**Important Instructions for this Test:**
As always, "NOTA" stands for "None Of These Answers" is correct. Good luck and have fun!

**1.** Evaluate: $\frac{d}{dx}\left[14\pi\sin\left(\sqrt{x}\right)\right]_{x=\pi^2}$

A: $-14$ B: $-7$ C: $7$ D: $14$ E: NOTA

**2.** Evaluate: $\lim\limits_{x\to\infty}\arctan\left(\frac{\sqrt{3}x^4}{15-3x^4}\right)$

A: $-\frac{\pi}{6}$ B: $\frac{\pi}{6}$ C: $-\frac{\pi}{3}$ D: $\frac{\pi}{3}$ E: NOTA

**3.** Evaluate: $\lim\limits_{h\to 0}\frac{\tan\left(\frac{\pi}{3}+5h\right)-\tan\left(\frac{\pi}{3}-3h\right)}{2h}$

A: $\frac{3}{4}$ B: $3$ C: $4$ D: $16$ E: NOTA

**4.** For the function $f(x)=1-x^2$, find $b$ such that $c=1$ is the value of $c$ guaranteed by the Mean Value Theorem for Derivatives on the interval $[0,b]$.

A: $-2$ B: $1$ C: $2$ D: $4$ E: NOTA

**5.** Let $f(x)=ax^2+bx+1-ab$ with $a>0$. If the sum of the solutions to $f(x)=0$ is 2025, find the minimum possible value of the product of the solutions to $f(x)=0$.

A: $0$ B: $45$ C: $90$ D: $2025$ E: NOTA

**6.** A continuous function $f(x)$ has the following properties: $\int_0^2 f(x)dx=20$, $\int_1^3 f(x)dx=25$, and $\int_0^3 f(x)dx=9$. Find $\int_1^2 f(x)dx$.

A: $54$ B: $36$ C: $14$ D: $4$ E: NOTA

**7.** The hyperbola $xy=25$ is tangent to the ellipse $\frac{x^2}{a^2}+y^2=1$. Find $a$ if $a>0$.

A: $50$ B: $25$ C: $5$ D: $2$ E: NOTA

**8.** If $f(x)=\cfrac{x}{1+\cfrac{x}{1+\cfrac{x}{1+\cfrac{x}{\cdots}}}}$ then find $f'(20)$.

A: $\frac{1}{10}$ B: $\frac{1}{11}$ C: $\frac{1}{8}$ D: $\frac{1}{9}$ E: NOTA

**9.** Let $f(x)=|3x^4-5x^3-15x^2+25x|$. Let $M$ be the number of relative maxima of $f(x)$, $N$ be the number of relative minima of $f(x)$, and $P$ be the number of points of inflection of $f(x)$. Find $M\cdot N\cdot P$.

A: $4$ B: $36$ C: $72$ D: $144$ E: NOTA

**10.** Find the number of zeroes at the end of $\frac{d^{2025}}{dx^{2025}}\left[e^{x^5}\right]_{x=0}$.

A: 405 B: 505 C: 2025 D: 2125 E: NOTA

**11.** Find the shortest distance between the curve $y = x^{\frac{3}{2}} - 1$ and the point $\left(\frac{1}{2}, -1\right)$.

A: $\frac{7}{108}$ B: $\frac{\sqrt{21}}{18}$ C: $\frac{13}{4}$ D: $\frac{\sqrt{13}}{2}$ E: NOTA

**12.** Evaluate: $\int_1^2 (12x^3 - 6x^2 + 8x - 3)dx$

A: 49 B: 48 C: 46 D: 44 E: NOTA

**13.** Find the area of the finite region bounded by the curves $f(x) = 20x^3$ and $g(x) = 25x^4$.

A: $\frac{16}{25}$ B: $\frac{64}{125}$ C: $\frac{256}{625}$ D: $\frac{1024}{3125}$ E: NOTA

**14.** The area between $f(x) = 4x^3 + 3x^2 + 2x + 1$ and the x-axis from $x = 1$ to $x = 2$ is approximated using Simpson's Rule with 20 intervals of equal width. Find the resulting value.

A: 27 B: 26 C: 25 D: 24 E: NOTA

**15.** Find the volume when the finite region bounded by the x-axis, the line $x = 1$, and the curve $y = x^{25}$ is revolved around the line $x = 1$.

A: $\frac{\pi}{351}$ B: $\frac{\pi}{325}$ C: $\frac{\pi}{300}$ D: $\frac{\pi}{276}$ E: NOTA

**16.** Evaluate: $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{\sqrt{4n^2 - k^2}}$

A: $\frac{\pi}{2}$ B: $\frac{\pi}{3}$ C: $\frac{\pi}{4}$ D: $\frac{\pi}{6}$ E: NOTA

**17.** A open-topped cylindrical container with radius 25 and height 100 units contains a spherical marble of radius $r < 25$. Water is poured into the cylinder until it reaches the top of the marble (the marble does not float). Find the value of $r$ that maximizes the volume of water poured into the cylinder.

A: $\frac{25}{4}$ B: $\frac{25\sqrt{2}}{4}$ C: $\frac{25}{2}$ D: $\frac{25\sqrt{2}}{2}$ E: NOTA

**18.** $\int_0^1 \frac{x^3}{\sqrt{x^4+1}}dx$ is a root of $y = 4x^2 + 4x + K$. Find $K$.

A: $-1$ B: 0 C: 1 D: 2 E: NOTA

**19.** Let $F(x)$ and $G(x)$ be continuous, differentiable functions such that there exists a continuous function $h(x)$ with $F'(x) = h(x)G(x)$ and $G'(x) = h(x)F(x)$. If $F(0) = 1$, $F(1) = \frac{25}{7}$, $G(0) = 0$, and $G(1) = \frac{24}{7}$, find $\int_0^1 h(x)dx$.

A: ln(3) B: ln(5) C: ln(6) D: ln(7) E: NOTA

**20.** If $f(x)$ is a continuous, differentiable function with $f(0)=20$, $f(1)=25$, and $f(x)>0$ for $0\le x\le 1$, what is $\int_0^1 \left(\frac{d}{d\left(\int_0^x f(t)dt\right)}[f(x)]\right)dx$?

A: $\ln(500)$ B: $5$ C: $\ln\left(\frac{4}{5}\right)$ D: $\ln\left(\frac{5}{4}\right)$ E: NOTA

**21.** Let $\mathcal{R}$ be the region in the $xy$-plane where the following integral converges:

$$\int_1^\infty \frac{t^{3x+y+1}+1}{\sqrt[2x]{t^{2x-y^2+2}+1}}\,dt$$

Find the area of $\mathcal{R}$.

A: $\frac{\pi}{\sqrt{3}}$ B: $\frac{\pi}{2}$ C: $\frac{\pi}{\sqrt{5}}$ D: $\frac{\pi}{\sqrt{6}}$ E: NOTA

**22.** Let continuous, twice-differentiable functions $y_1(x), y_2(x)$, and $y_3(x)$ satisfy the following initial value problems:

$$y_1' + e^x\ln(x)\,y_1 = 0;\quad y_1(1)=2$$
$$y_2' + \frac{e^x}{x}y_2 = 0;\quad y_2(1)=5$$
$$y_3' + \frac{e^x}{x^2}y_3 = 0;\quad y_3(1)=10$$

Evaluate:

$$\int_1^e \ln\left(\frac{y_1y_2^2}{y_3}\right)dx$$

A: $(e-1)(\ln(5)-e)+e^e$ B: $(e-1)(\ln(5)+e)-e^e$

C: $e\ln(5)-e^2+e^e$ D: $e\ln(5)+e^2-e^e$ E: NOTA

**23.** Consider the matrix:

$$\begin{bmatrix} x & x^3 \\ \frac{1}{x} & 2x \end{bmatrix}$$

If $x$ is positive and increasing at a rate of $\sqrt{11}$ units per minute, find the rate of change of the smaller angle between the two eigenvectors of this matrix when the eigenvectors are perpendicular.

A: $-1$ B: $-4$ C: $1$ D: $4$ E: NOTA

**24.** Which is larger, $\pi^{\sqrt{2}+\sqrt{3}}$ or $\left(\sqrt{2}+\sqrt{3}\right)^\pi$?

A: $\pi^{\sqrt{2}+\sqrt{3}}$ B: $\left(\sqrt{2}+\sqrt{3}\right)^\pi$
C: They are equal. D: Cannot be determined. E: NOTA

**25.** Let $f(x)$ and $g(x)$ both be continuous, differentiable functions. Let $g(x)$ be periodic with finite period $p$ and let $[a, b]$ be a closed finite interval on the real line. How many of the following statements are always true?

I. $\int_a^b \big(f(x) - g(x)\big)dx = \int_a^b f(x)dx - \int_a^b g(x)dx$

II. $\int_0^{np} g(x)dx = n\int_0^p g(x)dx$

III. $\lim_{n\to\infty} \int_a^b f(x)g(nx)dx = \frac{1}{p}\left(\int_a^b f(x)dx\right)\left(\int_0^p g(x)dx\right)$

A: 0 B: 1 C: 2 D: 3 E: NOTA

**26.** Evaluate:

$$\lim_{n\to\infty} \int_0^1 \frac{x^2}{(1 + |\sin(nx)|)^2}\,dx$$

A: $\frac{2}{9\pi}$ B: $\frac{2}{3\pi}$ C: $\frac{4}{9\pi}$ D: $\frac{4}{3\pi}$ E: NOTA

**27.** Consider the function:

$$f(x) = \sum_{k=0}^{2025} \left(\sin|x - k| + \cos\left|x - k + \frac{1}{2}\right|\right)$$

For how many real values of $x$ is $f(x)$ *NOT* differentiable?

A: 2025 B: 2026 C: 4050 D: 4052 E: NOTA

**28.** Evaluate:

$$\int_0^\infty e^{-\left(\frac{x^2}{4} + \frac{2025}{x^2}\right)}dx$$

*Hint:* $\int_{-\infty}^{\infty} e^{-u^2}du = \sqrt{\pi}$

A: $e^{-45}\sqrt{\pi}$ B: $2e^{-45}\sqrt{\pi}$
C: $e^{-90}\sqrt{\pi}$ D: $2e^{-90}\sqrt{\pi}$ E: NOTA

**29.** If $\sin(3 + \cos^2(x) + \sin^4(x)) = \frac{24}{25}$, find the value of $\cos(3 + \sin^2(x) + \cos^4(x))$.

A: $-\frac{7}{25}$ B: $\frac{7}{25}$ C: $-\frac{24}{25}$ D: $\frac{24}{25}$ E: NOTA

**30.** Evaluate: $\int_{-45}^{45} \sqrt{2025 - x^2}\,dx$

A: $\frac{2025\pi}{4}$ B: $\frac{2025\pi}{2}$ C: $2025\pi$ D: $4050\pi$ E: NOTA