All uppercase letter variables are positive integers unless otherwise stated. All fractions containing uppercase letter variables are in lowest terms. NOTA means "None of the Above."

Unless otherwise specified, all coins, dice, etc. are fair. The sides of a coin are heads and tails. A coin cannot flip a result other than those two. The N sides of a dN die are labeled 1 through N. The notation MdN refers to M dice, each with N sides. Decks of cards contain the standard 52 cards with no jokers, with Ace high.

$\sim\sim\sim\sim\sim\sim\sim\sim$ *Good luck, and have fun!* $\sim\sim\sim\sim\sim\sim\sim\sim$

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1) Legosi rolls 1d6, and Armosi rolls 1d4. Find the probability that Legosi's roll is greater than Armosi's.

A) $\frac{7}{12}$ B) $\frac{3}{5}$ C) $\frac{2}{3}$ D) $\frac{3}{4}$ E) NOTA

2) Legosi rolls 1d6, and Lestopsi rolls an unfair d4 such that the probability of rolling an n is equal to kn for some constant k for all $1 \leq n \leq 4$. What is the value of k?

A) $\frac{1}{4}$ B) $\frac{1}{10}$ C) $\frac{1}{16}$ D) $\frac{1}{24}$ E) NOTA

3) Given the facts of the previous problem, find the probability that Legosi's roll is greater than Lestopsi's.

A) $\frac{3}{8}$ B) $\frac{1}{2}$ C) $\frac{7}{12}$ D) $\frac{2}{3}$ E) NOTA

4) Legosi rolls 1d6, and Legono rolls 2d4. Find the probability that Legosi's roll is greater than Legono's maximum roll.

A) $\frac{23}{48}$ B) $\frac{1}{2}$ C) $\frac{13}{24}$ D) $\frac{9}{16}$ E) NOTA

5) Legosi rolls nd6. K'nexsi finds the smallest value of n such that the probability at least one n is rolled is greater than 0.5. What is the value of n?

A) 2 B) 3 C) 4 D) 5 E) NOTA

6) Legosi, Lidsi, and Lsuperegosi all flip 3 fair coins. Find the probability that all of them flip the same number of heads.

A) $\frac{1}{16}$ B) $\frac{7}{64}$ C) $\frac{1}{8}$ D) $\frac{21}{128}$ E) NOTA

7) Legosi's friends, some of whom appeared in the previous questions, have names based on the string `LEGOSI`. A continuous, non-empty substring of `LEGOSI` was chosen to be replaced with some other text. For each such substring, Legosi has two friends; for example, Legosi's friends for the substring `EGO` are Lidsi and Lsuperegosi. Find the remainder when the number of Legosi's friends is divided by 6.

A) 2 B) 3 C) 4 D) 5 E) NOTA

8) Which of the following statements regarding independence are true?

I) If A and B are independent, then $\sim A$ and $\sim B$ are independent.

II) An event is always independent of itself.

III) It is possible for two events to be both independent and mutually exclusive.

A) I ONLY B) I, II ONLY C) I, III ONLY D) II, III ONLY E) NOTA

9) Let $p(A) = 0.5$ and $p(B) = 0.6$. The smallest continuous range containing all possible values of $p(A \cup B)$ is $[r, s]$. Find $r + s$.

A) 1.7 B) 1.6 C) 1.5 D) 1.2 E) NOTA

10) The probability of rain on any given day is 0.4. The probability of rain the day after a day with rain is 0.8. Find the probability that two consecutive days contain no rain.

A) 0.36 B) 0.44 C) 0.52 D) 0.56 E) NOTA

11) In poker, a ''full house" is defined as a hand of five cards containing three cards of one rank and two cards of another rank (combining three-of-a-kind with a pair). Seven cards are selected from a deck: all four aces, and three kings. Find the probability that a randomly selected group of five cards from those seven form a full house.

A) $\frac{2}{3}$ B) $\frac{5}{7}$ C) $\frac{16}{21}$ D) $\frac{6}{7}$ E) NOTA

12) The Seattle Mariners and the St. Louis Cardinals are playing against each other in the World Series. This is a best-of-7 match where the first team to win 4 games wins the World Series. If both teams have an equal probability of winning any game, find the probability that all 7 games are needed to determine the winner of the World Series.

A) $\frac{1}{4}$ B) $\frac{5}{16}$ C) $\frac{21}{64}$ D) $\frac{3}{8}$ E) NOTA

13) Louis and Haru are playing a best-of-3 tennis match, where the first player to win 2 sets wins the match. The probability that Louis wins a set is $0.5 < p =$ **[DATA EXPUNGED]** < 1. Is the match more likely to end in 2 sets or 3 sets?

A) 2 sets

B) 3 sets

C) 2 sets if p is close to 1, 3 sets if p is close to 0.5

D) 3 sets if p is close to 1, 2 sets if p is close to 0.5

E) NOTA

14) Legosi and Louis draw a card from a deck (without replacement). Find the probability that the rank of Legosi's card is higher than the rank of Louis's card.

A) $\frac{8}{17}$ B) $\frac{12}{25}$ C) $\frac{25}{51}$ D) $\frac{1}{2}$ E) NOTA

15) Let the value of a letter be its position in the alphabet; for example, the values of the letters in `LOUIS` are 12, 15, 21, 9, and 19. Define the fishiness of a word as the sequence of greater-than and less-than signs that can go between the values of the word's letters; for example, since $12 < 15 < 21 > 9 < 19$, the fishiness of `LOUIS` is $<<><$. Determine the number of arrangements of the letters in `LEGOSI` (including '`LEGOSI`') that share the same fishiness as `LEGOSI`.

A) 12 B) 15 C) 16 D) 20 E) NOTA

16) Two integers between -4 and 5 (inclusive) are selected with replacement. Let the probability that their product is greater than 0 equal $\frac{A}{B}$. Find $A + B$.

A) 13 B) 61 C) 122 D) 141 E) NOTA

17) Two concentric circles have radii 1 and 2. Two points on the larger circle are selected. Find the probability that the chord connecting those two points passes through the smaller circle.

A) $\frac{1}{6}$ B) $\frac{1}{4}$ C) $\frac{1}{3}$ D) $\frac{1}{2}$ E) NOTA

18) There is a one-to-one correspondence between the rationals $\mathbb{Q}$ and the integers $\mathbb{Z}$, meaning that the two sets have equal cardinalities ($|\mathbb{Q}| = |\mathbb{Z}|$). Clearly, $|\mathbb{Q}| \geq |\mathbb{Z}|$, since every number n in $\mathbb{Z}$ can be written as a rational number, $\frac{n}{1}$. A popular argument to show $|\mathbb{Q}| \leq |\mathbb{Z}|$ involves writing all rational numbers $\frac{p}{q}$ such that $p + q = N$ in increasing order of p for all positive integers N greater than 1: $\frac{1}{1}, \frac{1}{2}, \frac{2}{1}, \frac{1}{3}, \frac{2}{2}, \frac{3}{1}, \frac{1}{4}, \cdots$. Find the positive difference between the numerator and denominator of the 2025^{th} element of this sequence.

A) 43 B) 44 C) 45 D) 46 E) NOTA

For Questions 19–21, consider the following fact pattern.
At Legosi's casino, Louis can bet all his money on the outcome of a fair coin flip, some on 'Heads' and the rest on 'Tails'. The 'Heads' bet pays with 2-to-1 odds, and the 'Tails' bet pays with 4-to-5 odds; for example, successful \$100 bets with those odds will return *profits* of \$200 and \$80, respectively. Let p be the proportion of Louis' bankroll he bets on Heads.

19) Find the value of p that will maximize Louis' expected profit on one flip of the coin.

A) $\frac{5}{24}$ B) $\frac{5}{14}$ C) $\frac{8}{9}$ D) $\frac{9}{10}$ E) NOTA

20) A bet (or a group of bets) is considered ''risk-free" if it is impossible to lose money on the bet. Louis's bet is risk free if p is on the interval $[a, b]$. Find $a + b$.

A) $\frac{11}{18}$ B) $\frac{2}{3}$ C) $\frac{7}{9}$ D) $\frac{5}{6}$ E) NOTA

21) For a specific value of p, Louis can guarantee that he will profit a fixed portion p' of his bankroll regardless of the outcome of the coin flip. Find p'.

A) $\frac{1}{9}$ B) $\frac{1}{8}$ C) $\frac{1}{5}$ D) $\frac{2}{9}$ E) NOTA

For Questions 22–30, consider the following fact pattern.
For each of three boxes, a random real number between 0 and 100 is uniformly randomly selected, and that amount of money (including fractional cents) is placed into the box. Legosi, Louis, and Haru then play a game. On his turn, Legosi selects a box, and the money inside is revealed. On Louis's and then Haru's turn, the player may select a new box, or the player may take the box held by a previous player, who will then select a new box and reveal the money inside. Unless otherwise stated, you may assume that each player wants to maximize the amount of money in their box at the end of the game, and that all players play perfectly. Round all money answers in this section to the nearest cent.

22) Find the expected value of the amount of money in the first box Legosi opens.

A) \$33.33 B) \$50.00 C) \$50.50 D) \$66.67 E) NOTA

23) Find the probability that Haru ends up with the smallest amount of money in his box.

A) $\frac{1}{24}$ B) $\frac{1}{18}$ C) $\frac{1}{16}$ D) $\frac{1}{12}$ E) NOTA

24) Suppose Legosi's box contains \$80. Find the expected value for the amount of money in Louis's box at the conclusion of the game if he decides to select a new box.

A) \$42.00 B) \$46.00 C) \$48.00 D) \$50.00 E) NOTA

25) Suppose Legosi's box contains \$80. Find the expected value for the amount of money in Louis's box at the conclusion of the game if he decides to take Legosi's box.

A) \$50.00 B) \$52.00 C) \$56.00 D) \$64.00 E) NOTA

26) Let Legosi's box contain $\$x$, and let Louis's expected winnings be $\$f(x)$. Find the maximum possible value of $\$f(x)$.

A) \$50.00 B) \$56.25 C) \$62.50 D) \$66.67 E) NOTA

27) Find the probability that Louis decides to take Legosi's box instead of opening an unopened box on his turn.

A) $\frac{1}{2}$ B) $\frac{3}{5}$ C) $\frac{5}{8}$ D) $\frac{2}{3}$ E) NOTA

28) Let the expected number of times that someone decides to take a box (instead of selecting an unopened box) on their turn during the game equal $\frac{A}{B}$. Find $A + B$.

A) 9 B) 19 C) 29 D) 47 E) NOTA

29) Find the expected amount of money in Haru's box at the conclusion of the game.
Hint: You may use the fact that the expected value of $\max(x, y)$ *for uniformly randomly distributed variables* $0 \leq x, y \leq 1$ *is* $\frac{2}{3}$.

A) \$66.67 B) \$70.83 C) \$72.92 D) \$75.00 E) NOTA

30) Find the probability that the real number selected before putting money in Legosi's box is 42.

A) $\frac{1}{10001}$ B) $\frac{1}{10000}$ C) $\frac{1}{101}$ D) $\frac{1}{100}$ E) NOTA