2025 Mu Alpha Theta National Convention
Theta CPAV Topic Test ANSWERS AND SOLUTIONS

ANSWERS:

1. C
2. D
3. B
4. C
5. A

6. B
7. A
8. D
9. B
10. D

11. C
12. C
13. A
14. D
15. B

16. D
17. C
18. D
19. A
20. D

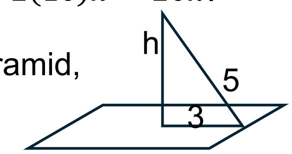
21. A
22. A
23. A
24. B
25. A

26. C
27. B
28. C
29. D
30. C

1. **C.** The leg length TS is 12 by the Pythagorean Theorem. Area = 0.5(12)(5) = 30.

2. **D.** Draw from C to a chord endpoint. We have a 6-8-10 Triple. Hypotenuse 10 is the radius of circle C. Circumference = $2(10)\pi = 20\pi$.

3. **B.** In a right pyramid, the height of the pyramid, the base apothem and the slant height form a right triangle. So height is 4. $V = \frac{1}{3}(Base\ area)h = \frac{1}{3}(36)(4)$ = 48.



4. **C.** Bases have sides 12 and 6 respectively. That is a ratio of 2:1. So volume is cubed, a ratio of 8:1.

5. **A.** The height will be 10 and the rectangle circumference will be the horizontal dimension of the rectangle in the picture, 10π. A=100π

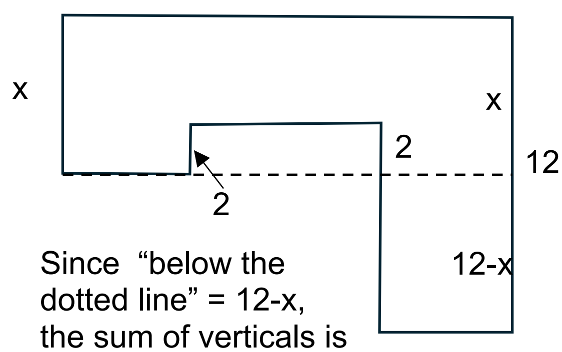
6. **B**. $A = \frac{1}{2}(h)(sum\ of\ bases)$.
$128 = \frac{1}{2}(8)(10 + b)$. Divide by 4.
$32 = 10 + b$. $b = 22$.

7. **A**. Let the diameter be the base of the triangle. Then the height will vary depending on where the inscribed angle's vertex lies. The area will be greatest when the vertex is greatest and that is the point that makes the height the radius. A = $\frac{1}{2}(10)(5) = 25$

8. **D**. Let the length of the long leg of the rectangle be L and the short leg length be W. Look at RT:
2L + W = 20. Now look at one of the vertical segments. 2W = L. So

4W + W = 20, and W = 4. L = 8.
The area of ΔRAT: use RT as base.
Area = $\frac{1}{2}(8)(20) = 80$.

9. **B.** Let the left-most vertical segment be x.



Since "below the dotted line" = 12-x, the sum of verticals is
x + 2 + 14-x + 12 = 28
Horizontal segments add to double 10. So Perimeter = 20 + 28 = 48.

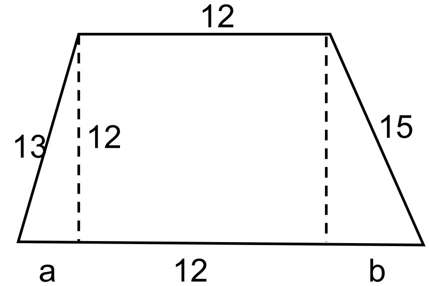
10. **D**. Let the heights be h. The square will have side length h. The equilateral triangle will have side length $\frac{2h}{\sqrt{3}}$. Area will be $\frac{1}{4}(side)^2\sqrt{3}$ = $\frac{1}{4}\left(\frac{4h^2}{3}\right)\sqrt{3}$.
$\frac{1}{3}h^2\sqrt{3}$. The ratio is 1: $\frac{1}{3}\sqrt{3}$.
Times 3 ... $\sqrt{3}$. Not a choice.
Divide by $\sqrt{3}$: $\sqrt{3}:1$.

11. **C.** If the missing sector is m degrees then the remaining sector has degree 360-m. So the area of this sector is $\frac{360-m}{360}(36\pi) = 30\pi$. Divide by π and reduce to get $\frac{360-m}{10} = 30$. Solve to get m = 60.

12. **C.** Let the original triangle have base b and height h. Then the new triangle will have base $1.2b$ and height $0.8h$. Original area is $\frac{1}{2}bh$ and new area is $\frac{1}{2}(1.2)(0.8)bh$ = $0.48bh$. That is a decrease of (0.50-0.48)bh = 0.02bh Divide by the original 0.50bh to get 0.02/0.5 = 2/50 = 4/100 = 4%.

13. **A.** Each side of the quadrilaterals is 10 since both have four equal sides. The square has area 100 and the rhombus area is bh = 96. Since b=10, h = 9.6. The difference in heights is 10 – 9.6 = 0.4.

14. **D.**



Drop the second height as shown to get the larger base parts to be a, 12 and b. Use the Pythagorean Th. to get a = 5. Label the second height length 12 and use the Pythorean Theorem to get b = 9. Both are triples, so you can avoid the computation if you noticed.
P = 12 + 15 + (5 + 9 + 12) + 13 =66

15. **B**. Although the diagram does not suggest it, we have a theorem that says a radius is perpendicular to a chord at its midpoint. Let the midpoint of the chord $\overline{SR}$ be M. Then $\overleftrightarrow{UM}$ is perpendicular to $\overline{SR}$ at M. $\overline{TM}$ is also perpendicular at M. So TU is perpendicular. So let ΔTRU have base length TU and height RM. Now consider ΔTRM. Use the

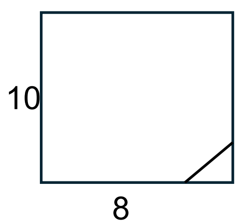
Pythagorean Theorem to get RM = $\sqrt{15^2 - 9^2}$ = 12. MU = $\sqrt{\sqrt{181}^2 - 9^2}$ = 10. That gives base TU = 12+10 = 22. Height is 9. Area = 99

16. **D**. Angle P has measure 30 and angle R has measure 60 so PR is a diameter since angle Q is a right angle. QR = 10 so PR = 20 and the circle has area 100π.

17. **C**. To get perimeter 10 with even integer length sides we try 2, 2, 6 and this cannot be a triangle. 4, 4 and 2 can, so we get the area of this triangle. Drop the altitude to the midpoint of the side which has length 2. That makes height $\sqrt{4^2 - 1^2}$ = $\sqrt{15}$. A = $\frac{1}{2}(2)\sqrt{15}$=$\sqrt{15}$

18. **D**. $4\pi r^2 = 36\pi$. r = 3. V = $\frac{4}{3}\pi(27)$ = 36π

19. **A**.



The original faces were 10x10, with area 100. Now three faces will have area 100 less the area of an isosceles right triangle with legs 2 each. That is $100 - \frac{1}{2}(2)(2) = 98$. Six of the faces will have total area 100(3) + 98(3) = 594. The seventh face will be an equilateral triangle with sides 2$\sqrt{2}$. Area = $\frac{1}{4}(8)\sqrt{3}$ = 2$\sqrt{3}$. That makes total area = $594 + 2\sqrt{3}$. This is $a + b\sqrt{c}$, and $a + b + c = 594 + 2 + 3 = 599$.

20. **D**. Area of a hexagon is $\frac{3}{2}side^2\sqrt{3}$. A = $\frac{3}{2}(64)\sqrt{3} = 96\sqrt{3}$.

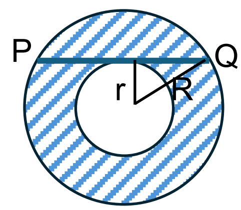
21. **A.** Consider one of the three circle segments bound by a chord and an arc. The arc has degree 2/12 of the circle. The area of one of these segments is $\frac{1}{6}(36\pi) - \frac{1}{4}(36)\sqrt{3}$ which comes from a 60 degree sector less an equilateral triangle of side length 6. This is $6\pi - 9\sqrt{3}$. To get the shaded region of the diagram, we will take the circle area, subtract the area of three segments, and subtract the area of the center triangle which has side length $6\sqrt{3}$. This is $36\pi - 3\left(6\pi - 9\sqrt{3}\right) - \frac{1}{4}(108\sqrt{3})$.This gives $18\pi + 27\sqrt{3} - 27\sqrt{3}$ = 18π. $a + b + c = 18 + 0 + 0 = 18$.

22. **A.** The leash has length 12 ft. Imagine Gary walking from the vertex where the leash is anchored, along the short wall, and then along the far wall until the leash stops him. He has traced two legs of a 30-60-90 triangle. The area of this triangle is $\frac{1}{2}\left(6\sqrt{3}\right)(6) = 18\sqrt{3}$. Now, the leash will keep Gary at a distance of 12' from the anchor point. He can walk a 60 degree sector of a circle of radius 12'. That is $\frac{1}{6}(144\pi)$ = 24π square feet. So the total he can reach is an area of $18\sqrt{3} + 24\pi$.

23. **A**. The ratio of the areas of ΔTLN and ΔPYZ is 10:40 = 1: 4. So the sides are in the ratio of 1:2. Let TN = x and PZ = 2x. Since ΔXZN is equilateral, $m\angle XZQ = 60°$. And ZQ =

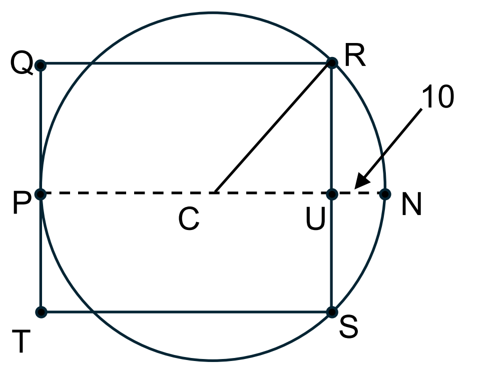
x. Same idea for ΔTNQ, QN = 2x and QT = $x\sqrt{3}$. So equilateral ΔTQP side to ΔTLN side is $x\sqrt{3} : x$. So areas of those triangles are 3:1. That makes the area of ΔTQP equal to 3 times 10 = 30.

24. **B**. Let R be the large circle radius, And r be the radius of the small circle.



By the Pythagorean Theorem, $5^2 = R^2 - r^2$. The area of the annulus is $(R^2 - r^2)\pi$ Which is 25π.

25. **A.**



Let C be the center of the circle. PC = CN = r. So the length of the side of the square is 2r -10 = PU = TS. That means RU = r-5. CU = r-10. CR = r. Using ΔCUR,

$(r-10)^2 + (r-5)^2 = r^2$.

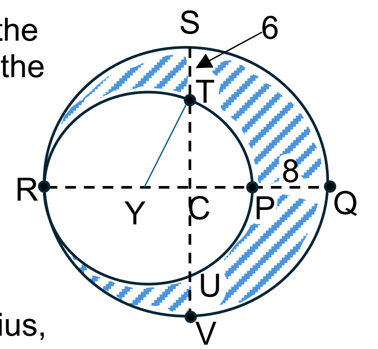
$r^2 - 20r + 100 + r^2 - 10r + 25 = r^2$

$r^2 - 30r + 125 = 0$.

$(r-25)(r-5) = 0$. The radius cannot be 5 if UN=10. So the radius is 25. PU = 2r-10=40 which is the length of the square side. The area of the square is 1600.

26. **C.** The center of the circle is C. RQ = 2r for r the radius of the big circle. That makes RP = 2r -8. That is the diameter of the small circle and the small radius is r-4. So the two radii have a difference of 4. SV = 2r so TU = 2r – 12. CT = r-6.

Let Y be the center of the small circle. Look at ΔCTY. TY = the small radius, or r-4. TC =r-6 and CY=4.



$16 + (r-6)^2 = (r-4)^2$.

$16 + r^2 - 12r + 36 = r^2 - 8r + 16$.

36= $4r$ and r=9. Area between the Two circles is $81\pi - 25\pi$ = 56π.

27. **B.** $6k\pi = 2\pi r$. Radius = 3k. Area = $9k^2\pi = 9k\pi$. Divide By $9k\pi$. k=1. Radius is 3k = 3.

28. **C.** Using the Pythagorean Th., RT = 10. In a rectangle, diagonals are congruent so US =10 which is the radius of the circle. The area in QI is $\frac{1}{4}(100\pi) = 25\pi$. So the area in QI is $25\pi - 48$. This is $a\pi - b$. $a + b = 25 + 48 = 73$.

29. **D.** Area = $\frac{1}{2}(8)(8) = 32$.

30. **C.** (UT)(TP)= (XT)(TL). (UT)(10)=(8)(20). UT = 16