2025 MAθ National Convention: Theta Cyphering

ANSWERS:

0. 19
1. 17
2. 8
3. -819
4. 6
5. 448
6. 14
7. 105
8. 820
9. 2.8 or $\frac{14}{5}$
10. 75

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0. Solution: Multiply numerator and denominator by the conjugate of $4-\sqrt{3}$.
$\frac{13(4+\sqrt{3})}{(4-\sqrt{3})(4+\sqrt{3})} = \frac{13(4+\sqrt{3})}{13} = 4+\sqrt{3}$ $= \sqrt{a}+\sqrt{b}$ when $a = 16, b = 3$.
$a + b$ = 19.

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1. Solution: The sum of each adjacent pair of rectangles gives the number above both. In the bottom row, 2+3=5, and 5 is above the 2 and 3 rectangles.

Now in the second diagram, I will use S for star and F for smiley face. In the second row, the first rectangle is 5+S and the second rectangle has F = S+3. Then we have 5+S+S+3=22. 2S=14 and S=7. Then F=10. The sum of S and F is then 17.

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2. Solution: $4\frac{1}{2} \times A\frac{B}{9} = 19.\ \frac{9}{2} \times \frac{9A+B}{9} = 19.$ The 9 terms cancel.

We have 9A+B=38, and we were given that $A - B = 2$. Add to get 10A=40 and A=4. So B=2 and $A + 2B$ = 8.

3. Solution: $a = (1@4)@1 = -7@1 =$ 49-2=47.
$b = (5@2)@4 = 21@4 = 441 - 8 = 433.$
$\sqrt{a}@b$ = 47-866= -819

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4. Solution: $4x + \frac{1}{x} = 4.\ 4x^2 - 4x + 1 = 0.\ (2x-1)^2 = 0.\ x = \frac{1}{2}.$
$8x^2 + \frac{1}{x^2}$ = A. $8\left(\frac{1}{4}\right) + 4$ = 6.

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5. Solution: $\left(\frac{x^3}{2} - \frac{2}{x}\right)^8$: A term of the expansion is $C(8,n) \cdot \left(\frac{x^3}{2}\right)^{8-n} \left(\frac{-2}{x}\right)^n$
For the term to be a constant, the powers of x must be equal and
cancel. So $3(8-n) = n.$ 24-3n=n. n=6. That makes the term C(8,6) $\left(\frac{x^3}{2}\right)^2 \left(\frac{-2}{x}\right)^6$
$= \frac{8!}{6!2!} \cdot \frac{x^6}{4} \cdot \frac{64}{x^6} = \frac{8(7)}{2} \cdot 16 = 28(16) = 448$

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6. Solution: The y-intercept is $y = |4 - |0+6|| = 2$. The x-intercepts occur when
$|4 - |x+6|| = 0.$ That is, when $|x+6| = 4$. This occurs at x= -2 or x = -10. The final answer will be the absolute value of these three values, which is 2 + 2 + 10 = 14.

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7. Solution: $log_2 8$=3. Horizontally, we then have 3 – a = -2 so a = -5.
Vertically we have 3 + c = 11 so c=8. The bottom row then is 8 – d = 4
so d=4. The sum of the squares of the three integers is 25+16+64 = 105.

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8. Solution: $f(x) = x^2 - 25^2 = (x-25)(x+25).\ f(2025) = 2000(2050).$
$f(75) = 50(100).\ \frac{f(2025)}{f(75)} = \frac{2000(2050)}{50(100)} = 20(41) = 820.$

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9. Solution:

$$\sum_{n=2}^{\infty}\left(\frac{4}{5}\right)^n = \frac{16}{25}+\frac{64}{125}+\cdots = \frac{a_1}{1-r} = \frac{\frac{16}{25}}{1-\frac{4}{5}} = \frac{16}{25-20} = \frac{16}{5}$$

$$\sum_{n=2}^{\infty} 8\left(\frac{1}{5}\right)^n = \frac{\frac{8}{25}}{1-\frac{1}{5}} \quad = \frac{8}{25-5} \quad = \frac{2}{5}\cdot\ \frac{16}{5}-\frac{2}{5}$$

Subtract to get 14/5 or 2.8.

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10. Solution: Let the graph be on the xy-plane with y-intercept (0,100) and x-intercepts (20,0) and (-20,0). Then the equation of the graph is $y = 100 - ax^2$. Use (20,0) to get $0 = 100 - 400a$, and $a = \frac{1}{4}$. Now use $y = 100 - \frac{1}{4}x^2$ with x=10. $y = 100 - 25 = 75$.
