*The answer choice **(E)** NOTA denotes that "none of these answers" are correct. All answers must be exact unless otherwise specified. Good luck, and have fun!*

1. Compute the number of permutations of the letters of FAMAT that do not contain adjacent vowels.

   (A) 30 **(B)** 36 **(C)** 48 **(D)** 60 **(E)** NOTA

2. Saathvik is 20 years older than Sharvaa. Sharvaa notices that his age (in years) is a divisor of Saathvik's age (in years). Saathvik remarks that this statement was also true six years ago. After how many (non-zero) years will this statement be true again?

   (A) 6 **(B)** 8 **(C)** 10 **(D)** 12 **(E)** NOTA

3. Simplify $\tan(\arctan(1) + \arctan(2) + \arctan(3) + \arctan(4))$.

   (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** NOTA

4. Complex numbers x and y satisfy $x^{20} = 1$ and $y^{25} = 1$. How many distinct possible values are there for xy?

   (A) 50 **(B)** 75 **(C)** 100 **(D)** 200 **(E)** NOTA

5. How many solutions does $4\sin^3(x) - 2\sin^2(x) - \sin(x) + \cos(2x) = 0$ have on the closed interval $[0, 2\pi]$?

   (A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** NOTA

6. Larissa randomly draws two chords of length 4 and $4\sqrt{2}$ in a circle with radius 4. What is the probability these two chords intersect?

   (A) $1/6$ **(B)** $1/4$ **(C)** $1/3$ **(D)** $5/12$ **(E)** NOTA

7. In a unit square ABCD, M is the midpoint of side BC, and N is the midpoint of side CD. Find the area of the intersection of the triangles AMD and ANB.

   (A) $1/5$ **(B)** $4/15$ **(C)** $2/5$ **(D)** $8/15$ **(E)** NOTA

8. Evaluate $\lim_{n\to\infty} \left(\sum_{i=1}^{n} \frac{1}{\sqrt{n(n+i)}} \right)$

   (A) $2\sqrt{2} - 2$ **(B)** $\sqrt{2} - 1$ **(C)** $2\sqrt{2}$ **(D)** $2 - \sqrt{2}$ **(E)** NOTA

9. Given that x and y are distinct positive integers, which of the following is not a possible value of

$$\frac{x+y}{\frac{x}{y}+\frac{y}{x}-2}\cdot\left(3y-3x+\frac{x^3-y^3}{xy}\right)$$

(A) 14 **(B)** 15 **(C)** 21 **(D)** 35 **(E)** NOTA

10. Consider the polynomial $x^3-9x^2+tx+165$, where t is a real number. At what value of t does this polynomial have three distinct roots in arithmetic progression?

(A) 0 **(B)** -27 **(C)** -37 **(D)** 37 **(E)** NOTA

11. The graph of $f(x)=\dfrac{x^2-8}{x+3}$ contains a local minimum at (p,q) and a local maximum at (r,s). Compute: $\dfrac{3s-2q}{r-p}$.

(A) 1 **(B)** 2 **(C)** 4 **(D)** 8 **(E)** NOTA

12. There are eight crewmates and two impostors on a ship. Every second, one person is randomly thrown off the ship. What is the probability that before the ship is emptied, there are always more crewmates than impostors on board?

(A) $\frac{4}{9}$ **(B)** $\frac{7}{15}$ **(C)** $\frac{3}{5}$ **(D)** $\frac{7}{10}$ **(E)** NOTA

13. Consider square $ABCD$, with point P in its interior such that $AP=1$, $BP=\sqrt{5}$, and $CP=3$. Compute the area of the square $ABCD$.

(A) 6 **(B)** 8 **(C)** 10 **(D)** 12 **(E)** NOTA

14. Consider the hyperbola $\dfrac{x^2}{9}-\dfrac{y^2}{36}=1$ with center A. For any two points B,C on the curve with $\overline{AB}\perp\overline{AC}$, let $r=\overline{AB}$ and $s=\overline{AC}$. Evaluate $\dfrac{1}{r^2}+\dfrac{1}{s^2}$.

(A) $\dfrac{1}{18}$ **(B)** $\dfrac{1}{15}$ **(C)** $\dfrac{1}{12}$ **(D)** $\dfrac{1}{6}$ **(E)** NOTA

15. The number 1234312 has three distinct prime factors. Compute their sum.

(A) 62 **(B)** 420 **(C)** 836 **(D)** 1114 **(E)** NOTA

16. Tim draws an equilateral triangle of area 1. He then randomly place a point within the triangle. What is the expected value of the area of the smallest polygon needed to enclose the midpoints of the triangle's sides and the point?

(A) $\frac{1}{4}$ **(B)** $\frac{5}{16}$ **(C)** $\frac{1}{3}$ **(D)** $\frac{7}{20}$ **(E)** NOTA

17. Alice and Bob are at opposite ends of a bridge. They both run to the opposite end and back at constant speeds. They know that the first time they meet they are 1000 meters away from Alice's starting point and that the second time they meet they are 400 meters away from Bob's starting point. Given that the length of the bridge is an integer, what is the total length of the bridge (in meters)?

(A) 2400 **(B)** 2600 **(C)** 2000 **(D)** 3000 **(E)** NOTA

18. Compute the derivative at zero of

$$3\sin(1\sin(4\sin(1\sin(5x)))).$$

(A) 0 **(B)** 14 **(C)** 30 **(D)** 60 **(E)** NOTA

For Questions 19 - 21 use the following information. Let S_1 be a cube of side length 2. Define S_n as the shape formed by connecting the centers of adjacent faces of S_{n-1} for $n \geq 2$.

19. Karthik recently discovered that S_2 is in fact an octahedron, a shape he doesn't know very much about. Help Karthik out by computing the volume of S_2.

(A) $\frac{2}{3}$ **(B)** 1 **(C)** $\frac{4}{3}$ **(D)** 2 **(E)** NOTA

20. Let R be the ratio of the surface area of S_5 to the surface area of S_3. If R can be written in the form $\frac{m}{n}$, where m and n are relatively prime positive integers, find $m+n$.

(A) 8 **(B)** 10 **(C)** 12 **(D)** 20 **(E)** NOTA

21. For his final project in Artistic Geometry, Karthik decides to paint S_1 in a unique way. He fills S_1 with white paint, removes the paint from the region contained by S_2, refills the region contained by S_3, and then repeats this pattern of filling one region and removing paint from another. Let V be the volume of paint contained within his cube as Karthik continues this process to infinity. If V can be written in the form $\frac{m}{n}$, where m and n are relatively prime positive integers, find $m+n$.

(A) 77 **(B)** 101 **(C)** 103 **(D)** 117 **(E)** NOTA

22. Which of these is closest to the average of the digits of the number 99999^3 when written in decimal notation?

(A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** NOTA

23. Let x, y be independent and uniformly random from $[0, 1]$. Compute the expected value of:

$$\int_0^x t^y \, dt.$$

(A) $\ln\left(\frac{5}{4}\right)$ **(B)** $\ln\left(\frac{4}{3}\right)$ **(C)** $\frac{1}{3}$ **(D)** $\ln\left(\frac{3}{2}\right)$ **(E)** NOTA

24. How many times does the graph of $y = \cos(\frac{101\pi x}{2})$ intersect the unit circle?

(A) 200 **(B)** 201 **(C)** 202 **(D)** 203 **(E)** NOTA

For Questions 25 - 26 we will be studying the Taxman game. Assume you start with a pot consisting of all the positive integers from 1 to N inclusive. Every time you remove a number, the taxman removes all of its other divisors which remain in the pot. The number you removed is added to your score, and the numbers the taxman removes are added to his score. This process is repeated until you have no more possible moves. The taxman then receives all remaining numbers. What makes the game interesting is that the taxman has to get something on every turn, so you can't pick a number that has no other divisors left in the pot. For example, with $N = 6$, a starting move of 4 would give the taxman 1 and 2. This also means 5 can never be picked because 1 has already been removed.

25. In the case of $N = 93$, what number is the best to remove on your first turn?

(A) 1 **(B)** 81 **(C)** 89 **(D)** 93 **(E)** NOTA

26. Compute the optimal score you can get for the case of $N = 10$.

(A) 33 **(B)** 34 **(C)** 40 **(D)** 41 **(E)** NOTA

27. Let $x(t)$ be a continuous function defined on nonnegative reals that solves the differential equation $dx = \lfloor x \rfloor \, dt$ for all noninteger t with initial condition $x(0) = 1$. Which of the following answer choices is closest to the smallest $t > 0$ at which $x(t)$ is equal to a billion?

(A) 20 **(B)** 200 **(C)** $2,000$ **(D)** $20,000$ **(E)** $200,000$

28. Albert wants to write down an ordering of the eight numbers $1, 2, \ldots, 8$ such that every number is larger than all the prime numbers that occurred before it in the ordering. Compute the number of ways that this could be done.

(A) 160 **(B)** 192 **(C)** 216 **(D)** 224 **(E)** NOTA

29. Let $f_t : \mathbb{R} \to \mathbb{R}$ for $t > 0$ be a function that rounds its input **down** to the nearest integer multiple of t. Compute

$$\int_0^{210} f_{10}(f_{21}(x))\,dx.$$

(A) 18000 **(B)** 18900 **(C)** 19000 **(D)** 21000 **(E)** NOTA

30. Evaluate $\int_0^1 \left(x - \sqrt{1-x^2}\right)^{2024} dx$

(A) $\frac{1}{2025}$ **(B)** $\frac{1}{2024}$ **(C)** $\frac{1}{2023}$ **(D)** $\frac{1}{401}$ **(E)** NOTA