1. B: Let x be the number of hours Alejandro works. Solve $300 + 9x \geq 500$ to find $x \geq \frac{200}{9}$ so the least integer x is 23.

2. C: Assume Jackson and Mary start at the origin, $A(0,0)$. Jackson goes to $B(3,2)$ and Mary goes to $C(-1,-2)$. . Then $BC = \sqrt{(3+1)^2 + (2+2)^2} = \sqrt{32} = 4\sqrt{2}$.

3. B: The height of the cylinder is $8\sin(60°) = 8\left(\frac{\sqrt{3}}{2}\right) = 4\sqrt{3}$. Let r be the radius of the base. Then $\pi r^2(4\sqrt{3}) = 48\pi\sqrt{3}$. Therefore, $r^2 = 12$ and $r = \sqrt{12} = 2\sqrt{3}$. Thus, the diameter of the base is $2(2\sqrt{3}) = 4\sqrt{3}$.

4. A: Let $DC = x$. Since ΔADC is a 30-60-90 triangle, $AC = x\sqrt{3}$. Then $BC = x\sqrt{3} - 20$. But since ΔDBC is a 45-45-90 triangle, $BC = x$. Therefore, $x = x\sqrt{3} - 20$ or $20 = x(\sqrt{3} - 1)$. Thus, $x = \frac{20}{\sqrt{3}-1} = \frac{20(\sqrt{3}+1)}{2} = 10(\sqrt{3} + 1)$.

5. B: There are $10 \cdot 9 \cdot 26 \cdot 25 = 58{,}500$ possible passwords.

6. C: Let x be the original price. Then $x(0.8)(1.05) = 50.40$. Thus, $x = \frac{50.40}{0.84} = 60$.

7. C: Let the dimensions of each pen be x units across and y units vertically. From using 360 total feet of fencing, $6x + 4y = 360$ or $y = \frac{360-6x}{4}$. We wish to maximize $A = xy = \frac{x(360-6x)}{4}$. This simplifies to $A(x) = 90x - \frac{3}{2}x^2$. The maximum of a quadratic occurs at $x = \frac{-90}{2\cdot -\frac{3}{2}} = 30$. Then $y = \frac{360-6\cdot 30}{4} = 45$ and the maximum area of one pen is $xy = 30 \cdot 45 = 1350$ square feet.

8. B: Since the sum of the three angles in arithmetic progression is 180 degrees, then $m\angle B = 60°$. Let the other two measures be $m\angle A = 60 - d$ and $m\angle C = 60 + d$. Since every measure is divisible by 5, then d must be divisible by 5. To maximize $m\angle C - m\angle A = 2d$, choose d to be the largest multiple of 5 less than 60, which is 55. Then the maximum difference is $2(55) = 110$ when the angles measure $5, 60,$ and 115 degrees.

9. E: Use the model $A = P(1.15)^t$ to represent the town's population A after t years, where P is the initial population. To triple, $A = 3P = P(1.15)^t$. Dividing by P gives $3 = 1.15^t$. Taking the natural log of both sides gives $\ln(3) = \ln(1.15^t)$. Use a log property to write $\ln(3) = t \cdot \ln(1.15)$ so $t = \frac{\ln(3)}{\ln(1.15)}$.

10. D: Freda's location can be modeled by $(x-1)^2 + (y-2)^2 = 25$ and Alex's location can be modeled by $(x-4)^2 + (y-6)^2 = 20$. Squaring and simplifying gives $x^2 - 2x + y^2 - 4y = 20$ and $x^2 - 8x + y^2 - 12y = -32$. Subtract the two equations to get $6x + 8y = 52$ so $y = \frac{26-3x}{4}$. Substitute into the first equation: $(x-1)^2 + \left(\frac{26-3x}{4} - 2\right)^2 = 25$. Multiply by 16 and expand to get $16x^2 - 32x + 16 + 324 - 108x + 9x^2 = 400$. This simplifies to the quadratic $25x^2 - 140x - 60 = 0$. Factor as $5(5x+2)(x-6) = 0$ so $x = 6, -\frac{2}{5}$. Since they meet at integer coordinates, $x = 6$. Then $(6-1)^2 + (y-2)^2 = 25$ so $y = 2$. The sum $x + y = 6 + 2 = 8$.

11. B: Let a and r be the initial term and constant ratio of Anthony's series, where $\frac{a}{1-r} = 24$. In Juliette's series, the first term is a^2 and the ratio is r^2 for a sum of $\frac{a^2}{1-r^2} = 720$. From the first equation, $a = 24(1-r)$. Substitute into the second to write $\frac{24^2(1-r)^2}{1-r^2} = 720$. Since $r \neq 1$ because $|r| < 1$ to converge, this simplifies to $\frac{576(1-r)}{1+r} = 720$ or $576 - 576r = 720 + 720r$. Solving for r gives $r = -\frac{1}{9}$.

12. C: The volume of a prism is the area of the base times the height. In this case, each base is a trapezoid of area $\frac{1}{2}(4+10) \cdot 20 = 140$. The height of the prism is 15. So the total volume is $140 \cdot 15 = 2100$.

13. D: Let the radius of the cylinder be r and the height be h. Then $\pi r^2 h = 16\pi^5$ and $2\pi r^2 + 2\pi rh = 8\pi^3 + 16\pi^4$. Simplify the two equations to $r^2 h = 16\pi^4$ and $r^2 + rh = 4\pi^2 + 8\pi^3$. Notice that when $r = 2\pi$ and $h = 4\pi^2$ then the two equations are satisfied. Now consider one cycle of the string. Unfold the cylinder to form its lateral face in the shape of a rectangle of width $2\pi r = 4\pi^2$ and height $\frac{h}{2} = 2\pi^2$. By the Pythagorean Theorem, the length of the string for one cycle is $\sqrt{(4\pi^2)^2 + (2\pi^2)^2} = \sqrt{20\pi^4} = 2\pi^2\sqrt{5}$. Thus, the total length of the string is $4\pi^2\sqrt{5}$.

14. D: There are $9 \cdot 8 = 72$ possible linear equations. If $b > 0$ then the line must pass through Quadrant II no matter the slope. This accounts for $4 \cdot 9 = 36$ possibilities. If $b = 0$, then $a < 0$ to pass through Quadrant II. This accounts for 4 more possibilities. If $b < 0$, then $a < 0$ as well to pass through Quadrant II, giving $3 \cdot 4 = 12$ more cases. The probability is $\frac{36+4+12}{72} = \frac{2}{3}$.

15. D: Write $x(4-x^2)^{\frac{1}{2}} = 4 - x^2$ or $(4-x^2)^{\frac{1}{2}}(x - (4-x^2)^{\frac{1}{2}}) = 0$. If $(4-x^2)^{\frac{1}{2}} = 0$, then $4 - x^2 = 0$ or $x = \pm 2$. If $x - (4-x^2)^{\frac{1}{2}} = 0$ then $x = \sqrt{4-x^2}$. Square both sides to get $x^2 = 4 - x^2$ or $2x^2 = 4$ so $x = \pm\sqrt{2}$. However, $x = -\sqrt{2}$ is extraneous since $t(-\sqrt{2}) < 0$ but $h(-\sqrt{2}) > 0$. Thus, there are only 3 solutions.

16. D: Rearrange to $I = \frac{V}{R} = \frac{21+i}{2-3i}$. Multiply numerator and denominator by the conjugate, $(2+3i)$. Then $(21+i)(2+3i) = 42 + 2i + 63i + 3i^2 = 42 + 65i - 3 = 39 + 65i$. Also $(2-3i)(2+3i) = 4 - 6i + 6i - 9i^2 = 4 + 9 = 13$. Thus, $I = \frac{39+65i}{13} = 3 + 5i$.

17. B: When they all work together, they work at a combined rate of $\frac{1}{6} + \frac{1}{9} + \frac{1}{12} = \frac{13}{36}$, so they can paint $\frac{13}{36}$ of the fence per hour. After 2 hours, they have completed $\frac{13}{36} \cdot 2 = \frac{13}{18}$ of the fence, leaving $\frac{5}{18}$ of the fence left to be painted. Alex and Bella have a combined rate of $\frac{1}{6} + \frac{1}{9} = \frac{5}{18}$, so it will take them an additional hour to finish the rest.

18. D: Suppose $T_1 = 2\pi\sqrt{\frac{\ell_1}{g}}$ and let $\ell_2 = 4\ell_1$. Then $T_2 = 2\pi\sqrt{\frac{\ell_2}{g}} = 2\pi\sqrt{\frac{4\ell_1}{g}} = 2\pi \cdot 2\sqrt{\frac{\ell_1}{g}} = 2T_1$. This shows that if the length is quadrupled, then the period will double.

19. A: Since $lcm(8,9) = 72$, then M is divisible by 72. Let $M = 100a + 10b + c$. Then $N = 100c + 10b + a$. Note $M - N = 99(a-c) = 198$ so $a - c = 2$ and $a = 2 + c$. The three-digit numbers divisible by 72 are: $M =$144, 216, 288, 360, 432, 504, 576, 648, 720, 792, 864, 936. The only possibility where the hundreds digit is 2 more than the ones digit is 432. The product of the digits of M is $4 \cdot 3 \cdot 2 = 24$.

20. C: To get from A to B, the player makes 10 total moves: 4 right and 6 left. Thus, there are $\binom{10}{4} = 210$ possible paths from A to B. From A to C there are $\binom{4}{2} = 6$ paths and from C to B there are $\binom{6}{2} = 15$ paths. Thus, there are $210 - 6 \cdot 15 = 120$ paths from A to B not through C.

21. C: The maximum occurs when $t = \frac{-48}{2 \cdot -16} = \frac{3}{2}$. Then $y\left(\frac{3}{2}\right) = -16\left(\frac{3}{2}\right)^2 + 48\left(\frac{3}{2}\right) + 28$
$= -36 + 72 + 28 = 64$.

22. C: Solve $-16t^2 + 48t + 28 = 0$. Divide by -4: $4t^2 - 12t - 7 = 0$. Factor as $(2t + 1)(2t - 7) = 0$ so $t = -\frac{1}{2}, t = \frac{7}{2}$. Since time must be positive, $t = \frac{7}{2}$.

23. E: By the Rational Root Theorem, the rational roots could be $x = \pm 1, \pm 5, \pm\frac{1}{2}, \pm\frac{5}{2}$. Use synthetic division to see that $x = 1$ is a root. Factor as $(x - 1)(2x^3 - 3x^2 + 8x + 5)$. Divide again to see that $x = -\frac{1}{2}$ is a root so factor as $(x - 1)(2x + 1)(x^2 - 2x + 5)$. The non-real roots are $x = \frac{2 \pm \sqrt{4 - 4 \cdot 5}}{2}$ which simplifies to $x = 1 \pm 2i$. Their product is $(1 + 2i)(1 - 2i) = 1 - 4i^2 = 1 + 4 = 5$.

24. A: Multiply by the LCD $Rr_1r_2r_3$ to get $r_1r_2r_3 = Rr_2r_3 + Rr_1r_3 + Rr_1r_2$. Rearrange to get: $r_1(r_2r_3 - Rr_3 - Rr_2) = Rr_2r_3$. Divide to find $r_1 = \frac{Rr_2r_3}{r_2r_3 - Rr_3 - Rr_2}$.

25. C: Let (a, b) be the upper right vertex of the rectangle where $b = -ka + k$. Then $(a, -b)$ is the lower right vertex so the height of the rectangle is $2b = 2k - 2ka$. The width of the rectangle is a. The area is then $A = a(2k - 2ka) = -2ka^2 + 2ka$. The maximum occurs when $a = \frac{-2k}{2 \cdot -2k} = \frac{1}{2}$. The maximum area is then $-2k\left(\frac{1}{2}\right)^2 + 2k\left(\frac{1}{2}\right) = -\frac{k}{2} + k = \frac{k}{2}$. So $\frac{k}{2} = 4$ and $k = 8$.

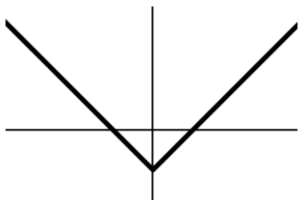
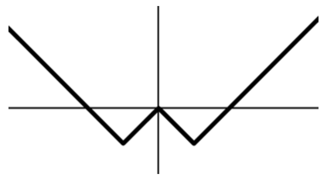
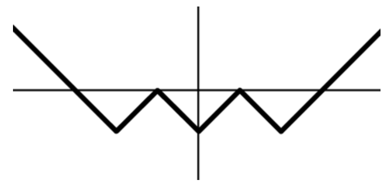
26. A: Given $6 = \frac{k(300)}{4}$ so $k = \frac{24}{300} = \frac{2}{25}$. Then $V = \frac{kT}{P} = \frac{\frac{2}{25} \cdot 960}{12} = \frac{1920}{300} = 6.4$.

27. D: Use the stars-and-bars technique for counting. With no restriction, imagine $12 + 4 = 16$ stars and 4 bars. There are $m = \binom{16}{4}$ ways to order the 4 bars among the 16 bars. For each employee to get at least one coin, then there are $12 - 5 = 7$ remaining coins to distribute among the 5 employees. Use $7 + 4 = 11$ stars and 4 bars to get $n = \binom{11}{4}$ ways. Thus, $m - n = \binom{16}{4} - \binom{11}{4}$.

28. B: Write the equation of the ellipse as $9(x - 8)^2 + 25y^2 = 900$ which expands and simplifies to $9x^2 - 144x + 25y^2 = 324$. The equation of the hyperbola can be written as $9x^2 - 15y^2 = 36$. Multiply the first equation by 3 and add 5 times the second equation to eliminate y^2:
$3(9x^2 - 144x + 25y^2) + 5(9x^2 - 15y^2) = 3(324) + 5(36)$ so $72x^2 - 432x - 1152 = 0$.
Divide by 72 to get $x^2 - 6x - 16 = 0$. Factor as $(x - 8)(x + 2) = 0$ so $x = 8, x = -2$. If $x = 8$, then $y = \pm 6$. If $x = -2$ then $y = 0$. The possible locations are $(-2, 0), (8, 6), (8, -6)$. The maximum distance is then $\sqrt{8^2 + 6^2} = 10$.

29. C: Sketch the first few iterations. Notice

$g_1(x) = |x| - 2$ $\quad$ $g_2(x) = \big||x| - 2\big| - 2$ $\quad$ $g_3(x) = \Big|\big||x| - 2\big| - 2\Big| - 2$

Continuing, notice that $g_n(x)$ has n relative minima and $n - 1$ relative maxima. When $y = -1$ is drawn, $g_1(x)$ intersects 2 times, $g_2(x)$ intersects 4 times, and $g_3(x)$ intersects 6 times. Continuing this pattern, $g_n(x)$ intersects $y = -1$ a total of $2n$ times. To have 100 solutions, then $2n = 100$ and $n = 50$.

30. D: Define the number of smartphones, tablets, laptops, and smartwatches with variables $a, b, c, d,$ respectively. We have following system:

$$\begin{cases} 100a + 150b + 300c + 50d = 31500 \\ 200a + 250b + 500c + 150d = 61500 \\ a + b + c + d = 280 \\ a = b + 100 \end{cases} \text{ or } \begin{cases} 2a + 3b + 6c + d = 630 \\ 4a + 5b + 10c + 3d = 1230 \\ a + b + c + d = 280 \\ a = b + 100 \end{cases}.$$

Substitute $a = b + 100$ into each of the first three equations: $\begin{cases} 5b + 6c + d = 430 \\ 9b + 10c + 3d = 830. \\ 2b + c + d = 180 \end{cases}$

Multiply the first by 3 and subtract the second: $6b + 8c = 460$.
Subtract the third from the first: $3b + 5c = 250$.
Eliminate b to get $2c = 40$ so $c = 20$. Then $b = 50$, $a = 150$, and $d = 60$.
Thus, $d - c = 60 - 20 = 40$.