2025 MAθ National Convention: Theta Cyphering

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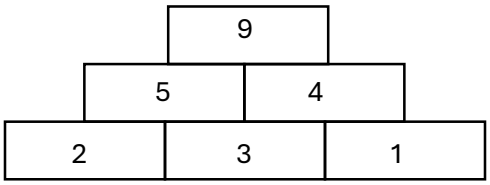
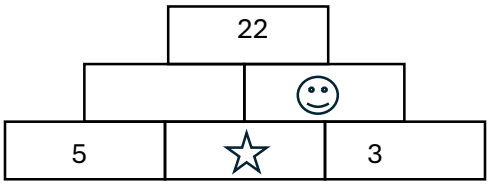
0. For $\frac{13}{4-\sqrt{3}} = \sqrt{a} + \sqrt{b}$. For a and b positive integers, give the value of $a + b$.

1.



Observe the "Rule" shown above.



Use the "Rule" to find the missing numbers.

Find the sum of ☆ and ☺.

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2. For A and B, single positive digits, $4\frac{1}{2} \times A\frac{B}{9} = 19$

and $A - B = 2$.

Find $A + 2B$.

3.

$x@y$	y=1	y=2	y=4
x=1	−1	−3	−7
x=4	14	12	8
x=5	23	21	17

The operation @, with a domain and range of integers, is defined by $x@y = x^2 - 2y$.

Some values for $x@y$ are given in the table above.

If $a = (1@4)@1$ and $b = (5@2)@4$, then give the value of $\sqrt{a}@b$.

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4. $4x + \frac{1}{x} = 4.$

Find the value of

$8x^2 + \frac{1}{x^2}$

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5. Find the constant term of the expansion of $\left(\frac{x^3}{2} - \frac{2}{x}\right)^8$.

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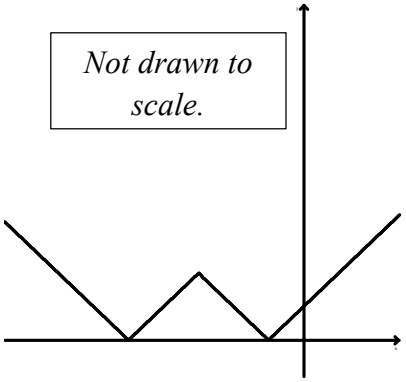
6. 

The graph of $y = |4 - |x + 6||$ is graphed to the left. The x-intercepts are (a, 0) and (b, 0). The y-intercept is (0, c). Give the sum of the absolute values of a, b and c: |a| + |b| + |c| = ?

7.

$log_2 8$	−		= −2
+		+	
	−		= 4
=		=	
11		9	

To the left, the diagram shows four equations. Two are written vertically, and two horizontally.

Find the integer values that belong in the three "empty" boxes and give the sum of their squares.

8. For $f(x) = x^2 - 25^2$, give the value of $\frac{f(2025)}{f(75)}$.

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9. Evaluate:

$$\sum_{n=2}^{\infty} \left(\frac{4}{5}\right)^n \quad - \quad \sum_{n=2}^{\infty} 8\left(\frac{1}{5}\right)^n$$

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10. A parabolic arch has its "feet" on the ground 40 ft apart. Its height at the maximum is 100 ft above ground. A pole is to be put 10 feet horizontally off of the axis of symmetry (dotted line). What is the height of the arch from the ground at that position, in feet?