Important Instructions For This Test: Good luck, have fun, and as always: "NOTA" stands for "None of These Answers is correct."

1. Which of the following is the negation of "If Nylah is a black belt, then Daniela is a brown belt"?

(A) If Nylah is not a black belt, then Daniela is not a brown belt.
(B) If Daniela is not a brown belt, then Nylah is not a black belt.
(C) Nylah is a black belt and Daniela is not a brown belt.
(D) If Nylah is a black belt, then Daniela is not a brown belt.
(E) NOTA

2. Which of the following is the negation of "There exists a real number x such that x is irrational"?

(A) There is no real number x such that x is irrational.
(B) There exists a real number x such that x is rational.
(C) For all real numbers x, x is rational.
(D) For all real numbers x, x is irrational.
(E) NOTA

3. Consider the statement "For all persons x, there exists a person y such that x is older than y." Which of the following is equivalent to this statement?

(A) Everybody is older than somebody.
(B) Someone is older than everyone.
(C) Everybody is younger than somebody.
(D) Someone is younger than everyone.
(E) NOTA

4. Consider the truth table below.

P	Q	??
T	T	F
T	F	F
F	T	T
F	F	F

Which of the following statements has this truth table?

(A) $P \wedge Q$ **(B)** $P \vee Q$ **(C)** $\neg(P \wedge Q)$ **(D)** $\neg P \wedge Q$ **(E)** NOTA

5. The decimal number 2025 has how many ones in its binary representation?

(A) 2 **(B)** 4 **(C)** 6 **(D)** 8 **(E)** NOTA

6. Suppose k is an integer. Evaluate $\lceil k - \frac{1}{2} \rceil + \lfloor k - \frac{1}{2} \rfloor$ in terms of k.

(A) $k + 1$ **(B)** k **(C)** $2k$ **(D)** $2k - 1$ **(E)** NOTA

7. Avion correctly found the greatest common divisor of 10,673 and 11,284 by using the Euclidean algorithm. Her work is below. However, she spilled coffee on her work, which blotted out the middle line. What is the missing middle line of her correct work?

$$11284 = 10673 \cdot 1 + 611$$
$$10673 = 611 \cdot 17 + 286$$
$$??$$
$$286 = 39 \cdot 7 + 13$$
$$39 = 13 \cdot 3 + 0$$

(A) $611 = 286 \cdot 2 + 39$ **(C)** $286 = 611 \cdot 1 - 325$ **(E)** NOTA
(B) $611 = 39 \cdot 15 + 26$ **(D)** $611 = 286 \cdot 3 - 247$

8. The value of `a` after the execution of the following loop is the reduced fraction $\frac{m}{n}$. Find $m \bmod n$.

```
a := 2
for i := 1 to 3
  a := a/2 + i/a
next i
```

(A) 1 **(B)** 169 **(C)** 289 **(D)** 313 **(E)** NOTA

9. Let N be the number of different six-letter strings, where each letter is one of the 26 letters of the English alphabet. For example, "`kqzwfp`" and "`lalala`" are both such six-letter strings. Compute $N \bmod 7$.

(A) 2 **(B)** 3 **(C)** 5 **(D)** 6 **(E)** NOTA

10. In an equilateral triangle of side length 1 inch, what is the minimum number of points that must be chosen in the triangle such that it is guaranteed that there are two points no more than $1/12$ of an inch apart?

(A) 101 **(B)** 122 **(C)** 145 **(D)** 170 **(E)** NOTA

11. In a math tournament, the individuals in places 1 through 10 were given trophies for places 1 through 10 in a random order. The probability that exactly 8 of the competitors received the correct trophy can be written as the fraction $N/10!$. Which of the following is N?

(A) 45 **(B)** 90 **(C)** 1814400 **(D)** 3628800 **(E)** NOTA

12. Compute 2^{2025} mod 43.

(A) 1 **(B)** 9 **(C)** 39 **(D)** 41 **(E)** NOTA

13. There are positive integers x that satisfy both $5x + 3 \equiv 2 \bmod 11$ and $6x + 1 \equiv 3 \bmod 7$. Let n be the smallest such positive integer. Compute n mod 9.

(A) 1 **(B)** 3 **(C)** 5 **(D)** 7 **(E)** NOTA

14. What is the number of positive integer divisors of 157201?

(A) 2 **(B)** 4 **(C)** 6 **(D)** 8 **(E)** NOTA

15. Suppose $325_k = 35_m$, where k and m are positive integers. Find the smallest possible value of m.

(A) 11 **(B)** 12 **(C)** 28 **(D)** 40 **(E)** NOTA

16. A fair coin is flipped 10 times. One of the flips is tails. What is the probability that all the other flips are also tails?

(A) $\frac{1}{1024}$ **(B)** $\frac{1}{1023}$ **(C)** $\frac{1}{512}$ **(D)** $\frac{1}{511}$ **(E)** NOTA

17. How many integers between 1 and 1000 are divisible by 12 or 18?

(A) 108 **(B)** 109 **(C)** 110 **(D)** 111 **(E)** NOTA

18. Let d be a digit. Suppose 33 divides $28577d245$ evenly. Which of these choices contains the value of d?

(A) 0 or 1 **(B)** 2 or 3 **(C)** 4 or 5 **(D)** 6 or 7 **(E)** NOTA

19. Let a and b be integers such that $2464a + 2947b = 7$. Compute the smallest possible value of $|a + b|$.

(A) 10 **(B)** 84 **(C)** 102 **(D)** 112 **(E)** NOTA

20. What is the coefficient of the xy^3z^3 term in the expansion of $(x + y + z)^7$?

(A) 35 **(B)** 70 **(C)** 105 **(D)** 140 **(E)** NOTA

21. In how many distinct ways can the letters of the word `RAZZMATAZZ` be arranged?

(A) 2025 **(B)** 25200 **(C)** 151200 **(D)** 3628800 **(E)** NOTA

22. Suppose a store sells six kinds of two-liter soft drinks: Fizz Cola, Cherry Fizz, Diet Fizz, Fizz Zero, Lemon Fizz, and Dr. Fizz. How many ways can we choose 10 two-liter bottles if at least one of them must be Diet Fizz, at least two must be Dr. Fizz, at least three must be Fizz Cola, and at most one can be Lemon Fizz?

(A) 15 **(B)** 21 **(C)** 105 **(D)** 210 **(E)** NOTA

23. How many of the following six statements are false?

Statement I: If n is an even integer, then 4 divides n^2.

Statement II: The product of a rational number and an irrational number is irrational.

Statement III: The product of any two consecutive integers is even.

Statement IV: For all integers a, b, and c, if a does not divide bc, then a does not divide b.

Statement V: If the sum of two primes is prime, then one of the primes must be 2.

Statement VI: If n is an odd integer, then $n^2 = 8k + 1$ for some integer k.

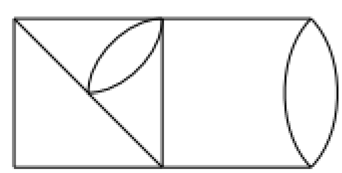
(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** NOTA

24. A tree is a graph that has no circuits. Suppose tree T is connected and has 20 vertices. How many edges must T have?

(A) 10 **(B)** 19 **(C)** 20 **(D)** 21 **(E)** NOTA

25. In a graph G, an *Euler path* is any path that uses each edge of the graph exactly once. An *Euler circuit* is an Euler path that begins and ends at the same vertex. A *Hamiltonian circuit* is a path that uses every vertex exactly once and begins and ends at the same vertex. Which of these does the graph G below have?

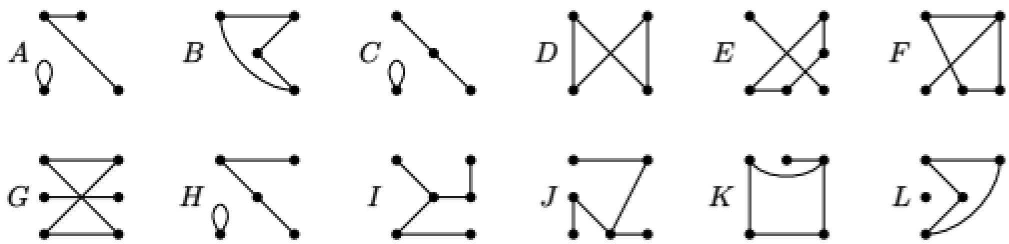
(A) only a Hamiltonian circuit
(B) only an Euler circuit
(C) both a Hamiltonian and an Euler circuit
(D) both an Euler path and an Euler circuit
(E) NOTA



26. Which of the following graphs G does not exist?

(A) G is a regular graph of degree 3 with 27 edges.
(B) G is connected graph with 22 vertices and 22 edges.
(C) G is a regular graph on 9 vertices and total degree 36.
(D) G has 5 vertices of degrees 1, 2, 2, 3, and 3.
(E) NOTA

27. How many *pairs* of isomorphic graphs are there among the twelve graphs A, B, C, D, E, F, G, H, I, J, K, and L, shown below?



(A) 0 **(B)** 2 **(C)** 4 **(D)** 6 **(E)** NOTA

28. How many four-digit integers have 4 or 7 as at least one of their digits?

(A) 1470 **(B)** 3906 **(C)** 5094 **(D)** 7530 **(E)** NOTA

29. Let $a = 405_6$ and $b = 11011_6$. What are the last two digits of $a^{2025}b^{2025}$ in base 6?

(A) 21 **(B)** 35 **(C)** 41 **(D)** 55 **(E)** NOTA

30. Fourteen points on a circle are split into seven pairs and each pair is joined by a segment. The probability that no two of these segments intersect is the reduced fraction m/n. Find $m + n$.

(A) 129 **(B)** 316 **(C)** 353 **(D)** 5041 **(E)** NOTA