

Instructions For This Test: All answers must be exact, all functions are real-valued, and all angles are in radians unless otherwise specified. Good luck, have fun, and as always: "NOTA" stands for "None of These Answers is correct." and "DNE" stands for "Does Not Exist."

1. Evaluate:

$$\int_{-2025}^{2025} 2025x \, dx$$

- (A) 4100625 (B) 0 (C) 4050 (D) 2025 (E) NOTA

2. Evaluate

$$\int_0^2 (x^3 + 3x^2 + 5x + 2) \, dx$$

- (A) 22 (B) 24 (C) 26 (D) 27 (E) NOTA

3. Evaluate

$$\int_{-1}^1 (20252025x^{2025} + 1) \sqrt{1-x^2} \, dx$$

- (A) π (B) $\frac{20252025}{2}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{2}$ (E) NOTA

4. Evaluate:

$$\int_0^{\frac{\pi}{2}} \ln(\tan(x)) \, dx$$

- (A) 0 (B) $\pi \ln(2)$ (C) $-\frac{\pi \ln(2)}{2}$ (D) $-\pi \ln(2)$ (E) NOTA

5. Evaluate:

$$\int_0^1 e^{x+e^x} \, dx$$

- (A) e^2 (B) $e - 1$ (C) $e^e - e$ (D) e^e (E) NOTA

6. Evaluate:

$$\int_0^{\sqrt{2}} \frac{x}{x^4 + 4} dx$$

(A) $\frac{\pi}{4}$

(B) $\frac{\pi}{2}$

(C) $\frac{\pi}{8}$

(D) $\frac{\pi}{16}$

(E) NOTA

7. Evaluate:

$$\int_0^{\pi} \sin(x) \sin(2x) dx$$

(A) $\frac{1}{6}$

(B) $\frac{2}{3}$

(C) $\frac{1}{3}$

(D) 0

(E) NOTA

8. Evaluate:

$$\int_0^{\frac{\pi}{2}} \frac{\sin(x)}{\cos(x) + 1} dx$$

(A) $2 \ln 2$

(B) $\frac{\ln 2}{2}$

(C) $\ln 2 + 1$

(D) $\ln 2$

(E) NOTA

9. Evaluate:

$$\int_0^{\infty} \frac{\arctan(x)}{x^2 + 1} dx$$

(A) $\frac{\pi^2}{8}$

(B) $\frac{\pi}{8}$

(C) $\frac{\pi^2}{4}$

(D) $\frac{\pi}{4}$

(E) NOTA

10. Evaluate:

$$\int_0^{\frac{\pi}{3}} \sqrt{\frac{1}{1 + \sin(x)} + \frac{1}{1 + \cos(x)} + \frac{1}{1 + \tan(x)} + \frac{1}{1 + \cot(x)} + \frac{1}{1 + \csc(x)} + \frac{1}{1 + \sec(x)}} dx$$

(A) 0

(B) π

(C) $\frac{\pi}{\sqrt{3}}$

(D) $\frac{\pi}{3}$

(E) NOTA

11. Evaluate:

$$\int_0^{\infty} \frac{1}{e^x + 1} dx$$

(A) 0

(B) $\ln 2$

(C) e

(D) Divergent

(E) NOTA

12. Evaluate:

$$\int_0^{\infty} \frac{x^2}{(x^2 + 1)^2} dx$$

(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{8}$

(D) $\frac{\pi}{16}$

(E) NOTA

13. Evaluate:

$$\int_0^{\pi} \frac{x \sin(x) + \sin(x)}{1 + \sin(x)} dx$$

(A) 1

(B) $\pi^2 - 2$

(C) $\frac{\pi}{2} - 1$

(D) $\frac{\pi^2}{2} - 2$

(E) NOTA

14. What is

$$a \mod b \equiv$$

if a and b are relatively prime integers, and

$$\int_1^2 \frac{2x^9 + x^6 + x^4}{(x^8 + x^5 + 3x^3)^2} dx = \frac{a}{b}?$$

(A) 2

(B) 23

(C) 50

(D) 51

(E) NOTA

15. Evaluate:

$$\int_0^3 \frac{x^{2025}}{-18 - 3x + 13x^2 + 7x^3 + x^4} dx$$

(A) 0

(B) $\frac{\pi}{2026}$

(C) $\frac{\pi}{2025}$

(D) $\frac{\pi}{2}$

(E) NOTA

Recall that the Gamma function $\Gamma(x)$ has the property $\Gamma(x) \cdot \Gamma(1-x) = \pi \csc(\pi x)$.

16. Evaluate:

$$\int_0^1 \ln(\Gamma(x)) dx$$

- (A) $\frac{\pi \ln 2}{2}$ (B) $\frac{2\pi \ln(2)}{5}$ (C) $\frac{\ln(2\pi)}{5}$ (D) $\frac{\ln(2\pi)}{2}$ (E) NOTA

17. Evaluate:

$$\int_0^1 \frac{1}{\Gamma(x)^2 + \Gamma(x)\Gamma(1-x)} dx = a\pi^n$$

- (A) $\frac{1}{\pi^2}$ (B) $\frac{2}{\pi^2}$ (C) $\frac{1}{\pi}$ (D) $\frac{2}{\pi}$ (E) NOTA

18. What is

$$a + b$$

if a and b are relatively prime integers and

$$\int_1^9 \left(\sqrt[3]{x - \sqrt{x^2 - 1}} + \sqrt[3]{x + \sqrt{x^2 - 1}} \right) dx = \frac{a}{b}?$$

- (A) 165 (B) 169 (C) 173 (D) 175 (E) NOTA

19. Evaluate:

$$\int_3^7 \frac{\ln(x+2)}{\ln(24+10x-x^2)} dx$$

- (A) $\sqrt{2}$ (B) $\sqrt{3+\sqrt{5}}$ (C) $\ln 7$ (D) 2 (E) NOTA

20. Evaluate

$$\int_0^{\ln(2)} \arctan(1+e^x) dx$$

- (A) $\frac{\pi}{2\sqrt{2}}$ (B) $\frac{3\pi \ln 2}{8}$ (C) $\sqrt{\frac{\pi}{2}} \ln 2$ (D) $\frac{\pi(\ln 2)^2}{4}$ (E) NOTA

21. Evaluate

$$\int_0^{\infty} \frac{1}{2^{2^{\lfloor x \rfloor}} + x - \lfloor x \rfloor} dx$$

(A) $\frac{\ln 2}{4}$

(B) $\frac{\ln 2}{2}$

(C) $\frac{3 \ln 2}{4}$

(D) $\ln 2$

(E) NOTA

22. Evaluate:

$$\int_0^1 \frac{\arctan(x)}{x\sqrt{1-x^2}} dx$$

(A) $\frac{\pi \ln(1+\sqrt{2})}{4}$

(B) $\frac{3\pi \ln 2}{4}$

(C) $\frac{\pi \ln(1+\sqrt{2})}{8}$

(D) $\frac{3\pi \ln 2}{8}$

(E) NOTA

23. Evaluate:

$$\int_0^{\infty} \frac{x^4}{(x^4 + x^2 + 1)^3} dx$$

(A) $\frac{\pi}{48\sqrt{3}}$

(B) $\frac{\pi}{36\sqrt{3}}$

(C) $\frac{\pi}{24\sqrt{3}}$

(D) $\frac{\pi}{12\sqrt{3}}$

(E) NOTA

24. Evaluate:

$$\int_0^{\frac{\pi}{2}} \ln^2(\tan(x)) dx$$

Hint:

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^3} = \frac{\pi^3}{32}$$

(A) $\frac{\pi^3}{6}$

(B) $\frac{\pi^3}{8}$

(C) $\frac{\pi^3}{16}$

(D) $\frac{\pi^3}{32}$

(E) NOTA

25. Evaluate:

$$\int_0^{\frac{\pi}{2}} \frac{\sin(x) + 2 \cos(x)}{1 + \sqrt{\sin(2x)}} dx$$

(A) $\frac{\pi}{2} - 1$

(B) $\pi - 2$

(C) $\frac{3\pi}{2} - 2$

(D) $\frac{3\pi}{2} - 3$

(E) NOTA

26. Evaluate:

$$a + b + c + d$$

where

$$\int_0^1 \frac{x \ln(1+x)}{x^2+1} dx = \frac{a \ln(2)^b + c\pi^2}{d}$$

given

$$\gcd(a, c, d) = 1$$

(A) 110

(B) 111

(C) 120

(D) 121

(E) NOTA

27. Evaluate:

$$\lim_{n \rightarrow \infty} \left\{ n \int_0^{\frac{\pi}{4}} e^x \tan^n x dx \right\}$$

(A) $\frac{e^{\frac{\pi}{4}}}{2}$

(B) $\frac{e^{\frac{\pi}{4}}}{4}$

(C) $\frac{e^{\frac{\pi^2}{16}}}{2}$

(D) $\frac{e^{\frac{\pi^2}{16}}}{4}$

(E) NOTA

28. Evaluate:

$$\lim_{n \rightarrow \infty} \left\{ n \int_0^1 \frac{x^n}{x^3+1} dx \right\}$$

(A) $\frac{12}{25}$

(B) $\frac{9}{19}$

(C) $\frac{27}{55}$

(D) $\frac{6}{11}$

(E) NOTA

29. Evaluate:

$$\int_0^1 \frac{x\sqrt{x} \ln(x)}{x^2 - x + 1} dx$$

(A) $\frac{\sqrt{3}\pi^2}{18} - 2$

(B) $\frac{\sqrt{3}\pi^2}{9} - 4$

(C) $\frac{\sqrt{3}\pi^2}{18} - 4$

(D) $\frac{2\sqrt{3}\pi^2}{9} - 4$

(E) NOTA

30. Evaluate:

$$\int_0^\infty \frac{\sqrt[\phi]{x} \arctan(x)}{(x^\phi + 1)^2} dx$$

if $\phi = \frac{\sqrt{5}+1}{2}$

(A) $\frac{\pi}{5\phi}$

(B) $\frac{\pi}{4\phi}$

(C) $\frac{\pi}{3\phi}$

(D) $\frac{\pi}{2\phi}$

(E) NOTA