Answer Key

0A: 2

0B: $\frac{\sqrt{5}}{5}$

0C: $\frac{\sqrt{5}+3}{3}$

1A: 360

1B: 38

1C: $-39e\sin(e)$

2A: 3

2B: 7

2C: 4

3A: $\frac{\sqrt{3}}{3}$

3B: 9

3C: $\frac{9}{2}$

4A: $-\frac{3}{2}$

4B: 6

4C: $72-18\sqrt{2}+18\sqrt{6}$

5A: 5

5B: 300

5C: 100

6A: 2

6B: 3

6C: 13

7A: 3

7B: $\pi+9$

7C: -1

8A: 2

8B: e^4

8C: $\frac{1}{9}$

9A: 9

9B: 81

9C: 69

Solutions

0A: Get ready for a Musical Adventure! The number of songs Mauithedog10 listens to over time can be modeled by $f(x) = 2x - 1$. Let A be the value of x where $f(x)$ intersects the line $g(x) = x + 1$

Solution:

$$f(x) = g(x) \quad \rightarrow \quad 2x - 1 = x + 1 \quad \rightarrow \quad x = 2$$

This gives us $A = \boxed{2}$.

0B: Get ready for a Musical Adventure! The number of songs Mauithedog10 listens to over time can be modeled by $f(x) = Ax$. If θ is the angle $f(x)$ makes with the positive x-axis, let B be the value of $\cos\theta$.

Solution: Draw a Picture!! We have that if the line has slope A, then the line forms a triangle with the x-axis from the points $(0,0), (1,0), (1,A)$. This gives that $\cos\theta = \frac{1}{\sqrt{1+A^2}}$.

Using $A = 2$ gives $B = \cos\theta = \frac{1}{\sqrt{5}} = \boxed{\dfrac{\sqrt{5}}{5}}$.

0C: Get ready for a Musical Adventure! The number of songs Mauithedog10 listens to over time can be modeled by $f(x) = \frac{1}{B}x^2 + 1$. Let C be the area in the first quadrant between $y = f(x)$ and $x = 1$.

Solution: $C = \int_0^1 \frac{1}{B}x^2 + 1 \, dx = \frac{1}{3B}x^3 + x\Big|_0^1 = \frac{1}{3B} + 1$.

Using $B = \frac{\sqrt{5}}{5}$ gives $C = \frac{1}{3\frac{\sqrt{5}}{5}} + 1 = \frac{5}{3\sqrt{5}} + 1 = \frac{\sqrt{5}}{3} + 1 = \boxed{\dfrac{\sqrt{5}+3}{3}}$.

1A: Consider the musical artist Phoebe B. and imagine you are arranging the letters PHOEBEB. Let A be the number ways are there to arrange the letters in PHOEBEB such that the P and H are next to each other.

Solution: To keep P and H together, treat them as a "grouped letter". This gives 6 letters, PH, O, E, B, E, B. The number of ways to arrange 6 letters is 6!. However, there a doubles of two of the letters, E and B, that cause overcounting. To account for this we divide by 2! twice, giving $\frac{6!}{(2!)(2!)}$. Finally, PH can be in any order (PH or HP) so we multiply by 2.
This gives $A = \frac{2(6!)}{(2!)(2!)} = \frac{720}{2} = \boxed{360}$.

1B: A vinyl record spins θ radians. If $\tan\theta = \frac{A}{60}$, let

$$B = \frac{1 + \cos^2\theta}{1 - \sin^2\theta}$$

Solution:

$$B = \frac{1 + \cos^2\theta}{1 - \sin^2\theta} = \frac{1 + \cos^2\theta}{\cos^2\theta} = \sec^2\theta + 1 = \tan^2\theta + 2 = \left(\frac{A}{60}\right)^2 + 2$$

Using $\frac{A}{60} = 6$ gives $B = 6^2 + 2 = \boxed{38}$.

1C: Charli XCX is going Vroom Vroom in her car which has a displacement given by $x(t) = \cos(t^B e^t)$. Let C be the velocity of the car when $t = 1$.

Solution: The velocity is given by

$$x'(t) = -\sin(t^B e^t)(Bt^{B-1}e^t + t^B e^t) \quad \rightarrow \quad C = x'(1) = -\sin(e)(Be + e) = -(B+1)e\sin(e)$$

.

Using $B = 38$ gives $C = \boxed{-39e\sin(e)}$.

2A: Bad Bunny hops according to a strange sequence. Let

$$A = \frac{1}{\ln(2^{15})}\left(\ln(2^0) + \ln(2^1) + \ln(2^2) + \cdots + \ln(2^9)\right)$$

Note that A **must be expressed as an integer**.

Solution:

$$A = \frac{1}{\ln(2^{15})}(\ln(2^0 \cdot 2^1 \cdot \cdots \cdot 2^9)) = \frac{1}{\ln(2^{15})}(\ln(2^{0+1+\cdots+9})) = \frac{\ln(2^{\frac{9\cdot 10}{2}})}{\ln(2^{15})} = \frac{\ln(2^{45})}{\ln(2^{15})} = \frac{45\ln 2}{15\ln 2} = \boxed{3}$$

.

2B: The Beatles are pointing on their album cover according to two vectors, given by $\vec{v}(t) = \hat{\imath} + \hat{\jmath}$ and $\hat{u}(t) = (A+1)\hat{\imath} - A\hat{\jmath}$. Let B be the tangent of the smaller angle between $\vec{v}(t)$ and $\vec{u}(t)$.
Solution: Using the formula for the cosine of the angle between two vectors, we have that

$$\begin{aligned}
\cos^2(\theta) &= \left(\frac{\vec{v}\cdot\vec{u}}{\|\vec{v}\|\|\vec{u}\|}\right)^2 \\
\frac{1}{B^2+1} &= \left(\frac{\langle 1,1\rangle \cdot \langle A+1, -A\rangle}{(\sqrt{2})(\sqrt{(A+1)^2 + A^2})}\right)^2 \\
\frac{1}{B^2+1} &= \frac{1}{2(2A^2+2A+1)} \\
4A^2 + 4A + 2 &= B^2 + 1 \\
(2A+1)^2 &= B^2 \\
2A + 1 &= |B|
\end{aligned}$$

Note that $\hat{u}(t) = \langle 4, -3\rangle$ lies above the vector $\langle 4, -4\rangle$, which is perpendicular to $\langle 1, 1\rangle$, so the angle is acute and B is positive. Using $A = 3$ gives that $B = \boxed{7}$.

2C: Take The Eagles to the limit! Let

$$C = \lim_{x\to\infty}\left(\sqrt{x^2 + Bx} - \sqrt{x^2 - x}\right)$$

Solution:

$$
\begin{aligned}
C &= \lim_{x\to\infty}\left(\sqrt{x^2+Bx}-\sqrt{x^2-x}\right)\\
&= \lim_{x\to\infty}\left((\sqrt{x^2+Bx}-\sqrt{x^2-x})\frac{\sqrt{x^2+Bx}+\sqrt{x^2-x}}{\sqrt{x^2+Bx}+\sqrt{x^2-x}}\right)\\
&= \lim_{x\to\infty}\left(\frac{x^2+Bx-x^2+x}{\sqrt{x^2+Bx}+\sqrt{x^2-x}}\right)\\
&= \lim_{x\to\infty}\left(\frac{(B+1)x}{\sqrt{x^2(1+\frac{B}{x})}+\sqrt{x^2(1-\frac{1}{x})}}\right)\\
&= \lim_{x\to\infty}\left(\frac{B+1}{\sqrt{1+\frac{B}{x}}+\sqrt{1-\frac{1}{x}}}\right)\\
&= \frac{B+1}{2}
\end{aligned}
$$

Using $B=7$ gives $A=\boxed{4}$.

3A: Olivia Rodrigo has déjà vu! She keeps doing the same processes over and over again: She first starts with an equilateral triangle with side length 1. This triangle has area A_0. She then creates a new triangle by halving the length of all the side lengths. This triangle has area A_1. She then continues this to get A_2, A_3 and so on to infinity. Let $A = A_0+A_1+A_2+A_3+\ldots$, or the infinite sum of all the areas of the triangles.

Solution: When you scale the side lengths of a 2D shape by some constant c, the area then scales by c^2. Hence, by halving the side lengths, each new triangle has $1/4$ the area of the previous triangle. This gives, by the formula for an infinite geometric sequence, that

$$
\begin{aligned}
A = A_0+A_1+A_2+\ldots &= A_0+\left(\frac{1}{4}\right)A_0+\left(\frac{1}{4}\right)^2A_0+\ldots\\
&= A_0\left(1+\frac{1}{4}+\left(\frac{1}{4}\right)^2+\ldots\right)\\
&= A_0\left(\frac{1}{1-1/4}\right)=\frac{4}{3}A_0
\end{aligned}
$$

A_0 is the area of an equilateral triangle with side length 1, which is $1^2\sqrt{3}/4=\sqrt{3}/4$, which gives that $A=\boxed{\sqrt{3}/3}$

3B: Harry Styles draws a Fine Line that can be written as $\vec{r}(t)=\vec{r_0}+\vec{v}t$. This line is parallel to both $(A^2)x+(\sqrt{3})y+(A\sqrt{3})z=10$ and $(A^2\sqrt{3})x+2y-Az=15$. If the z-coordinate of $\vec{v}$ is 1, let B be the x-coordinate of $\vec{v}$.

Solution: If the line is parallel to both planes, then it must be perpendicular to both the planes normal vectors. This means that the direction vector $\vec{v}$ must be a scalar multiple of the cross-product of the normal vectors, as this is in the direction of a vector that is perpendicular to both. This gives:

$$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A^2 & \sqrt{3} & A\sqrt{3} \\ A^2\sqrt{3} & 2 & -A \end{vmatrix} = \hat{\imath}\begin{vmatrix} \sqrt{3} & A\sqrt{3} \\ 2 & -A \end{vmatrix} - \hat{\jmath}\begin{vmatrix} A^2 & A\sqrt{3} \\ A^2\sqrt{3} & -A \end{vmatrix} + \hat{k}\begin{vmatrix} A^2 & \sqrt{3} \\ A^2\sqrt{3} & 2 \end{vmatrix}$$

$$= \hat{\imath}(-A\sqrt{3} - 2A\sqrt{3}) - \hat{\jmath}(-A^3 - 3A^3) + \hat{k}(2A^2 - 3A^2)$$

$$= (-3A\sqrt{3})\hat{\imath} + (4A^3)\hat{\jmath} + (-A^2)\hat{k}$$

Therefore, $\vec{v} = c\langle -3A\sqrt{3}, 4A^3, -A^2\rangle$ for some scalar c. We are given that $-cA^2 = 1$, hence $c = -1/A^2$, which gives that the x-coordinate of $\vec{v}$ is $3\sqrt{3}/A$.
Using $A = \sqrt{3}/3$ gives that $B = \boxed{9}$.

3C: The Weeknd is blinded by the lights, and starts walking in a straight line directly away from a lamppost of height H meters at a speed of $|B|$ m/s. If The Weeknd has a height of of $H/3$ meters, let C be how fast is the length of his shadow changing when he is 10 meters away.

Solution: Draw a Picture!! We have that The Weeknd and his shadow make a right triangle, with legs $H/3$ and $S(t)$, where $S(t)$ is the length of the shadow at time t. However, the shadow and the lamppost also make a right triangle with legs of length H and $D(t) + S(t)$, where $D(t)$ is the distance between the lamppost and The Weeknd. By the drawing, these triangles are similar, so by similar triangles the ratio of the legs are equal. This gives,

$$\frac{S(t)}{H/3} = \frac{S(t) + D(t)}{H}$$

$$H \cdot S(t) = \frac{H}{3}(S(t) + D(t))$$

$$\frac{2H}{3}S(t) = \frac{H}{3}D(t)$$

$$S(t) = \frac{1}{2}D(t)$$

Hence, this gives that $S'(t) = \frac{1}{2}D'(t) = \frac{1}{2}B$. Note this is independent of the distance between The Weeknd and the lamppost. Using $B = 9$ gives that $S'(t) = \boxed{9/2}$.

4A: Sabrina Carpenter is getting a Taste of the fundamentals of Calculus. Consider the following function f given by the table:

x	$f(x)$	$f'(x)$
-1	0	-1
0	-1	-2
1	-2	-3

Assume that f^{-1} exists and is continuous on $[-1, 1]$. Let A be the approximation of $f^{-1}(1)$ using the value of $\int_{-1}^{1}(f^{-1})'(x)\,dx$, approximated by a left Riemann sum with 2 sub-intervals of length 1.

Solution: We have by the Fundamental Theorem of Calculus that

$$\begin{aligned}
f^{-1}(1) &= f^{-1}(-1) + \int_{-1}^{1} (f^{-1})'(x)\, dx \\
&= 0 + \int_{-1}^{1} (f^{-1})'(x)\, dx \\
&\approx (f^{-1})'(-1) \cdot 1 + (f^{-1})'(0) \cdot 1 && \text{left Riemman sum} \\
&= \frac{1}{f'(f^{-1}(-1))} + \frac{1}{f'(f^{-1}(0))} && \text{formula for derivative of inverse} \\
&= \frac{1}{f'(0)} + \frac{1}{f'(-1)} \\
&= -\frac{1}{2} - 1 \\
&= -\frac{3}{2}
\end{aligned}$$

This gives that $A = \boxed{-\frac{3}{2}}$.

4B: After wrapping up on set, Dylan is Calling After Me to solve the following determinant:

$$B = \begin{vmatrix} A & -3 & -A \\ -5 & -A & -3 \\ -2 & 3A & 2 \end{vmatrix}$$

Solution: There are multiple ways to find this determinant. The Rule of Sarrus method is used here, which involves multiplying down left to right three diagonals and then subtracting the diagonals running up right to left. This gives

$$B = (-2A^2 - 18 + 15A^2) - (-2A^2 - 9A^2 + 30) = 24A^2 - 48$$

Using $A = -\frac{3}{2}$ gives that $B = 24 \cdot \frac{9}{4} - 48 = 54 - 48 = \boxed{6}$.

4C: Bruno Mars needs to study angles at the APT if he wants to get good at astronomy. Given triangle ΔPQR with $\overline{PQ} = \overline{QR} = B$ and $\angle Q = \frac{\pi}{4} + \frac{\pi}{3}$, let $C = (\overline{PR})^2$. **Note C is the square of the side length**.

Solution: By Law of Cosines,

$$\begin{aligned}
(\overline{PR})^2 &= (\overline{PQ})^2 + (\overline{QR})^2 - 2(\overline{PQ})(\overline{QR}) \cos\left(\frac{\pi}{4} + \frac{\pi}{3}\right) \\
C &= 2B^2 \left(1 - \cos\left(\frac{\pi}{4} + \frac{\pi}{3}\right)\right) \\
C &= 2B^2 \left(1 - \cos\left(\frac{\pi}{4}\right) \cos\left(\frac{\pi}{3}\right) + \sin\left(\frac{\pi}{4}\right) \sin\left(\frac{\pi}{3}\right)\right)\Bigg) && \text{addition formula for cosine} \\
C &= 2B^2 \left(1 - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} + \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2}\right) \\
&= B^2 \left(\frac{4 - \sqrt{2} + \sqrt{6}}{2}\right)
\end{aligned}$$

Using $B = 6$ gives $C = \boxed{72 - 18\sqrt{2} + 18\sqrt{6}}$

5A: Billie Eilish wants to be THE GREATEST at calculus. Help her with the following problem: Let $A = V/\pi$, where V is the volume of the solid obtained from rotating the function $f(x) = 2xe^{-x+1}$ on the domain $[1, \infty)$ around the x-axis. **Note that A is not the value of the volume.**

Solution: We have

$$\begin{aligned} V &= \int_1^\infty \pi (f(x))^2 \, dx \\ &= \int_1^\infty 4x^2 \pi e^{-2x+2} \, dx \\ &= 4e^2 \pi \int_1^\infty x^2 e^{-2x} \, dx \end{aligned}$$

This integral can be solved using integration by parts. Using the Tabular Method, we want to differentiate x^2 and integrate e^{-2x} as this will help reduce down the integral to 0:

S	D	I
$+$	x^2	e^{-2x}
$-$	$2x$	$-\frac{1}{2}e^{-2x}$
$+$	2	$\frac{1}{4}e^{-2x}$
$-$	0	$-\frac{1}{8}e^{-2x}$

Using the Tabular Method with this table (multiplying along the diagonals and accounting for the sign) gives

$$\begin{aligned} 4e^2\pi \int_1^\infty x^2 e^{-2x} \, dx &= 4e^2\pi \left(-\frac{1}{2}x^2 e^{-2x} - \frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x} \right) \Big|_1^\infty \\ &= 4e^2\pi \left((0 - 0 - 0) - (-\frac{1}{2}e^{-2} - \frac{1}{2}e^{-2} - \frac{1}{4}e^{-2}) \right) \\ &= 4e^2\pi \left(\frac{5}{4}e^{-2} \right) \\ &= 5\pi \end{aligned}$$

This gives that $A = \boxed{5}$.

5B: In her study of Supernovas and orbits, Chappell Roan considers the following conic:

$$9x^2 + 4y^2 - 18x + 8y + 13 - 36A^2 = 0$$

A quadrilateral is formed so that the diagonals of the quadrilateral are the axes of the conic. Let B be the area of this quadrilateral.

Solution: Factoring the conic by completing the square gives:

$$\begin{aligned} 9(x-1)^2 - 9 + 4(y+1)^2 - 4 + 13 - 36A^2 &= 0 \\ 9(x-1)^2 + 4(y+1)^2 &= 36A^2 \\ \frac{(x-1)^2}{(2A)^2} + \frac{(y+1)^2}{(3A)^2} &= 1 \end{aligned}$$

This gives an ellipse with major and minor axis lengths $6A$ and $4A$ (note you must multiply them by 2 to get the full axes length). The area of any quadrilateral can be found by multiplying the length of the diagonals and dividing by 2. Thus the area formed is $\frac{1}{2}(6A)(4A) = 12A^2$.
Using $A = 5$ gives that $B = 12 \cdot 25 = \boxed{300}$

5C: Not all questions can be music themed, and not all areas can be found without knowing linear algebra. A convex region in the xy-plane with positive area k is redrawn so that every point (x, y) now has the coordinate $(3kx - y, Bx + 2y)$. This new region has area B^2. Let $C = k$.

Solution: We have that the new region is formed by multiplying by the matrix $\begin{bmatrix} 3k & -1 \\ B & 2 \end{bmatrix}$. When applying a linear transformation, the area of regions scaled by the determinant of the matrix. This gives that

$$B^2 = k(6k + B)$$

Solving this for k gives

$$\begin{aligned} 6k^2 + Bk - B^2 &= 0 \\ (2k + B)(3k - B) &= 0 \\ k &= \begin{cases} B/3 & \text{if } B > 0 \\ -B/2 & \text{if } B < 0 \end{cases} && \text{Note } k \text{ must be positive} \end{aligned}$$

Using $B = 300$ gives that $C = \boxed{100}$.

6A: : Not all questions can be music themed, and not all integrals can be solved without useful bounds.

$$\pi \ln(k) = \int_0^\pi \frac{x|\cos(x)|}{1 + \sin(x)}\, dx$$

Let $A = k$. **Note A is not the value of the whole integral.**

Solution: The text hints that one must use a bounds trick. Adding the integral to itself and using the substitution $x \to \pi - x$ gives

$$\begin{aligned} 2A &= \int_0^\pi \frac{x|\cos(x)|}{1 + \sin(x)}\, dx - \int_\pi^0 \frac{(\pi - x)|\cos(\pi - x)|}{1 + \sin(\pi - x)}\, dx \\ 2A &= \int_0^\pi \frac{x|\cos(x)|}{1 + \sin(x)}\, dx + \int_0^\pi \frac{(\pi - x)| - \cos(x)|}{1 + \sin(x)}\, dx \\ 2A &= \int_0^\pi \frac{x|\cos(x)|}{1 + \sin(x)}\, dx - \int_0^\pi \frac{x|\cos(x)|}{1 + \sin(x)}\, dx + \pi \int_0^\pi \frac{|\cos(x)|}{1 + \sin(x)}\, dx \\ A &= \frac{\pi}{2} \int_0^\pi \frac{|\cos(x)|}{1 + \sin(x)}\, dx \end{aligned}$$

This can be solved by splitting the integral over the absolute value:

$$\begin{aligned}
\frac{\pi}{2}\int_0^{\pi} \frac{|\cos(x)|}{1+\sin(x)}\,dx &= \frac{\pi}{2}\left(\int_0^{\pi/2} \frac{\cos(x)}{1+\sin(x)}\,dx + \int_{\pi/2}^{\pi} \frac{-\cos(x)}{1+\sin(x)}\,dx\right) \\
&= \frac{\pi}{2}\left(\ln(1+\sin(x))\Big|_0^{\pi/2} - \ln(1+\sin(x))\Big|_{\pi/2}^{\pi}\right) \\
&= \frac{\pi}{2}((\ln 2 - 0) - (0 - \ln 2)) \\
&= \frac{\pi}{2}(2\ln 2) \\
&= \pi \ln 2
\end{aligned}$$

Hence $A = \boxed{2}$.

6B: Rather than undefined, Natasha Bedingfield prefers the term *Unwritten*. If

$$f(x) = \frac{3Ax+1}{2x-5}$$

Let B be the value of x **not** in the domain of $f^{-1}(x)$.

Solution: $f^{-1}(x)$ can be found in a number of ways. The first utilizes the fact that rational functions of this form operate the same as 2x2 matrices via a homomorphism, and thus have the same inverse. This gives immediately that (swapping the diagonal and negative the off-diagonal, noting the determinant does not change the value as it cancels out in the fraction):

$$\begin{aligned}
f(x) = \frac{3Ax+1}{2x-5} \quad &\rightarrow \quad A = \begin{bmatrix} 3A & 1 \\ 2 & -5 \end{bmatrix} \\
A^{-1} = \frac{1}{-15A-2}\begin{bmatrix} -5 & -1 \\ -2 & 3A \end{bmatrix} \quad &\rightarrow \quad f^{-1}(x) = \frac{-5x-1}{-2x+3A}
\end{aligned}$$

The normal method is using the fact that the inverse is the function reflected across the diagonal, which can be done by swapping x and y and solving for y:

$$\begin{aligned}
x &= \frac{3Ay+1}{2y-5} \\
2xy - 5x &= 3Ay + 1 \\
2xy - 3Ay &= 5x + 1 \\
y &= \frac{5x+1}{2x-3A}
\end{aligned}$$

Which gives

$$f^{-1}(x) = \frac{5x+1}{2x-3A}$$

Note that this is the same as before. In both cases, the value of x not in the domain is the root of the denominator, which is $3A/2$.

Using $A = 2$ gives $B = \boxed{3}$.

6C: ABBA is trying to train their angle eyes by practicing their trigonometry. They are interested in finding the maximum value of

$$B\sqrt{3}\sin(\theta) + 2\cos(\theta + \pi/3)$$

Let C be **the square of this maximum value.**

Solution: First we rewrite using the addition formula for cosine:

$$\begin{aligned}
B\sqrt{3}\sin(\theta) + 2\cos(\theta + \pi/3) &= B\sqrt{3}\sin(\theta) + 2\cos(\theta)\cos(\pi/3) - 2\sin(\theta)\sin(\pi/3) \\
&= B\sqrt{3}\sin(\theta) + \cos(\theta) - \sqrt{3}\sin(\theta) \\
&= (B\sqrt{3} - \sqrt{3})\sin(\theta) + \cos(\theta) \\
&= (B-1)\sqrt{3}\sin(\theta) + \cos(\theta)
\end{aligned}$$

Let ϕ be such that $\cos(\phi) = \frac{(B-1)\sqrt{3}}{\sqrt{1+3(B-1)^2}}$ and $\sin(\phi) = \frac{1}{\sqrt{1+3(B-1)^2}}$. Then,

$$\begin{aligned}
&= \sqrt{1+3(B-1)^2}\left(\frac{(B-1)\sqrt{3}}{\sqrt{1+3(B-1)^2}}\sin(\theta) + \frac{1}{\sqrt{1+3(B-1)^2}}\cos(\theta)\right) \\
&= \sqrt{1+3(B-1)^2}(\cos(\phi)\sin(\theta) + \sin(\phi)\cos(\theta)) \\
&= \sqrt{1+3(B-1)^2}\sin(\theta + \phi)
\end{aligned}$$

This has maximum value $\sqrt{1+3(B-1)^2}$. This gives that $C = 1 + 3(B-1)^2$.

Using $B = 3$ gives $C = 1 + 3(3-1)^2 = 1 + 12 = \boxed{13}$.

7A: Avril Lavigne always makes things too complicated! If $Re(z)$ and $Im(z)$ gives the real and imaginary parts of complex number z respectively, let

$$A = |Re(\sqrt{3-4i})| + |Im(\sqrt{3-4i})|$$

Solution: One can simplify the problem by first considering solutions to the equation

$$a + bi = \sqrt{3-4i}$$

Squaring both sides gives that

$$a^2 + 2(ab)i - b^2 = 3 - 4i$$

Matching up real and imaginary parts then gives

$$a^2 - b^2 = 3 \quad (1)$$

$$2ab = -4 \quad (2)$$

Substituting the value of b from (2) into (1) gives

$$\begin{aligned}
a^2 - \left(\frac{-4}{2a}\right)^2 &= 3 \\
a^2 - \frac{4}{a^2} &= 3 \\
a^4 - 3a^2 - 4 &= 0 \\
(a^2 - 4)(a^2 + 1) &= 0
\end{aligned}$$

and since a is a real number, we have that $|a| = 2$. It can then be seen that $|b| = 1$. Hence, $A = 1 + 2 = \boxed{3}$.

7B: In an argument to settle who was the ultimate Pop Girl of the 2024 Summer, Dr. Santos gives Mr. Snampal the following differential equation: Let $y(x)$ be a function defined on the positive real numbers such that for all $x > 0$,

$$\left(\frac{y'}{x^2} - \frac{2y}{x^3}\right) = \frac{1}{x^2+1}$$

Given that $y(1) = A + \pi/4$, let $B = y(\sqrt{3})$.

<u>Solution</u>: Note that the left hand side is the result of a product rule (those who have taken differential equations will notice this is simply a first order linear equation in which the integrating factor has already been multiplied through). Hence, we can "undo" the product rule and get

$$\begin{aligned}\left(\frac{y}{x^2}\right)' &= \frac{1}{x^2+1}\\ \frac{y}{x^2} &= \int \frac{1}{x^2+1}\,dx\\ \frac{y}{x^2} &= \arctan(x) + C\\ y &= x^2\arctan(x) + Cx^2\end{aligned}$$

Note that we will not run into any domain issues since we are only defining y on the domain $x > 0$. Setting $A + \pi/4 = y(1)$ then gives:

$$\begin{aligned}A + \pi/4 &= \arctan(1) + C\\ A &= C\end{aligned}$$

And so we have that

$$B = y(\sqrt{3}) = 3(\arctan(\sqrt{3}) + A) = 3(\pi/3 + A) = \pi + 3A$$

Using $A = 3$ gives $B = \boxed{\pi + 9}$.

7C: Not all questions can be music themed, and not all roots can be found without useful tricks. Let $B = n\pi + m$ for integers n, m. If p, q are the complex roots to the polynomial $nx^2 + mx + 1$, then let

$$C = \frac{1}{p-1} + \frac{1}{q-1}$$

<u>Solution</u>: First, note that sums and products of roots can be read off easily from polynomials. For example, $(x-a)(x-b) = x^2 - (a+b) + ab$, so the second and third coefficients give the sum and product. This is an application of Vieta's Sums.
Now, note that if p is a root of the polynomial $p(x)$, then $p-1$ is a root of the polynomial $p(x+1)$. Hence, $p-1$ and $q-1$ are roots of

$$\begin{aligned}n(x+1)^2 + m(x+1) + 1 &= n(x^2 + 2x + 1) + mx + m + 1\\ &= nx^2 + (2n+m)x + (n+m+1)\\ &= n\left(x^2 + \frac{2n+m}{n}x + \frac{n+m+1}{n}\right)\end{aligned}$$

Putting this all together, we have

$$C = \frac{1}{p-1} + \frac{1}{q-1}$$
$$C = \frac{(q-1)+(p-1)}{(p-1)(q-1)}$$
$$C = -\frac{\frac{2n+m}{n}}{\frac{n+m+1}{n}}$$
$$C = -\frac{2n+m}{n+m+1}$$

Using $B = \pi + 9$ gives $n = 1, m = 9$, giving $C = -\frac{2+9}{1+9+1} = \boxed{-1}$.

8A: In order to test her true Solar Power, Lorde is studying orbits and considers the following (possibly degernate) conic:

$$3Ax^2 + xy + 2Ay^2 - 3Ax + 2y - 5A = 0$$

The conic is then rotated counterclockise by an acute angle θ so that the conic no longer has an xy term. Given that $\cos(4\theta) = 3/5$, and that A **is positive**, find A.

Solution: For a conic with equation $Ax^2 + Bxy + Cy^2 + \dots$, If the angle eliminates the xy term after rotating, we know that $\cot(2\theta) = \frac{A-C}{B}$. From this we know that in the problem that

$$\cot(2\theta) = \frac{3A - 2A}{1} = A$$

Draw a Picture!! One could deduce that $|\cos(2\theta)| = \frac{A}{\sqrt{A^2+1}}$ (the sign does not matter) by drawing a right triangle with side lengths $A, 1$, and $\sqrt{A^2+1}$. Using the double angle formula for cos gives

$$\cos(4\theta) = 2\cos^2(2\theta) - 1 = 2\left(\frac{A^2}{A^2+1}\right) - 1 = \frac{2A^2 - A^2 - 1}{A^2+1} = \frac{A^2-1}{A^2+1}$$

So

$$\frac{3}{5} = \frac{A^2-1}{A^2+1}$$
$$3A^2 + 3 = 5A^2 - 5$$
$$8 = 2A^2$$

Since we know A is positive, this gives $A = \boxed{2}$.

8B: Taylor Swift can do anything with a Broken Heart. After thinking she was the problem (it's me), she solves the following calculus problem: Let

$$B = \lim_{x\to 0}\left(e^{Ax^2}\right)^{\left(\left(\int_0^x e^{-t^2}\sin(t)\,dt\right)^{-1}\right)}$$

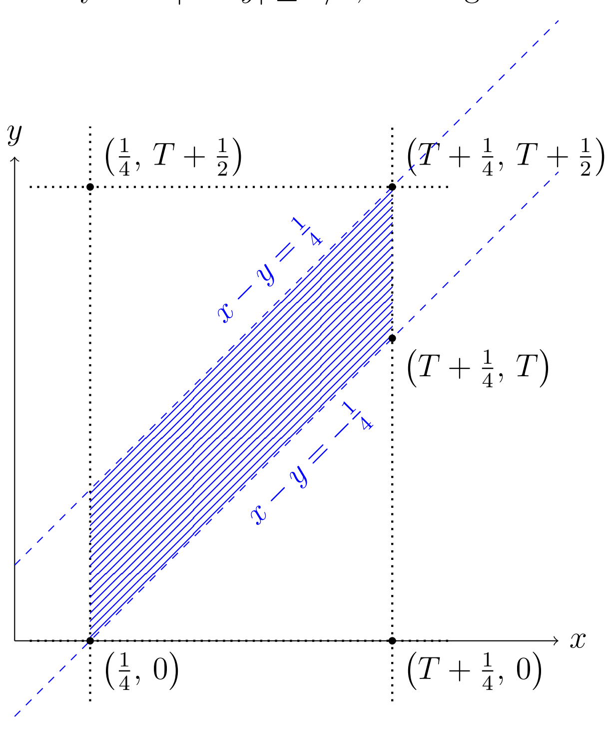
Solution: Note that the base goes to 1 and the integral goes to 0 since you are integrating from 0 to 0 in the limit. Since the exponent is one over the limit, this is of the type 1^∞. In this case is it best to apply the logarithm to evaluate:

$$
\begin{aligned}
B &= \lim_{x\to 0}\left(e^{Ax^2}\right)^{\left(\left(\int_0^x e^{At^2}\sin(t)\,dt\right)^{-1}\right)} \\
\ln B &= \lim_{x\to 0}\frac{\ln(e^{Ax^2})}{\int_0^x e^{-t^2}\sin(t)\,dt} && \text{allowed since } \ln x \text{ is continuous} \\
&= \lim_{x\to 0}\frac{Ax^2}{\int_0^x e^{-t^2}\sin(t)\,dt} \\
&= \lim_{x\to 0}\frac{2Ax}{e^{-x^2}\sin(x)} && \text{by L'Hospitals Rule and FTC} \\
&= \lim_{x\to 0}\frac{2A}{-2xe^{-x^2}\sin(x)+e^{-x^2}\cos(x)} && \text{by L'Hospitals Rule again} \\
&= 2A \\
B &= e^{2A}
\end{aligned}
$$

Using $A = 2$ gives $B = \boxed{e^4}$.

8C: The members of Magdelena Bay are Killing Time waiting for each other to arrive. Let $T = |\ln(B)|$. Two band members are going to a meeting that begins at 9 AM. One person will arrive anywhere from 9 AM to $(T + 1/2)$ hours after 9 AM, while the other person will arrive anywhere from **9:15 AM** to T hours after **9:15 AM**. Assume both people arrive at a random time uniformly on their respective intervals and are independent of each other. Let C be the probability that the two arrive within 15 minutes of each other.

Solution: The is a problem in geometric probability. To solve this, let y be the time the first person arrives and x the time the second person arrives. We can then graph all possible arrival times, letting 0 be 9 AM and each unit denoting one hour. This gives a square bounded by $x = 1/4, T + 1/4$ and $y = 0, T + 1/2$. We then want to find the probability that $|x - y| \le 1/4$, which gives the region



Hence the probability is the area of the blue region over the area of the whole rectangle. Here we find this by subtracting the areas of the triangles:

$$C = 1 - \frac{2 \cdot \frac{1}{2} \cdot T^2}{T(T + \frac{1}{2})} = 1 - \frac{T}{T + \frac{1}{2}} = \frac{\frac{1}{2}}{T + \frac{1}{2}} = \frac{1}{2T + 1}$$

Using $B = e^4$ gives that $T = 4$, giving $C = \boxed{\frac{1}{9}}$.

9A: The 1975 had a Change of Heart, and are more interested in maximization than regular Precalculus. If (x, y, z) lies on a sphere of radius 3, let A be the maximum value of $xy + xz + yz$.

Solution: If (x, y, z) lies on a sphere of radius 3, we have that $x^2 + y^2 + z^2 = 3^2$. Using this, we can find the maximum value using Cauchy Swartz:

$$\begin{aligned} xy + xz + yz = \langle x, z, y\rangle \cdot \langle y, x, z\rangle &\leq ||\langle x, z, y\rangle|| \cdot ||\langle y, x, z\rangle|| \\ &\leq (\sqrt{x^2 + z^2 + y^2})(\sqrt{y^2 + x^2 + z^2}) \\ &\leq (\sqrt{3^2})(\sqrt{3^2}) = 9 \end{aligned}$$

This gives that $A = \boxed{9}$.

9B: The 1975 would Love It If We Made It optimal. If they wanted a cone with surface area $2A$, let $B = \pi V^2$, where V is the maximum volume that such a cone could have.

Solution: Let r be the radius of the cone and h be the height of the cone.

Remark. One can find the surface area of a cone by "unraveling" it. This results in a portion of a circle with radius $\sqrt{r^2 + h^2}$ (the slant height) and circumference $2\pi r$. This means it has total area $\frac{r}{\sqrt{r^2+h^2}} \cdot \pi(\sqrt{r^2 + h^2})^2$ (the fraction of the total area of the full circle) which gives $\pi r\sqrt{r^2 + h^2}$, plus the base which is πr^2.

We have that $\pi r^2 + \pi r\sqrt{r^2 + h^2} = 2A$ and that $V = \frac{1}{3}\pi r^2 h$. We use the first equation to solve for h to get the volume in terms of one variable.

$$\begin{aligned} \pi r^2 + \pi r\sqrt{r^2 + h^2} &= 2A \\ r\sqrt{r^2 + h^2} &= \frac{2A}{\pi} - r^2 \\ \sqrt{r^2 + h^2} &= \frac{2A}{\pi r} - r \\ r^2 + h^2 &= \left(\frac{2A}{\pi r} - r\right)^2 \\ &= \frac{4A^2}{\pi^2 r^2} - \frac{4A}{\pi} + r^2 \\ h &= \sqrt{\frac{4A^2}{\pi^2 r^2} - \frac{4A}{\pi}} \end{aligned}$$

This gives that

$$V = \frac{1}{3}\pi r^2 \sqrt{\frac{4A^2}{\pi^2 r^2} - \frac{4A}{\pi}} = \frac{1}{3}\sqrt{4A^2 r^2 - 4A\pi r^4}$$

We can optimize this by taking the derivative and setting it to 0:

$$\begin{aligned} V' = 0 &= \frac{1}{3} \cdot \frac{8A^2r - 16A\pi r^3}{2\sqrt{4A^2r^2 - 4A\pi r^4}} \\ 0 &= 8A^2r - 16A\pi r^3 \\ 16A\pi r^3 &= 8A^2r \\ r^2 &= \frac{A}{2\pi} \end{aligned}$$

Plugging this value back into volume gives

$$V = \frac{1}{3}\sqrt{\frac{4A^3}{2\pi} - \frac{4\pi A^3}{4\pi^2}} = \frac{1}{3}\sqrt{\frac{2A^3}{\pi} - \frac{A^3}{\pi}} = \sqrt{\frac{A^3}{9\pi}}$$

Which gives the value of B as

$$B = \pi V^2 = \pi \cdot \frac{A^3}{9\pi} = \frac{1}{9}A^3$$

Using $A = 9$ gives that $B = \frac{1}{9} \cdot 9^2 = \boxed{81}$.

9C: The 1975 are done learning About You and moving on to Somebody Else. Matty is standing at (24, B) and must get to $(x, 20)$, where $x > 24$, while stopping along the line $y = 1$ along the way. They travel a distance D doing so. If $D^2 = 12025$, let C be the maximum value of x.

Solution: If x is the maximum value possible, this means that at this x the minimum distance traveled will be D. This is because if x was any larger, than traveling such a distance would no longer be possible, which makes this value of x a maximum. So we find the x such that the minimum distance needed to complete the trip is D.

The shortest trip would involve traveling in a straight line from $(24, B)$ to $y = 1$, then traveling back up in a straight line to $(x, 20)$. Imagine reflecting the end point over the line $y = 1$ to get $(x, -19)$. Then this trip can be rephrased as traveling from $(24, B)$ to $(x, -19)$ (imagining that after crossing $y = 1$, one would start traveling back up). This rephrases the problem as the shortest distance from $(24, B)$ to $(x, -19)$. This gives

$$\begin{aligned} D^2 &= (x - 24)^2 + (B + 19)^2 \\ 12025 &= (x - 24)^2 + (B + 19)^2 \\ 12025 - (B + 19)^2 &= (x - 24)^2 \\ \sqrt{12025 - (B + 19)^2} &= x - 24 && \text{Note } x > 24 \text{ so the root must be positive} \\ \sqrt{12025 - (B + 19)^2} + 24 &= x \end{aligned}$$

Using $B = 81$ gives that $x = \sqrt{12025 - 100^2} + 24 = \sqrt{2025} + 24 = \boxed{69}$.
