Theta Bowl – Answers and Solutions
2025 Mu Alpha Theta National Convention

0A 12
0B –20
0C –4
0D –9

1A $-\dfrac{1}{4}$
1B 18
1C $-\dfrac{271}{26}$
1D $(-\infty,-1)\cup\left(-\dfrac{1}{2},1\right)$

2A $(-\infty,5]$
2B –18
2C 2
2D 8

3A 134
3B 15
3C 969
3D 686

4A 57
4B $4+4\sqrt{3}$
4C $(1,-2025)$
4D $[0,2]$

5A 150
5B 9
5C 24(%)
5D $10\sqrt{22}$

6A 10
6B 8
6C $\left(0,\dfrac{1}{5}\right)\cup\left(1,5\sqrt{5}\right)$
6D 600

7A 42
7B –10
7C $\dfrac{5}{8}$
7D 28

8A 18
8B $72+154i$
8C 258
8D –27

9A 12
9B 27
9C 2,000,000
9D 6

10A 5
10B *RICSH*
10C 2520
10D $\dfrac{1}{4}$

11A $\dfrac{2}{9}$
11B 1154
11C 4
11D $\dfrac{89}{45}$

12A 32
12B 7
12C $2,\dfrac{1}{2}$
12D –3

13A 5
13B 4
13C –61
13D 808

Question 0

$f(x)=x^3+3x^2-9x-15$

$A=f(-3)=-27+3(9)-9(-3)-15=-27+27+27-15=12$

$B=f(1)=1+3-9-15=-20$

$C=f(-1)=-1+3+9-15=-4$

$D=-9$

Question 1

$f(x)=\dfrac{1+2x-x^2-2x^3}{(x-2)^2}=\dfrac{-(2x+1)(x+1)(x-1)}{(x-2)^2}$

Intercepts: $\left(-\frac{1}{2},0\right),(-1,0),(1,0),\left(0,\frac{1}{4}\right)$. $A=-\frac{1}{2}-1+1+\frac{1}{4}=-\frac{1}{4}$

Slant asymptote: Division gives us $y=-2x-9$. $B=|(-2)(-9)|=18$

Intersection: $\dfrac{1+2x-x^2-2x^3}{x^2-4x+4}=-2x-9\rightarrow 1+2x-x^2-2x^3=-2x^3-x^2+28x-36\rightarrow$

$x=\dfrac{37}{26},\ y=2\left(\dfrac{37}{26}\right)-9=-\dfrac{154}{13}$. $C=\dfrac{37}{26}-\dfrac{154}{13}=-\dfrac{271}{26}$

$f(x)>0$: Using sign analysis with intervals $(-\infty,-1)\cup\left(-1,-\frac{1}{2}\right)\cup\left(-\frac{1}{2},1\right)\cup(1,2)\cup(2,\infty)$,

we find $D=(-\infty,-1)\cup\left(-\frac{1}{2},1\right)$

Question 2

The domain for g is $(-\infty,5]$, so we know that 5 is the right-most limit. The value of g is substituted into f, so we know that $x^2+1\geq 1$. This simplifies to $x^2\geq 0$, which is true for all real numbers, so $A=(-\infty,5]$.

$\dfrac{1}{5}=\dfrac{4}{2-x}\rightarrow 2-x=20\rightarrow B=-18$

We can tell by the formula for $k(x)$ that the difference in the terms will create an arithmetic sequence. $k(3)=k(2)-6\rightarrow k(2)=11$; $k(2)=k(1)-4\rightarrow k(1)=15$. We see know that the first three terms are 15, 11, and 5; so the next terms will be –3 and –13. $C=k(1)+k(5)=15-13=2$

We have to separate this into cases.

(1) the exponent = 0: $|x+1|=0\rightarrow x=-1$

(2) the base = 1: $|x-2|-2=1\rightarrow|x-2|=3\rightarrow x-2=\pm 3\rightarrow x=5$ (or –1, but it's not distinct from case (1))

(3) the base = –1 and the exponent is even: $|x-2|-2=-1\rightarrow|x-2|=1\rightarrow x-2=\pm 1\rightarrow x=3$ or 1, both distinct and both create an even exponent.

$D = -1+5+3+1 = 8$

Question 3

Let $\angle UMD$ be split into three angles of measure *x* and $\angle UDM$ be split into three angles of measure *y*. Now we have $3x+3y+111=180 \to x+y=23$. Now, $m\angle MOD = 180-2(x+y) =$ $180-2(23)=134=A$

$32{,}432{,}400 = 2^4 3^4 5^2 7^1 11^1 13^1 = (9)(10)(11)(12)(13)(14)(15)$; $B=15$

The largest three-digit number is 999; $(17)(3)=51$; $\frac{999}{51} \approx 19.6 \to (15)(51)=969=C$

The integers less than 1000 that are:

divisible by 7: $\frac{999}{7} \approx 142.7 \to 142$ divisible by 5: $\frac{999}{5} \approx 199.8 \to 199$

divisible by 35: $\frac{999}{35} \approx 28.5 \to 28$ divisible by neither 7 nor 5: $999-142-199+28=686=D$

Question 4

Volume ratios $T:T+M+T+M+B = 1^3:2^3:3^3 = 1:8:27 \to T:M:B = 1:8-1:27-8 = 1:7:19$.
Since the volume for *T* is 3, the volume of part *B* is $(19)(3)=57=A$.

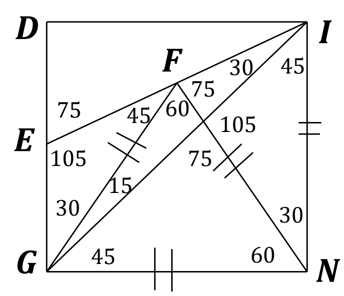
$IG=8$, so half that distance is 4. The height of the triangle will be $4\sqrt{3}$. $(a,b)=\left(5, 4\sqrt{3}-1\right) \to$ $a+b=B=4+4\sqrt{3}$

This will be easier if we sketch the points. However, the 1st, 5th, 9th, 13th, etc., moves all have an *x*-coordinate of 1, and these moves are four moves apart. The *y*-coordinate is the negative of the move number: the 1st move is (1, –1) and the 5th move is (1, –5). Since $2025=4(506)+1$, the 2025th move places the point at $(1,-2025)=C$.

If the line has a slope of 0, the line is horizontal. If the line has a slope of 2, the line will pass through the origin. $D=[0,2]$

Question 5

We can label the 60° and 45° angles from the equilateral triangle and square, respectively. We also know that the triangle and have the same side lengths. This creates isosceles triangle *INF*. $m\angle INF = (180°-30°) \div 2 = 75°$. From here, we can do some more angle chasing to get $m\angle DEI + m\angle EIG + m\angle EFG =$ $75°+30°+45°=150°=A$



$$\frac{37}{11}=3+\frac{4}{11}=3+\frac{1}{\frac{11}{4}}=3+\frac{1}{2+\frac{3}{4}}=3+\frac{1}{2+\frac{1}{\frac{4}{3}}}=3+\frac{1}{2+\frac{1}{1+\frac{1}{3}}};\ a+b+c+d=3+2+1+3=9=B$$

There are 25 combination pairs in our sample space, of which six give us a product that is 40 or greater: (5, 9), (7, 7), (7, 9), (9, 5), (9, 7), (9, 9). $\frac{6}{25}=24\%=C$

If the diagonal's length is $8\sqrt{11}$, then the side length is $4\sqrt{22}$ and the apothem is $2\sqrt{22}$. The semiperimeter is $8\sqrt{22}$, so our required total is $10\sqrt{22}=D$

Question 6

This is a 5-12-13 right triangle. Since E is the centroid, we can extend $\overline{OE}$ to create $\overline{ON}$, where N is the midpoint of the hypotenuse. By centroid properties, we know that $OE:ON=2:1$. Then, the area of $\triangle LEV$ is one-third that of $\triangle LOV$. $A=\frac{1}{3}\left[\frac{1}{2}(5)(12)\right]=10$

Substituting in −1, 1, and 2 for x, we get:

$-1+b-c+d=-3\rightarrow b-c+d=-2$

$1+b+c+d=9\rightarrow b+c+d=8$

$8+4b+2c+d=12\rightarrow 4b+2c+d=4$

Adding the first two equations, we get $2b+2d=6\rightarrow b+d=3$

Subtracting twice the second equation from the third, we get $2b-d=-12$

This gives us $b=-3, d=6, c=5.$

$B=-3+6+5=8$

$2\log_5 x-\log_x 125<1\rightarrow 2\log_5 x-\frac{3}{\log_5 x}-1<0\rightarrow 2\left(\log_5 x\right)^2-\log_5 x-3<0\rightarrow$

$\left(2\log_5 x-3\right)\left(\log_5 x+1\right)<0.$ Solving, we get $\log_5 x=\frac{3}{2}\rightarrow x=5\sqrt{5}$ and $\log_5 x=-1\rightarrow x=\frac{1}{5}$

Upon sign analysis, we get our solution: $\left(0,\frac{1}{5}\right)\cup\left(1,5\sqrt{5}\right)=C$

Let x be the distance from the accident to the end of the trip, and let R be the former rate of the truck. The normal time for the trip would be $\frac{x}{R}+1\rightarrow\frac{x+R}{R}$. Let's consider the time for each trip:

$1+\frac{1}{2}+\frac{x}{\frac{3}{4}R}=\frac{x+R}{R}+3\tfrac{1}{2}$ and $1+\frac{90}{R}+\frac{1}{2}+\frac{x-90}{\frac{3}{4}R}=\frac{x+R}{R}+3.$ Multiplying through by 3R, we get the system of equations $x=9R$ and $x-90=7.5R.$ Solving, we get $R=60$ and $x=540.$ Since the truck drove one hour at $R=60,$ the total distance, in miles, is $600=D.$

Question 7

Let the vertex of the parabola be on the y-axis be located at (0, 50) and the "feet" of the bridge be at (–75, 0) and (75, 0). This way, we can just use simple vertex form. Substituting (75, 0), we get $0 = a(75)^2 + 50 \rightarrow$ $a = -\frac{2}{225}$. Now, our equation is $y = -\frac{2}{225}x^2 + 50$. Substituting $x = 30$, we get that the height will be exactly 42 feet, so $A = 42$.

The only ways to get x^4 from these particular terms is using x^0 from the first factor and x^4 from the second factor *or* x^3 from the first factor and x^1 from the second factor. To get these, we have $\left[\binom{5}{0}(1)^5(-2x^3)^0\right]\left[\binom{8}{4}(1)^4(x)^4\right] + \left[\binom{5}{1}(1)^4(-2x^3)^1\right]\left[\binom{8}{1}(1)^7(x)^1\right]$. The coefficients will be $(1)(70) + (-10)(8) = -10 = B$

$\det X = ad - bc$. To get an even determinant, we need both products to be odd or both products to be even. There is a 1/2 chance of picking an odd number from the set, and there is also a 1/2 chance of picking an even number from the set. The only way to get an odd product is to multiply two odds; so to get all four numbers as odd, that would be $\left(\frac{1}{2}\right)^4 = \frac{1}{16}$. Using complementary counting, the probability of an even product will be 1 – P(odd) = $1 - \frac{1}{4} = \frac{3}{4}$; so, the probability of getting an even product will be $\left(\frac{3}{4}\right)^2 = \frac{9}{16}$. The total probability will be $\frac{1}{16} + \frac{9}{16} = \frac{10}{16} = \frac{5}{8} = C$.

$x^2 + 4y^2 = 4 \rightarrow \frac{x^2}{4} + \frac{y^2}{1} = 1$, so we know the major axis is along the x-axis. Our outer ellipse will therefore be in the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. We know that it passes through (4, 0), so $a = 4$. The ellipse also passes through (2, 1), which is a vertex of the rectangle. Substituting in (2, 1) for (x, y), we have $\frac{4}{16} + \frac{1}{b^2} = 1 \rightarrow \frac{1}{b^2} = \frac{3}{4} \rightarrow b^2 = \frac{4}{3}$. Our equation becomes $\frac{x^2}{16} + \frac{3y^2}{4} = 1 \rightarrow x^2 + 12y^2 = 16$. $D = 28$

Question 8

Point H must be (–1, –3). We can plot points and find the sum of the areas of two triangles, or we can use the shoelace method: $\frac{1}{2}\begin{vmatrix} 1 & 4 & -1 & -2 & 1 \\ 3 & 0 & -3 & 0 & 3 \end{vmatrix} = \frac{1}{2}\left|[(0-12+0-6)-(12+0+6+0)]\right| = \frac{1}{2}\left|(-18-18)\right| = 18$

$$\sqrt{z} = \frac{25}{1-2i} + \frac{15}{2+i} = \frac{50+25i+15-30i}{2+i-4i-2i^2} = \frac{65-5i}{4-3i} = \frac{5(13-i)}{4-3i} \cdot \frac{4+3i}{4+3i} = \frac{5(55+35i)}{25} = 11+7i$$

$$B = (11+7i)^2 = 121 + 154i - 49 = 72 + 154i$$

$(3^{2x-1})(4^{3x+1}) = 6^{x+3} \rightarrow (3^{2x-1})(2^{6x+2}) = (3^{x+3})(2^{x+3})$. Dividing, we get $\frac{3^{2x-1}}{3^{x+3}} = \frac{2^{x+3}}{2^{6x+2}} \rightarrow 3^{x-4} = 2^{-5x+1}$

$(x-4)\ln 3=(-5x+1)\ln 2 \rightarrow x\ln 3+5x\ln 2=\ln 2+4\ln 3 \rightarrow x(\ln 3+\ln 32)=\ln 2+\ln 81 \rightarrow$
$x(\ln 96)=\ln 162 \rightarrow x=\dfrac{\ln 162}{\ln 96}$. $C=162+96=258$

For this to have an infinite number of solutions, one equation must be a linear combination of the other two. We'll use $m(\text{eq. }1)+n(\text{eq. }2)=\text{eq. }3$. This gives us $3m+2n=1$, $-2m+n=-10$, $-4m+5n=g$, and $4m+fn=1$. Pairing the first two equations, we have $3m+2n=1$ and $4m-2n=20$, giving us $m=3$ and $n=-4$. Solving for *g* and *f*, $-12-20=g=-32$ and $21-4f=1 \rightarrow f=5$. $D=5-32=-27$

Question 9

$a_1+a_1r+a_1r^2=37$ and $\dfrac{a_1}{1-r}=64 \rightarrow a_1=64-64r$. Substituting, we get:

$64-64r+64r-64r^2+64r^2-64r^3=37 \rightarrow 64r^3=27 \rightarrow r=\dfrac{3}{4}$

$\dfrac{a_1}{\frac{1}{4}}=64 \rightarrow a_1=16$. $a_2=16\left(\dfrac{3}{4}\right)=12=A$

Let $EFG=x$ and $HIJ=y$. Then, $9(EFGHIJ)=4(HIFEFG) \rightarrow 9(1000x+y)=4(1000y+x) \rightarrow$ $8996x=3991y \rightarrow 692x=307y$. 692 and 307 have no common factors, so $x=307$ and $y=692$. $B=3+0+7+6+9+2=27$

9.1×10^7 is the distance from the sun to its corresponding vertex, and 9.3×10^7 is the distance from the other vertex to the sun. If we subtract 9.1×10^7 from 9.3×10^7, that will give us the distance between the foci. $C=10^7(9.3-9.1)=0.2\times 10^7=2\times 10^6=2{,}000{,}000$

The numbers end in 2, 4, 6, or 8. $(2)(4)(6)(8)=384$. This sequence occurs ten times, so we need to know the last digit of 4^{10}. $4^{10}=16^5$, and powers of 6 always end in 6. $D=6$

Question 10

Dimes	2	4	6	8	10
Pennies	22	17	12	7	2
Nickels	24	21	18	15	12

$A=5$

There are $5!=120$ permutations of *CHRIS*. For the words beginning with *C*, there 4! ways to arrange the remaining for letters. The same goes for those beginning with *H* and *I*, totaling 72 ways. There are 3! words beginning with *RC* and 3! words beginning with *RH*. That takes us to 84 arrangements. The 85th word will be *RICHS*, and the 86th word will be *RICSH*. $B=RICSH$

$(abc+abd+acd+bcd)^{10} \leftrightarrow a^{10}b^{10}c^{10}d^{10}\left(\dfrac{1}{a}+\dfrac{1}{b}+\dfrac{1}{c}+\dfrac{1}{d}\right)^{10}$, so we are looking for the coefficient of $a^{-2}b^{-6}c^{-1}d^{-1}$ in the expansion of $\left(\dfrac{1}{a}+\dfrac{1}{b}+\dfrac{1}{c}+\dfrac{1}{d}\right)^{10}$. $\dfrac{10!}{2!6!1!1!}=\dfrac{(10)(9)(8)(7)}{2}=2520=C$

Seven of the 10 gift cards will be chosen before eighth contestant draws. There are ${}_{10}C_7 = 120$ ways for the first seven drawings to occur. Next, we have to address having one of each gift card remaining. That will be $\left({}_5C_4\right)\left({}_3C_2\right)\left({}_2C_1\right) = (5)(3)(2) = 30.$ The probability that one of each card remaining is $\dfrac{30}{120} = \dfrac{1}{4} = D$

Question 11

A fact about the two imaginary cube roots of 1 is that they are each other's square and each other's reciprocal. Using Vieta's formulas, we know that $\omega^3 = 1$ and $\omega + \omega^2 + 1 = 0.$ Since $\omega^3 = 1,$ we know that $\omega + \omega^2 + \omega^3 = 0.$ The values from the die, $\{1, 2, 3, 4, 5, 6\}$, can be written in the form $3k, 3k+1,$ and $3k+2.$ In fact, there are two values for each of these forms, and we need one of each of them to satisfy our equation, as 1, 2, and 3 are also of the form $3k, 3k+1,$ and $3k+2.$ Our probability will be

$(3!)\left(\dfrac{{}_2C_1}{6}\right)\left(\dfrac{{}_2C_1}{6}\right)\left(\dfrac{{}_2C_1}{6}\right) = \dfrac{6(8)}{216} = \dfrac{2}{9} = A$. The eight combinations are $\{1, 2, 3\}, \{1, 2, 6\}, \{1, 3, 5\},$

$\{1, 5, 6\}, \{2, 3, 4\}, \{2, 4, 6\}, \{3, 4, 5\},$ and $\{4, 5, 6\}.$

$x = 3 + 2\sqrt{2}.$ $\dfrac{1}{x} = \dfrac{1}{3+2\sqrt{2}} \square \dfrac{3-2\sqrt{2}}{3-2\sqrt{2}} = 3 - 2\sqrt{2}.$ Now, $\left(x + \dfrac{1}{x}\right)^2 = 6^2 = 36 = x^2 + \dfrac{1}{x^2} + 2,$

so $x^2 + \dfrac{1}{x^2} = 34.$ Squaring this, we get $34^2 = 1156 = x^4 + \dfrac{1}{x^4} + 2,$ so $x^4 + \dfrac{1}{x^4} = 1156 - 2 = 1154 = B.$

$\left|\dfrac{x^2-5x+4}{x^2-4}\right| \le 1 \rightarrow \dfrac{x^2-5x+4}{x^2-4} \le 1$ and $\dfrac{x^2-5x+4}{x^2-4} \ge -1.$ These inequalities simplify to

$\dfrac{x^2-5x+4-x^2+4}{x^2-4} \le 0$ and $\dfrac{x^2-5x+4+x^2-4}{x^2-4} \ge 0,$ which simplify further to $\dfrac{5x-8}{(x+2)(x-2)} \ge 0$ and

$\dfrac{x(2x-5)}{(x+2)(x-2)} \ge 0.$ The solution for the first inequality is $\left(-2, \dfrac{8}{5}\right] \cup (2, \infty),$ and the solution for the second inequality is $(-\infty, -2) \cup [0, 2) \cup \left[\dfrac{5}{2}, \infty\right).$ The intervals common to each are $\left[0, \dfrac{8}{5}\right] \cup \left[\dfrac{5}{2}, \infty\right).$

$C = \left(\dfrac{8}{5}\right)\left(\dfrac{5}{2}\right) = 4$

First, pull this apart using partial fractions: $\dfrac{2}{n(n-1)} = \dfrac{A}{n} + \dfrac{B}{n-1} \rightarrow 2 = A(n-1) + Bn.$ When $n = 1, B = 2;$ when $n = 0, A = -2.$ This leads us to $2\sum_{n=2}^{90}\left(\dfrac{1}{n-1} - \dfrac{1}{n}\right) =$

$$2\left[\left(1-\frac{1}{2}\right) + \left(\frac{1}{2}-\frac{1}{3}\right) + \left(\frac{1}{3}-\frac{1}{4}\right) + \ldots + \left(\frac{1}{89}-\frac{1}{90}\right)\right] = 2\left(1-\frac{1}{90}\right) = 2\left(\frac{89}{90}\right) = \frac{89}{45} = D$$

Question 12

$\log\left(64\sqrt[24]{2^{x^2-40x}}\right)=0 \to 64\sqrt[24]{2^{x^2-40x}}=1 \to \sqrt[24]{2^{x^2-40x}}=\frac{1}{64} \to 2^{x^2-40x}=\left(\frac{1}{64}\right)^{24} \to x^2-40x=\log_2\left(\frac{1}{64}\right)^{24}=$
$24(-6)=-144 \to x^2-40x+144=0 \to (x-36)(x-4)=0.$ $x=36$ or 4. $A=36-4-32$

We are given that $a_1+8d=55$ and $1900<\frac{25}{2}\left(2a_1+24d\right)<2000.$ Now we can see that
$1900<25\left(a_1+12d\right)<2000 \to 1900<25\left(a_1+8d\right)+25\left(4d\right)<2000 \to$
$1900<25\left(55\right)+100d<2000 \to \frac{525}{100}<d<\frac{625}{100}$. Since d is an integer, $d=6$, so $a_1=55-8(6)=7=B$

$2x^2-5x+\sqrt{2x^2-5x+11}=1 \to 2x^2-5x+11+\sqrt{2x^2-5x+11}=12 \to a^2+a-12=0 \to$
$(a+4)(a-3)=0.$ A positive square root can't equal −4, so $\sqrt{2x^2-5x+11}=3 \to 2x^2-5x+2=0 \to$
$(2x-1)(x-2)=0.$ $C=2$ or $\frac{1}{2}$

$\sqrt[3]{\frac{\sqrt{112}-\sqrt{9072}}{\sqrt{112}}}-\sqrt[4]{\frac{\sqrt{1008}-\sqrt{448}}{\sqrt{112}}} \to \sqrt[3]{\frac{\sqrt{112}-\sqrt{112}\sqrt{81}}{\sqrt{112}}}-\sqrt[4]{\frac{\sqrt{112}\sqrt{9}-\sqrt{112}\sqrt{4}}{\sqrt{112}}} \to$
$\sqrt[3]{1-\sqrt{81}}-\sqrt[4]{\sqrt{9}-\sqrt{4}} \to \sqrt[3]{-8}-\sqrt[4]{3-2} \to -2-1=-3=D$

Question 13

A pentagonal prism has 5 sides, 7 faces, 15 edges, and 10 vertices. $A=\frac{15+10+2(5)}{7}=5$

We have three cases to consider: $x^2<10^2+24^2$, $24^2<10^2+x^2$, and $10^2<x^2+24^2$. The first case gives us $x^2<676 \to x<26$. The second case gives us $x^2>476$. Since $21<\sqrt{476}<22$, that means that 22 will be the lower boundary for that. The third case gives us $x^2>-476$, which is true for all real numbers. We have four values that can work: 22, 23, 24, and 25. $B=4$

Using the given roots, we have $\left(x-1\right)\left(x-2\right)\left(x-3\right)\left(x-n\right)=0 \to \left(x^3-6x^2+11x-6\right)\left(x-n\right)=0$
$\to x^4+\left(-6-n\right)x^3+\left(11+6n\right)x^2+\left(-6-11n\right)x+6n=0.$ The x^3 coefficient is 0, so $-6-n=0 \to n=-6$.
For the constant, $c=6(-6)=-36.$ $C=-25-36=-61$

$200p+4q=2004 \to 50p+q=501.$ Vieta's formulas tell us that $p+q=109.$ Combining these, we get $49p=392 \to p=8$ and $q=101.$ f is the product of the roots, $808=D.$