1	A	11	D	21	B
2	B	12	B	22	B
3	B	13	A	23	A
4	A	14	A	24	A
5	C	15	D	25	C
6	B	16	D	26	B
7	E	17	C	27	C
8	C	18	D	28	B
9	B	19	E	29	B
10	C	20	C	30	E

1) Going case by case (Legosi rolling a 1 through 6), $\frac{1}{6}\left(0+\frac{1}{4}+\frac{2}{4}+\frac{3}{4}+1+1\right)=\frac{7}{12}$. $\boxed{\text{A}}$

2) Given that the sum of the probabilities must be 1, $1k+2k+3k+4k=1$, so $k=\frac{1}{10}$. $\boxed{\text{B}}$

3) Going case by case (Legosi rolling a 1 through 6), $\frac{1}{6}\left(0+\frac{1}{10}+\frac{3}{10}+\frac{6}{10}+1+1\right)=\frac{1}{2}$. $\boxed{\text{B}}$

4) There are n^2 ways to roll two dice such that the maximum is n. Going case by case (Legosi rolling a 1 through 6), $\frac{1}{6}\left(0+\frac{1}{16}+\frac{4}{16}+\frac{9}{16}+1+1\right)=\frac{23}{48}$. $\boxed{\text{A}}$

5) The probability that no n's are rolled is $\left(\frac{5}{6}\right)^n$. We want to find the first n such that this is less than 0.5. Since $n=3$ gives $\frac{125}{216}>\frac{125}{250}$ and $n=4$ gives $\frac{625}{1296}<\frac{625}{1250}$, the desired n is 4. $\boxed{\text{C}}$

6) The probability that n heads are rolled by any single person is $\frac{\binom{3}{n}}{8}$. Since all people are independent, these probabilities are multiplied. Going case by case (0 heads through 3), $\left(\frac{1}{8}\right)^3+\left(\frac{3}{8}\right)^3+\left(\frac{3}{8}\right)^3+\left(\frac{1}{8}\right)^3=\frac{7}{64}$. $\boxed{\text{B}}$

7) For lengths ℓ 1 through 6, there are $7-\ell$ substrings that can be replaced, or 21 substrings in total. This is equal to 42 friends, which has a remainder of 0 when divided by 6. $\boxed{\text{E}}$

8) By De Morgan's, $P(A'\cap B')=P((A\cup B)')=1-P(A\cup B)=1-P(A)-P(B)+P(A\cap B)=1-P(A)-P(B)+P(A)\cdot P(B)=(1-P(A))(1-P(B))=P(A')\cdot P(B')$, so I is true. $P(A)\cdot P(A)=P(A)$ only if $P(A)=0$ or $P(A)=1$, so II is false. If $P(A)=0$, then $P(A\cap B)=P(A)\cdot P(B)=0$ and $P(A\cup B)=P(A)+P(B)=P(B)$, so the events are independent and mutually exclusive and III is true. $\boxed{\text{C}}$

9) The extreme values of $P(A\cup B)$ are $0.6+0.5=1.1\to 1$ and 0.6. These sum to 1.6. $\boxed{\text{B}}$

10) $P(RR)=0.4\cdot 0.8=0.32$, so $P(R'R)=0.4-0.32=0.08$. $P(RR')=0.4\cdot 0.2=0.08$, so the last possibility, $P(R'R')=1-0.32-0.08-0.08=0.52$. $\boxed{\text{C}}$

11) There are two possible full houses, aces full of kings and kings full of aces. $\binom{4}{3}\binom{3}{2}+\binom{3}{3}\binom{4}{2}=12+6=18$ full houses are possible. There are $\binom{7}{5}=21$ ways to pick 5 cards from 7. $\frac{18}{21}=\frac{6}{7}$. Alternatively, complimentary counting can be used, since the only way a full house is not selected is if all four aces are selected. $\boxed{\text{D}}$

12) The series must be decided in the seventh game, so no team has four wins after six games; therefore, both teams have three wins after six games. The number of ways this is possible is $\binom{6}{3}=20$. Out of $2^6=64$ possible outcomes, this is a probability of $\frac{20}{64}=\frac{5}{16}$. $\boxed{\text{B}}$

13) Let $q = 1 - p$. The probabilities that Legosi and Louis win in straight sets are, respectively, p^2 and q^2, and the probabilities that Legosi and Louis win in three sets are, respectively, $2p^2q + 2pq^2 = 2pq(p+q) = 2pq$. By AM-GM, $\frac{p^2+q^2}{2} \geq \sqrt{p^2q^2} = pq$, with equality at $p = q = 0.5$. Since $p > 0.5$, the inequality can be made strict, and the match is more likely to end in 2 sets. $\boxed{\text{A}}$

14) If the cards have different ranks, then there is a bijection between cases where Legosi has the higher card and cases where Legosi has the lower card that can be created by swapping Legosi's and Louis's cards. The probability that a second card has the same rank as the first is $\frac{4-1}{52-1} = \frac{1}{17}$, so the probability Legosi's card is higher is $\frac{1}{2} \cdot \frac{16}{17} = \frac{8}{17}$. $\boxed{\text{A}}$

15) LEGOSI's fishiness is $><<<>$. Without loss of generality, let the numbers of the letters be 1 through 6; `LEGOSI` is 412563. The second slot is less than four others, so it is 1 or 2. If it is 2, then the sixth slot (not necessarily greater than the second slot) is 1 (x2xxx1). For each of the four possibilities for the first slot, the remaining slots are determined, so there are 4 possibilities in this case.
Now let the second slot be 1 (x1xxxx). We can look at subcases for placement of the 2. If it is in the sixth slot (x1xxx2), then selection of the first slot fixes the remaining slots for 4 cases. If it is in the first slot (21xxxx), then selection of the sixth slot fixes the remaining slots for 3 cases (since the sixth slot can't be 6). If it is in the third slot (x12xxx), we make subcases for the placement of the 3. If it goes in the first slot (312xxx), then the remaining slots are determined by the sixth slot, for 3 cases. If it goes in the sixth slot (x12xx3), then the remaining slots are determined by the first slot, for 3 cases. If it goes in the third slot (x123xx), then half of the cases will follow the last greater than sign, for 3 cases. That's $4 + 4 + 3 + 3 + 3 + 3 = 20$ total cases. Hooray for casebashing! $\boxed{\text{D}}$

16) The integers are either both positive (5 possibilities) or both negative (4 possibilities). Since there are ten possible integers, the probability is $\left(\frac{5}{10}\right)^2 + \left(\frac{4}{10}\right)^2 = \frac{41}{100}$. $41 + 100 = 141$. $\boxed{\text{D}}$

17) For a point on the outer circle, draw both tangents to the inner circle, then draw radii of the inner circle to the points of tangency, then draw the radius of the outer circle to the initial point. The radii are respectively a leg and the hypotenuse of a right triangle whose other leg is part of the tangent lines. Since the radii have lengths 1 and 2, this is a $30 - 60 - 90$ right triangle. The angle between the tangent lines is therefore $60°$. That angle subtends a $120°$ arc; if the second point is on that arc, then the chord will pass through the inner circle. $\frac{120}{360} = \frac{1}{3}$. $\boxed{\text{C}}$

18) The number of fractions whose parts sum to $N + 1$ is N. Since $2016 = \frac{63 \cdot 64}{2}$, the 2016^{th} fraction will be the last one whose parts sum to 64. The 2025^{th} fraction is therefore the ninth one whose parts sum to 65, $\frac{9}{55}$. $55 - 9 = 46$. $\boxed{\text{D}}$

19) The coin is fair, so the highest expected profit is if all the money is bet on the side that pays better odds. $\boxed{\text{E}}$

20) Suppose Louis bets x of his bankroll on heads and y on tails. If heads is flipped, then the profit is $2x - y$. If tails is flipped, then the profit is $\frac{4y}{5} - x$. These are both non-negative, so $2x \geq y$ and $\frac{4y}{5} \geq x$. Combining these gives $\frac{5x}{4} \leq y \leq 2x$. Since $y = 1 - x$, $\frac{9x}{4} \leq 1 \leq 3x$, so $\frac{1}{3} \leq x \leq \frac{4}{9}$. $\frac{1}{3} + \frac{4}{9} = \frac{7}{9}$. $\boxed{\text{C}}$

21) Set $2x - y = \frac{4y}{5} - x$. Then $3x = \frac{9y}{5}$, or $x = \frac{3y}{5} = \frac{3}{5}(1 - x)$. $5x = 3 - 3x$, so $x = \frac{3}{8}$. $y = \frac{5}{8}$, so the profit is $\frac{1}{8}$. $\boxed{\text{B}}$

22) The expected value of a uniformly randomly selected real between 0 and 100 is $\frac{0+100}{2} = 50$. Note that in this section, the notation $\overline{U(a,b)}$ will be used to denote the expected value of a uniform distribution with minimum a and maximum b; $\overline{U(a,b)} = \frac{a+b}{2}$ and $\overline{U(0,100)}$ was found here. $\boxed{\text{B}}$

23) Haru will open the last unopened box only if both previously revealed boxes have values under \$50. This occurs with probability $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$. Haru will be guaranteed to not have the smallest amount of money in his box if it is at least \$50, which occurs with probability $\frac{1}{2}$. Otherwise, all three of them have identically distributed boxes ($U(0,50)$), so by symmetry the probability Haru's box has the least money is $\frac{1}{3} \cdot \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{24}$. $\boxed{\text{A}}$

24) If Louis opens a new box, there is a $\frac{4}{5}$ chance he will keep it, since its value will be less than 80. There is a $\frac{1}{5}$ chance he will open the last unopened box, since Haru would take Louis's box if it has value more than 80. $\frac{4}{5} \cdot \overline{U(0,80)} + \frac{1}{5} \cdot \overline{U(0,100)} = 0.8 \cdot 40 + 0.2 \cdot 50 = 42$. $\boxed{\text{A}}$

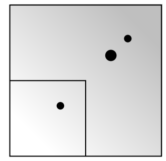
25) If Louis takes Legosi's box, then Haru's action will be based on the value of Legosi's new box. There is a $\frac{4}{5}$ chance Haru will take Louis's box, since its value (80) will be greater than the value of Legosi's new box. There is a $\frac{1}{5}$ chance he will take Legosi's new, higher-valued box and Louis ends up with the \$80. $\frac{4}{5} \cdot \overline{U(0,100)} + \frac{1}{5} \cdot 80 = 0.8 \cdot 50 + 0.2 \cdot 80 = 56$. $\boxed{\text{C}}$

26) Using ideas from before, $f(x)$ is the maximum of $\frac{x}{100} \cdot \overline{U(0,100)} + \frac{100-x}{100} \cdot x$ (choosing to take Legosi's box) and $\frac{x}{100} \cdot \overline{U(0,x)} + \frac{100-x}{100} \cdot \overline{U(0,100)}$ (choosing to open a new box). These simplify to $\frac{x}{2} + \left(1 - \frac{x}{100}\right)x = -\frac{x^2}{100} + \frac{3x}{2}$ and $\frac{x^2}{200} - \frac{x}{2} + 50$. The latter of these has a vertex at $x = 50$ and maxima of \$50 at $x = 0$ and $x = 100$. The former of these has a vertex at $x = 75$ and a maximum of \$56.25. $\boxed{\text{B}}$

27) The previous problem showed that if Legosi's box contains \$50 or higher, Louis should take Legosi's box. Now suppose Legosi's box contains less than \$50. If Louis chooses an unopened box, then Haru will take it if the revealed value is at least \$50 and Louis would end up with the last unopened box with distribution $U(0,100)$. If the revealed value is less than \$50, then that will be Louis's box at the end of the game, with distribution $U(0,50)$. The expected value of this strategy is $\frac{1}{2} \cdot \overline{U(0,100)} + \frac{1}{2} \cdot \overline{U(0,50)} = 0.5 \cdot 50 + 0.5 \cdot 25 = 37.5$. If Louis instead takes Legosi's box, then Haru would not take Louis's box on his turn, since (at minimum) the expected value of opening the last unopened box is higher. Thus, if Legosi's box has value greater than 37.5, Louis should take it. The range $[37.5, 100]$ gives a $\frac{5}{8}$ chance of Louis taking Legosi's box. $\boxed{\text{C}}$

28) In the $\frac{3}{8}$ of cases where Louis does not take Legosi's box, Louis will open an unopened box; in $\frac{1}{2}$ of those cases, the value of this box will be at least \$50 and Haru will take Louis's box. The remaining $\frac{5}{8}$ of cases where Louis takes Legosi's box can be broken into subcases. In $\frac{1}{2}$ of cases where the value of Legosi's box is at least \$50, Haru will take someone's box. In the remaining $\frac{1}{8}$ of cases, Haru will only take Legosi's box if Legosi's new box has a value of at least \$50. $\frac{3}{8} \cdot \frac{1}{2} + \frac{1}{2} \cdot 2 + \frac{1}{8} \cdot \frac{3}{2} = \frac{11}{8}$. $11 + 8 = 19$. $\boxed{\text{B}}$

29) Using the hint, consider the unit square, which represents dollars this problem scaled down by a factor of 100. If Legosi or Louis have a box with value of $\frac{1}{2}$ or greater, then Haru will take the maximum of Legosi's or Louis's values; otherwise Haru will get a value in $U(0,1)$. As previously shown, this latter case has probability $\frac{1}{4}$ and expected value $\frac{1}{2}$.



We can use mass points to determine the centroid of the larger region. $\frac{1}{4} \cdot \left(\frac{1}{3}, \frac{1}{3}\right) + \frac{3}{4} \cdot (x, x) = \left(\frac{2}{3}, \frac{2}{3}\right)$, so $x = \frac{7}{9}$. Thus, the expected value of Haru's value is $\frac{1}{4} \cdot \frac{1}{2} + \frac{3}{4} \cdot \frac{7}{9} = \frac{17}{24}$. Converting back to dollars, this is \$70.83. $\boxed{\text{B}}$

30) There is an uncountably infinite number of possible real numbers to choose, so the probability of selecting any one given real number is 0. $\boxed{\text{E}}$