, decreasing its volume by $C\pi$ cubic centimeters per second. When the ball has a radius of 4 centimeters, let A be the negative rate of change of the radius of the ball in centimeters per second.

B. Satsuki is blinded by ambition, and knows only that she walks along a curve that is a solution to the differential equation $\frac{dy}{dx} = \frac{y^2+y}{x^2-x}$. She eventually finds that she is walking along a line, and stops at the point (A, B).

C. Senketsu and Junketsu both pull a fiber along the x-axis, starting from the origin. Alone, Senketsu would make the shirt follow the path $x = -A\left(\frac{3t^2}{2} + \frac{4t}{3}\right)$, and Junketsu would make the shirt have velocity $v = -B\left(\frac{t^2}{240} + \frac{t}{45} + \frac{1}{20}\right)$. Let C be the position of the fiber on the x-axis at $t = 6$ if Senketsu's and Junketsu's efforts are additive.

D. Kamina's MIGHT grows! When he has n MIGHT, the number of seconds it takes for him to gain 1 MIGHT is $\left|\frac{n^2B^n}{n!}\right|$. It will take Kamina D seconds to become INFINITELY MIGHTY!!! after magically being reduced to 0 MIGHT.

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A. Points P and R are at the points $(1, 0)$ and $(-1, 0)$, respectively. At time t, point Q is at the point $(\cos t, \sin t)$. Let A be the rate of change of the area of triangle PQR when $t = \frac{2\pi}{3}$.

B. At time t', a hexagon has side length $t'^2 + 6t'$. t' is increasing at a rate of 3 units per second. Let B be the rate of increase of the area of the hexagon when $t' = 4$.

C. An ellipse has the equation $x^2 + cy^2 = 1$. c is increasing at 3 units per second. Let C equal the rate of change of the area of the ellipse when $c = 9$.

D. A piece of wire of length 16 is cut, and the two pieces are bent into a square and an equilateral triangle. The sum of the areas of the two pieces is minimized when the perimeter of the triangle is $\frac{d_1\sqrt{d_2}-d_3}{d_4}$ in simplest form. Let $D = d_1 + d_2 + d_3 + d_4$.

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