Q	A	B	C	D
0	2	e^3	$\frac{35}{24}$	e^{24}
1	$\frac{10}{3}$	4	114	512
2	$\frac{361\sqrt{3}}{48}$	$\frac{32}{3}$	$\frac{64\pi}{15}$	128π
3	-120	32	1680	$-\frac{51}{32}$
4	$\frac{\ln 2}{2}$	$4-\pi$	27	-305
5	$3\sqrt{11}$	$\frac{198}{61}$	$\frac{441}{541}$	1980
6	-1	$\frac{2\pi}{9}$	9	$\frac{2}{3}$
7	e^7	12	$\frac{5e^9+1}{3}$	$2e^9$
8	215	382	$\frac{\sqrt[3]{15}}{2}$	$-\frac{5}{16}$
9	$\frac{8\sqrt{3}}{9}$	$\frac{\pi}{6}$	$\frac{4\pi\sqrt{3}}{9}$	$\frac{32\pi}{81}$
10	$20e^{26}$	8	$\frac{16}{9}$	$-\frac{125}{16}$
11	40	$10\sqrt{610}$	$\frac{64}{3}$	$2+2\sqrt[3]{2}$
12	-1	-2	64	$6e^2$
13	$-\frac{1}{2}$	$5040\sqrt{3}$	$-\frac{\pi}{18}$	128

0) **A.** Using the formula for the sum of an infinite geometric series, this is $\frac{1}{1-1/2} = \boxed{2}$.

B. Using the Maclaurin series for e^x, this is $\boxed{e^3}$.

C. Since $\sum[x_i] \cdot \sum[y_j] = \sum\sum[x_i y_j]$, this can be split up into two infinite geometric series. $\frac{1}{1-1/5} \cdot \frac{1}{1-1/7} = \frac{5}{4} \cdot \frac{7}{6} = \boxed{\frac{35}{24}}$.

D. Similar to in part **C**, the sum can be split up into two e^x sums. $e^{11} \cdot e^{13} = \boxed{e^{24}}$.

1) **A.** Deriving using the Product Rule, $f'(x) = 7(x-1)^6(x-4)^2 + 2(x-1)^7(x-4) = (x-1)^6(x-4)(9x-30) = 3(x-1)^6(x-4)(3x-10)$. We are interested where this changes sign, so eliminate $3(x-1)^6$. $(x-4)(3x-10)$ changes from positive to negative at $x = \boxed{\frac{10}{3}}$.

B. Similar to in part **A**, we are interested in where $(x-4)(3x-10)$ changes from negative to positive. This occurs at $x = \boxed{4}$.

C. Deriving using the Product Rule (and now also the Chain Rule) again, $f''(x) = 18(x-1)^5(x-4)(3x-10) + 3(x-1)^6(3x-10) + 9(x-1)^6(x-4)$. Plugging in $x = 2$ gives $18 \cdot 1 \cdot (-2) \cdot (-4) + 3 \cdot 1 \cdot (-4) + 9 \cdot 1 \cdot (-2) = 144 - 12 - 18 = \boxed{114}$.

D. $f(x)$ is a monic degree-9 polynomial, so $\frac{f(2x)}{f(x)}$ is a rational function where both the numerator and denominator have degree 9, making us only care about the leading coefficient. $\frac{(2x)^9}{x^9} = \boxed{512}$.

2) **A.** The slopes of the tangent lines must be $\pm\sqrt{3}$. $f'(x) = -2x$, and this equals $\pm\sqrt{3}$ when $x = \pm\frac{\sqrt{3}}{2}$, so $y = \frac{13}{4}$. The tangent line is thus $y = -x\sqrt{3} + \frac{19}{4}$. The height of the equilateral triangle is $\frac{19}{4}$, so its side length is $\frac{19}{2\sqrt{3}}$ and its area is $\boxed{\frac{361\sqrt{3}}{48}}$.

B. $\int_{-2}^{2}(4-x^2)\,dx = 2\int_0^2(4-x^2)\,dx = 8x - \frac{2x^3}{3}\Big]_0^2 = \boxed{\frac{32}{3}}$.

C. The distance from the x-axis to the graph is the diameter of the semicircle d, and the area of the semicircle cross-section is $\frac{\pi}{2} \cdot \left(\frac{d}{2}\right)^2 = \frac{\pi d^2}{8}$. Integrating, $\int_{-2}^{2}\frac{\pi}{8}(4-x^2)^2\,dx = \frac{\pi}{4}\int_0^2(x^4 - 8x^2 + 16)\,dx = \frac{\pi}{4}\left(\frac{x^5}{5} - \frac{8x^3}{3} + 16x\right)\Big]_0^2 = \boxed{\frac{64\pi}{15}}$.

D. The region is symmetric about the y-axis, so its centroid has an x-coordinate of 0. The distance from the centroid to the axis of rotation is 6. By the Theorem of Pappus, $2\pi \cdot \frac{32}{3} \cdot 6 = \boxed{128\pi}$. The Shell Method can also be used.

3) **A.** $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$, so $\sin(x^2) = x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \cdots$. When taking the sixth derivative and evaluating at $x = 0$, only the x^6 term will be relevant. $-\frac{6!}{3!} = -5! = \boxed{-120}$.

B. Note that the first two derivatives of the well-known series $\frac{1}{1-x}$ are $\frac{1}{(1-x)^2}$ and $\frac{2}{(1-x)^3}$. Looking at series representations, $\frac{1}{1-x} = \sum x^n$, $\frac{1}{(1-x)^2} = \sum nx^{n-1}$, and $\frac{2}{(1-x)^3} = \sum n(n-1)x^{n-2} = 2 + 6x + 12x^2 + 20x^3 + \cdots$. The desired quantity is half this. When taking the 20^{th} derivative and evaluating at $x = 0$, only the x^{20} term will be relevant. This term is $\frac{1}{2} \cdot 22 \cdot 21x^{20}$, and its 20^{th} derivative evaluated at $x = 0$ is $\frac{22!}{2} = 21! \cdot 11$. $11 + 21 = \boxed{32}$.

C. $e^x = 1 - x + \frac{x^2}{2} - \cdots$, so $e^{-x^2} = 1 + x^2 + \frac{x^4}{2} + \cdots$. When taking the eighth derivative and evaluating at $x = 0$, only the x^8 term will be relevant. $\frac{8!}{4!} = 8 \cdot 7 \cdot 6 \cdot 5 = \boxed{1680}$.

D. Noticing that the denominator factorizes, consider partial fraction decomposition. Let $\frac{6x-16}{x^2-6x+8} = \frac{P}{x-2} + \frac{Q}{x-4}$. Multiplying by the quadratic, $6x-16 = P(x-4)+Q(x-2)$. When $x = 4$, $8 = 2Q$ and $Q = 4$. When $x = 2$, $-8 = -2P$ and $P = 4$. Thus, we are deriving $\frac{2}{x-2} + \frac{4}{x-4}$. Notice that these can be written as the sums of infinite geometric series. $\frac{2}{x-2} + \frac{4}{x-4} = -\frac{1}{1-x/2} - \frac{1}{1-x/4} = -\left(1 + \frac{x}{2} + \frac{x^2}{4} + \frac{x^3}{8} + \cdots\right) - \left(1 + \frac{x}{4} + \frac{x^2}{16} + \frac{x^3}{64} + \cdots\right)$. When taking the 4^{th} derivative and evaluating at $x = 0$, only the x^4 term will be relevant. This term is $-\frac{x^4}{16} - \frac{x^4}{256}$, and its fourth derivative evaluated at $x = 0$ is $-\frac{24}{16} - \frac{24}{256} = -\frac{3}{2} - \frac{3}{32} = \boxed{-\frac{51}{32}}$.

4) **A.** Let $u = x^2 + 16$. Then $du = 2x\ dx$. $\frac{1}{2}\int_{16}^{32} \frac{du}{u} = \boxed{\frac{\ln 2}{2}}$.

B. The integral equals $\int_0^4 \left(1 - \frac{16}{x^2+16}\right)\ dx = 4 - 16\int_0^4 \frac{dx}{x^2+16}$. Let $x = 4u$. Then $dx = 4\ du$. $\int_0^1 \frac{4}{16u^2+16}\ du = \frac{1}{4}\int_0^1 \frac{du}{u^2+1} = \frac{\arctan u}{4}\Big]_0^1 = \frac{\pi}{16}$. Plugging this into the quantity from before gives the original integral as having value $\boxed{4 - \pi}$.

C. The integral equals $\int_0^4 \left(1 - \frac{16}{x+16}\right)\ dx = 4 - 16\int_0^4 \frac{dx}{x+16}$. Evaluating, $\int_0^4 \frac{dx}{x+16} = \ln|x+16|\big]_0^4 = \ln 20 - \ln 16 = \ln 5 - 2\ln 2$. Plugging back in, the original integral has value $4 + 32\ln 2 - 16\ln 5$. $4 + 32 + 2 - 16 + 5 = \boxed{27}$.

D. The integral equals $\int_0^4 \left(x - 16 + \frac{256}{x+16}\right)\ dx$. Split up the integral to $\int_0^4 (x-16)\ dx + 256\int_0^4 \frac{dx}{x+16}$. The first integral evaluates to $\frac{x^2}{2} - 16x\Big]_0^4 = -56$. The second integral can be solved as in part **C** and has value $256\ln 5 - 512\ln 2$. The original integral has value $-56 - 512\ln 2 + 256\ln 5$. $-56 - 512 + 2 + 256 + 5 = \boxed{-305}$.

5) **A.** The horizontal component of Croix's movement is 50. The vertical component is $\sqrt{60^2 - 50^2} = 10\sqrt{11}$. The time it takes for the vertical distance between Croix and Ursula to decrease to zero is $\frac{330}{10\sqrt{11}} = \frac{33}{\sqrt{11}} = \boxed{3\sqrt{11}}$.

B. Ursula's position is $(50t, 0)$, and Croix's position is $(0, 330 - 60t)$. The distance between them is $D = (50t)^2 + (330 - 60t)^2$. Deriving, $2DD' = 100(50t) - 120(330 - 60t) = 0$. Solving $50t - 12(33 - 6t) = 0$ gives $122t = 396$ and $t = \boxed{\frac{198}{61}}$.

C. $\vec{v_0}$ is pointing in the direction of the positive y-axis, so let $\vec{v_0} = \langle 0, 1\rangle$. Croix moves to $(0, -210)$ while Ursula goes to $(100, 0)$. $\vec{v_2}$ is in the direction of the segment connecting these two points, so let $\vec{v_2} = \langle 100, 210\rangle$. $\vec{v_0} \cdot \vec{v_2} = 210 = 1 \cdot \sqrt{100^2 + 210^2} = 10\sqrt{541}\cos\theta$. This solves to $\cos\theta = \frac{21}{\sqrt{541}}$, so $\cos^2\theta = \boxed{\frac{441}{541}}$.

D. Croix will have to travel 330 feet north at 60 feet per second, which will take $\frac{330}{60} = \frac{11}{2}$ seconds. In that time Ursula will travel $\frac{11}{2} \cdot 50 = 275$ feet, so Croix needs to be this far east of Ursula before making her turn north. Croix gains 10 feet of horizontal distance on Ursula every second, so she must travel east for $\frac{55}{2}$ seconds. The total distance Croix travels is $\frac{55}{2} \cdot 60 + 330 = 1650 + 330 = \boxed{1980}$ feet.

6) **A.** Rewrite the equation as $x^3+y^3-2xy=0$. Deriving implicitly, $3x^2+3y^2\frac{dy}{dx}-2y-2x\frac{dy}{dx}=0$, so $\frac{dy}{dx}=\frac{2y-3x^2}{3y^2-2x}$. When $x=y=1$, this is $\boxed{-1}$.

B. $2y-3x^2 3y^2-2x=1$. Multiplying, $2y-3x^2=3y^2-2x$, so $3x^2+3y^2-2x-2y=0$. Completing the square, $\left(x-\frac{1}{3}\right)^2+\left(y-\frac{1}{3}\right)^2=\frac{2}{9}$. The area of this circle is $\boxed{\frac{2\pi}{9}}$.

C. The maximum value of y occurs when $\frac{dy}{dx}=0$, so $2y=3x^2$. Plugging this in, $x^3+y^3=3x^3$, so $y^3=2x^3$ and $x=\frac{y}{\sqrt[3]{2}}$. Solving $2y=\frac{3y^2}{\sqrt[3]{4}}$ gives $y=\frac{2\sqrt[3]{4}}{3}$. $2+4+3=\boxed{9}$.

D. Converting to polar, $r^3\cos^3\theta+r^3\sin^3\theta=2r^2\cos\theta\sin\theta$ and $r=\frac{2\sin\theta\cos\theta}{\sin^3\theta+\cos^3\theta}$. Integrating, $\frac{1}{2}\int_0^{\frac{\pi}{2}}\left(\frac{2\sin\theta\cos\theta}{\sin^3\theta+\cos^3\theta}\right)^2\,d\theta=2\int_0^{\frac{\pi}{2}}\frac{\sin^2\theta\cos^2\theta}{(\sin^3\theta+\cos^3\theta)^2}\,d\theta=2\int_0^{\frac{\pi}{2}}\frac{\tan^2\theta\sec^2\theta}{(\tan^3\theta+1)^2}\,d\theta=2\int_0^{\infty}\frac{u^2}{(u^3+1)^2}\,du=\frac{2}{3}\int_1^{\infty}\frac{dv}{v^2}=-\frac{2}{3v}\Big]_1^{\infty}=\boxed{\frac{2}{3}}$.

7) **A.** $\begin{bmatrix}2&0\\0&5\end{bmatrix}^n=\begin{bmatrix}2^n&0\\0&5^n\end{bmatrix}$, so $e^{X_a}=\begin{bmatrix}e^2&0\\0&e^5\end{bmatrix}$. The product of the nonzero entries of this is $\boxed{e^7}$.

B. Note that X_b is nilpotent with $\begin{bmatrix}2&1\\-4&-2\end{bmatrix}^2=\begin{bmatrix}0&0\\0&0\end{bmatrix}$. Thus, $e^{X_b}=\begin{bmatrix}1&0\\0&1\end{bmatrix}+\begin{bmatrix}2&1\\-4&-2\end{bmatrix}=\begin{bmatrix}3&1\\-4&-1\end{bmatrix}$. The product of the entries of this is $\boxed{12}$.

C. It can be shown inductively that $\begin{bmatrix}1&2\\4&8\end{bmatrix}^n=9^{n-1}\begin{bmatrix}1&2\\4&8\end{bmatrix}$ for $n\geq 1$. While $\sum_{k=0}^{\infty}9^{k-1}\begin{bmatrix}1&2\\4&8\end{bmatrix}^k=\frac{1}{9}\begin{bmatrix}e^9&2e^9\\4e^9&8e^9\end{bmatrix}$, this uses $\begin{bmatrix}\frac{1}{9}&\frac{2}{9}\\\frac{4}{9}&\frac{8}{9}\end{bmatrix}$ in the $k=0$ term instead of the identity matrix. Making the adjustments, $e^{X_c}=\begin{bmatrix}\frac{e^9+8}{9}&\frac{2e^9-2}{9}\\\frac{4e^9-4}{9}&\frac{8e^9+1}{9}\end{bmatrix}$. The sum of the entries of this is $\boxed{\frac{5e^9+1}{3}}$.

D. Noting the given diagonalization of X_d, recall that if $M=UDU^{-1}$, then $M^n=UD^nU^{-1}$. This will prove useful in computing X_d^k. The sum of the eigenvalues of X_d is 11 (its trace), and the product of the eigenvalues is 18 (its determinant), so $\lambda_1=2$ and $\lambda_2=9$. Thus, $e^{X_d}=\sum_{k=0}^{\infty}\frac{1}{k!}\cdot\left(-\frac{1}{7}\right)\begin{bmatrix}-1&3\\1&4\end{bmatrix}\begin{bmatrix}2^k&0\\0&9^k\end{bmatrix}\begin{bmatrix}4&-3\\-1&-1\end{bmatrix}$. Moving the sum in, this is $-\frac{1}{7}\begin{bmatrix}-1&3\\1&4\end{bmatrix}\begin{bmatrix}e^2&0\\0&e^9\end{bmatrix}\begin{bmatrix}4&-3\\-1&-1\end{bmatrix}=-\frac{1}{7}\begin{bmatrix}-e^2&3e^9\\e^2&4e^9\end{bmatrix}\begin{bmatrix}4&-3\\-1&-1\end{bmatrix}=-\frac{1}{7}\begin{bmatrix}-3e^9-4e^2&3e^2-3e^9\\4e^2-4e^9&-4e^9-3e^2\end{bmatrix}=\begin{bmatrix}\frac{3e^9+4e^2}{7}&\frac{3e^9-3e^2}{7}\\\frac{4e^9-4e^2}{7}&\frac{4e^9+3e^2}{7}\end{bmatrix}$. The sum of the entries of this is $\boxed{2e^9}$.

8) **A.** Using $u=x^2+9$, $y(4)=19+\int_0^4 6x\sqrt{x^2+9}\,dx=19+3\int_9^{25}\sqrt{u}\,du=19+\left[2u^{3/2}\right]_9^{25}=19+250-54=\boxed{215}$.

B. Separating, $\frac{y+2}{y^2+4y+1}\,dy = \frac{x^2+4x+1}{x+2}\,dx$. The left side can be integrated with $u = y^2+4y+1$; the right side simplifies to $x+2-\frac{3}{x+2}$. Integrating, $\frac{1}{2}\ln|y^2+4y+1| = \frac{x^2}{2}+2x-3\ln|x+2|+C$, so $y^2+4y+1 = K\frac{e^{x^2+4x}}{(x+2)^6}$ (taking note of the initial condition to resolve the absolute values). When $x=0$ and $y=1$, $6=\frac{K}{64}$, so $K=384$. When $x=-1$, $y^2+4y+1=\frac{384}{e^3}$, so using the quadratic formula, $y=-2+\sqrt{3+\frac{384}{e^3}}$. $-2+3+384-3=\boxed{382}$.

C. $y^3+3y^2y'=e^x$. Since $3y^2=(y^3)'$, multiply both sides by e^x so $e^xy^3+3e^xy^2y'=e^{2x}$. Since e^x is its own derivative, the left side can be recognized as the Product Rule derivative of e^xy^3, so $e^xy^3=\frac{e^{2x}}{2}+C$ and $y^3=\frac{e^x}{2}+\frac{C}{e^x}$. Plugging in the point $(0,0)$, $C=-\frac{1}{2}$, so $y^3=\frac{e^x}{2}-\frac{1}{2e^x}$. When $x=2\ln 2$, $y^3=\frac{4}{2}-\frac{1}{8}=\frac{15}{8}$, so $y=\boxed{\frac{\sqrt[3]{15}}{2}}$.

D. Let $y=x^r$. Then $x^2\cdot r(r-1)x^{r-2}+6x\cdot rx^{r-1}+6x^r=0$. The left side simplifies to $x^r(r^2+5r+6)=0$, so $r=-2$ or $r=-3$ and the general solution is $y=\frac{C_1}{x^2}+\frac{C_2}{x^3}$. Plugging in $(1,4)$ gives $C_1+C_2=4$, and plugging in $\left(2,-\frac{1}{2}\right)$ gives $\frac{C_1}{4}+\frac{C_2}{8}=-\frac{1}{2}$. $2C_1+C_2=-4$, so $C_1=-8$ and $C_2=12$ and $y=\frac{12}{x^3}-\frac{8}{x^2}$. When $x=4$, this is $\frac{12}{64}-\frac{8}{16}=\frac{3}{16}-\frac{1}{2}=\boxed{-\frac{5}{16}}$.

9) **A.** The diagonal of the cube is 2, so its side length is $\frac{2}{\sqrt{3}}$ and its volume is $\boxed{\frac{8\sqrt{3}}{9}}$.

B. Since the spheres are identical, the maximum radius of each sphere would be half that of the original sphere, and the volume would be one eighth of the original volume, or $\frac{4\pi}{3}\cdot\frac{1}{8}=\boxed{\frac{\pi}{6}}$.

C. If the height of the cylinder is $2x$, then the radius is $\sqrt{1-x^2}$ and the volume is $2\pi x(1-x^2)$. The derivative of this equals 0 when $-3x^2+1=0$, or $x=\frac{1}{\sqrt{3}}$. The volume of the sphere is $\boxed{\frac{4\pi\sqrt{3}}{9}}$.

D. The base of the cone would have radius $\sqrt{1-x^2}$ and thus area $\pi(1-x^2)$. The height of the pyramid is $1+x$, so the volume is $\frac{\pi}{3}(-x^3-x^2+x+1)$. The derivative of this equals 0 when $-3x^2-2x+1=0$, or $x=\frac{1}{3}$ (ignore the $x=-1$ solution). The volume of the pyramid is $\boxed{\frac{32\pi}{81}}$.

10) **A.** $f'(x)=\frac{1}{x}$. This equals 20 when $x=\frac{1}{20}$, so $a=26+\ln 20$ and $e^a=\boxed{20e^{26}}$.

B. $g'(x)=2x+4b$. This equals 20 when $x=10-2b$. $g(10-2b)=(4b^2-40b+100)+(40b-8b^2)+61=161-4b^2$. Plugging into the line, $161-4b^2=20(10-2b)+25$, so $-4b^2+40b-64=0$. Solving $b^2-10b+16=0$ gives a larger solution of $b=\boxed{8}$.

C. $h'(x)=2cx+4$. This equals 20 when $x=\frac{8}{c}$. $h\left(\frac{8}{c}\right)=\frac{64}{c}+\frac{32}{c}+61=\frac{96}{c}+61$. Plugging into the line, $\frac{96}{c}+61=\frac{160}{c}+25$, so $\frac{64}{c}=36$ and $c=\boxed{\frac{16}{9}}$.

D. $j(x)$ will be tangent to $y=20x+25$ when $j(x)-(20x+25)$ has a horizontal tangent on the x-axis. $\frac{d}{x}=20x+25$, so $20x^2+25x-d=0$. This is a quadratic, so its vertex must be on the x-axis. $x^2+\frac{5x}{4}-\frac{d}{20}=0$, so $\left(x+\frac{5}{8}\right)^2=\frac{d}{20}+\frac{25}{64}=0$. $d=-\frac{500}{64}=\boxed{-\frac{125}{16}}$.

11) **A.** $\vec{r_1}''(t) = \langle 2t, 2\rangle$, and $\vec{v_2}'(t) = \langle t^2+4, 2t\rangle$. When $t = 2$, these accelerations are $\langle 4, 2\rangle$ and $\langle 8, 4\rangle$ and have magnitudes of $2\sqrt{5}$ and $4\sqrt{5}$, respectively. The product of their magnitudes is $\boxed{40}$.

B. $\vec{r_1}'(t) = \langle t^2-1, 2t\rangle$, and $\vec{v_2}(t)$ is given. When $t = 3$, these velocities are $\langle 8, 6\rangle$ and $\langle 21, 13\rangle$, so the speeds are $\sqrt{8^2+6^2} = 10$ and $\sqrt{21^2+13^2} = \sqrt{610}$. The product of their speeds is $\boxed{10\sqrt{610}}$.

C. $\vec{r_1}'(t) = \langle t^2-1, 2t\rangle$, so the speed of the particle is $\sqrt{(t^2-1)^2+(2t)^2} = \sqrt{t^4-2t^2+1+4t^2} = \sqrt{t^4+2t^2+1} = t^2+1$. Arc length is the integral of this. $\int_4^5 (t^2+1)\,dt = \frac{t^3}{3}+t\Big]_4^5 = \frac{140}{3} - \frac{76}{3} = \boxed{\frac{64}{3}}$.

D. $\vec{v_2}(t)$ is perpendicular to a line with slope -2, which can be represented by the vector $\langle 1, -2\rangle$ (or for ease of calculation, $\langle 3, -6\rangle$). The dot product of these must equal 0. $t^3+12t-6t^2-24 = 0$. Recognizing the $\{1, -6, 12\}$ pattern of coefficients, $t^3-6t^2+12t-8-16 = 0$, so $(t-2)^3 = 16$ and $t = \boxed{2+2\sqrt[3]{2}}$

12) **A.** $V = \frac{4\pi r^3}{3}$, so $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$. Plugging in values, $-C\pi = 64\pi \cdot A$, so $A = -\frac{C}{64}$.

B. This is a separable differential equation. $\frac{dy}{y^2+y} = \frac{dx}{x^2-x}$. Recognizing both of these as needing partial fractions, $\int \left(\frac{1}{y} - \frac{1}{y+1}\right)\,dy = \int \left(\frac{1}{x-1} - \frac{1}{x}\right)\,dx$. This solves to $\ln\left|\frac{y}{y+1}\right| = \ln\left|\frac{x-1}{x}\right| + C$, so $\frac{y}{y+1} = \frac{K(x-1)}{x}$. Solving $xy = K(xy+x-y-1)$ for y gives $y = \frac{K(x-1)}{x-K(x-1)}$. This is a line when the denominator has no x terms, so $K = 1$ and $y = x-1$. Thus, $B = A-1$.

C. Senketsu gives the position directly, so $x_S = -A(54+8) = -62A$. Junketsu gives the velocity, but this can be integrated to get position, noting that the constant will be zero since the pulling starts from the origin. $x_J = -B\left(\frac{t^3}{720} + \frac{t^2}{90} + \frac{t}{20}\right)$. Plugging in $x = 6$ gives $x_J = -B\left(\frac{216}{720} + \frac{36}{90} + \frac{6}{20}\right) = -B$. The net position of the shirt is the sum of these two actions, so $C = -62A - B$.

D. This is solved with index-shifting. $\mathrm{D} = \sum_{n=0}^{\infty} \frac{n^2|B|^n}{n!} = \sum_{n=1}^{\infty} \frac{n|B|^n}{(n-1)!} = \sum_{n=0}^{\infty} \frac{(n+1)|B|^{n+1}}{n!} = |B|\left(\sum_{n=0}^{\infty} \frac{n|B|^n}{n!} + \sum_{n=0}^{\infty} \frac{|B|^n}{n!}\right) = |B|\left(e^{|B|} + \sum_{n=1}^{\infty} \frac{|B|^n}{(n-1)!}\right) = |B|(e^{|B|} + |B|e^{|B|}) = e^{|B|}(B^2+|B|)$.

The question is self-referential, and a system of three equations must be solved to determine the values of A, B, and C; B must be found to calculate D. Plugging $C = -64A$ and $B = A-1$ into the equation from part **C** gives $-64A = -62A-(A-1)$, or $A = \boxed{-1}$. It follows that $B = \boxed{-2}$ and $C = \boxed{64}$. That value of B can be plugged in to obtain $D = \boxed{6e^2}$.

13) **A.** PR is the base of the triangle and has length 2. The height of the triangle is $\sin t$. Thus, the area of the triangle is $\sin t$. The rate of change of the area of the triangle is therefore $\cos t$, and at $t = \frac{2\pi}{3}$ that is equal to $\boxed{-\frac{1}{2}}$.

B. The area of the hexagon is $\frac{3(t'^2+6t')^2\sqrt{3}}{2} = \frac{3\sqrt{3}}{2} \cdot (t'^4 + 12t'^3 + 36t'^2)$. Deriving, the rate of change of area is $\frac{3\sqrt{3}}{2} \cdot (4t'^3 + 36t'^2 + 72t') \cdot \dfrac{dt'}{dt} = 6t'\sqrt{3}(t'^2 + 9t' + 18)\frac{dt'}{dt}$. When $t' = 4$, this is $24\sqrt{3} \cdot 70 \cdot 3 = \boxed{5040\sqrt{3}}$

C. Rewrite the equation of the ellipse as $\frac{x^2}{1^2} + \frac{y^2}{(1/\sqrt{c})^2} = 1$. The area of the ellipse is $\frac{\pi}{\sqrt{c}}$. Deriving, the rate of change of area is $-\frac{\pi}{2c^{3/2}}\frac{dc}{dt}$. When $c = 9$, this is $-\frac{\pi}{54} \cdot 3 = \boxed{-\frac{\pi}{18}}$.

D. Let the triangle have perimeter x, so the square has perimeter $16 - x$. The side lengths are $\frac{x}{3}$ and $4 - \frac{x}{4}$, respectively. The total area is $\frac{3\left(\frac{x}{3}\right)^2\sqrt{3}}{4} + \left(4 - \frac{x}{4}\right)^2 = \frac{x^2\sqrt{3}}{12} + \left(\frac{x^2}{16} - 2x + 16\right)$. Deriving, $\frac{x\sqrt{3}}{6} + \frac{x}{8} - 2 = 0$, so $x\left(\frac{\sqrt{3}}{6} + \frac{1}{8}\right) = 2$. $x = \frac{48}{4\sqrt{3}+3} = \frac{64\sqrt{3}-48}{13}$. $64 + 3 + 48 + 13 = \boxed{128}$.