Answer Key:

1. D
2. D
3. E
4. B
5. B

6. A
7. B
8. C
9. C
10. A

11. D
12. C
13. D
14. B
15. A

16. E
17. B
18. C
19. B
20. A

21. E
22. A
23. B
24. B
25. B

26. C
27. C
28. A
29. D
30. B

Solutions:

1. D: $y = x^2 + 4x + 7 \longrightarrow y = (x+2)^2 + 3$, so the vertex is $(-2, 3)$.

2. D: $x^2 - 2x + y^2 + 8y - 8 = 0 \longrightarrow (x-1)^2 + (y+4)^2 = 25$, so $r = 5$ and thus, the circumference is 10π.

3. E: $9x^2 - 54x - 16y^2 + 160y - 463 = 0 \longrightarrow \frac{(x-3)^2}{16} - \frac{(y-5)^2}{9} = 1$. The center of this hyperbola is $(3, 5)$ and it opens left and right. $a = 4$, $b = 3$, and hence, $c = \sqrt{4^2 + 3^2} = 5$. Thus, the foci are $(-2, 5)$ and $(8, 5)$, neither of which is provided as an answer.

4. B: $9x^2 + 36x + 4y^2 - 8y + 4 = 0 \longrightarrow \frac{(x+2)^2}{4} + \frac{(y-1)^2}{9} = 1$, so the larger radius has length 3. Thus, the major axis has length 6.

5. B: $y = \frac{3}{4}x - 7 \longrightarrow 3x - 4y - 28 = 0$ and the center of the circle is $(-3, 2)$. So, the shortest distance between the center of the circle and the line is $\frac{|3(-3)+(-4)(2)-28|}{\sqrt{3^2+(-4)^2}} = 9$. Subtracting the radius, $9 - 6 = 3$.

6. A: The length of the latus rectum is four times the focal length p. By inspection, $\frac{1}{4p} = \frac{3}{8}$ and hence, $4p = \frac{8}{3}$.

7. B: $4x^2 - 48x + 25y^2 + 100y + 144 = 0 \longrightarrow \frac{(x-6)^2}{25} + \frac{(y+2)^2}{4} = 1$, so $a = 5, b = 2$, and hence, $A = 10\pi$.

8. C: Substituting, $x^2 + (2x^2 - 6)^2 = 4 \longrightarrow 4x^4 - 23x^2 + 32 = 0$, a quadratic in x^2. Since the discriminant is $(-23)^2 - (4)(4)(32) = 17 > 0$, the polynomial has two distinct real solutions in x^2 and hence, four distinct real solutions in x. Thus, there are four intersection points whose x-coordinates which are exactly the four roots of the polynomial. Thus, our answer is the product of the roots which is $\frac{32}{4} = 8$.

9. C: $a = 2, b = 4$, and hence, the slopes of the asymptotes are $\pm\frac{4}{2} = \pm 2$. Using the fact that both lines pass through the center $(-5, 7)$, it can be deduced that the asymptotes are $y = 2x + 17$, $y = -2x - 3$.

10. A: $25x^2 - 50x + 36y^2 - 875 = 0 \longrightarrow \frac{(x-1)^2}{36} + \frac{y^2}{25} = 1$, so $a = 6, b = 5$, and hence, $c = \sqrt{6^2 - 5^2} = \sqrt{11}$. Thus, since the larger radius has length 6, the eccentricity $e = \frac{\sqrt{11}}{6}$.

11. D: $x^2 - y^2 = 0 \longrightarrow y^2 = x^2 \longrightarrow y = \pm x$. Thus, the graph is the two intersecting lines $y = x$ and $y = -x$.

12. C: The center of the ellipse is the midpoint of the vertices, which is $\left(-\frac{9}{2}, 1\right)$. Thus, the length of the minor axis is twice the distance between $\left(-\frac{9}{2}, 1\right)$ and $\left(-3, -\frac{1}{8}\right)$, which is $2\sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{9}{8}\right)^2} = \frac{15}{4}$.

13. D: $y = \frac{1}{4}x^2 - \frac{1}{2}x - \frac{7}{4} \longrightarrow y = \frac{1}{4}(x-1)^2 - 2$. So, $\frac{1}{4p} = \frac{1}{4}$ and hence, $p = 1$. Thus, since the center is $(1, -2)$ and the parabola opens upward, the directrix is the horizontal line 1 unit below, which is $y = -3$.

14. B: The described locus of points is a hyperbola with foci at $(-2\sqrt{2},-2)$ and $(2\sqrt{2},-2)$. So, this hyperbola has center $(0,-2)$ and opens left and right. Observe that the positive difference between the distances from a vertex to the two foci must be $(2a+c)-c=4$, and hence, $a=2$. Then, since $c=2\sqrt{2}$, $b=\sqrt{(2\sqrt{2})^2-2^2}=2$. Thus, the hyperbola has equation $\frac{x^2}{4}-\frac{(y+2)^2}{4}=1 \longrightarrow x^2-y^2-4y-8=0$.

15. A: The length of a latus rectum of an ellipse is equal to twice the square of the length of the smaller radius divided by the length of the larger radius. $16(x+3)^2+5(y-4)^2=80 \longrightarrow \frac{(x+3)^2}{5}+\frac{(y-4)^2}{16}=1$, so $a=\sqrt{5}$, $b=4$, and hence, the length of a latus rectum is $\frac{2(\sqrt{5})^2}{4}=\frac{5}{2}$.

16. E: To maximize the height, the truck must be centered in the region with its top two vertices on the top of the semi-ellipse. Let the origin be at the center of the ellipse. Then, the top right vertex will have an x-coordinate of $\frac{8}{2}=4$. The smaller radius of the ellipse has length $14-10=4$ and the larger radius has length $\frac{12}{2}=6$, so the ellipse has equation $\frac{x^2}{36}+\frac{y^2}{16}=1$. Substituting and solving for y, $\frac{4^2}{36}+\frac{y^2}{16}=1 \longrightarrow y^2=\frac{80}{9} \longrightarrow y=\frac{4\sqrt{5}}{3}$. Adding 10 gives us the maximum height of $\frac{30+4\sqrt{5}}{3}$.

17. B: $7-\frac{55}{8}=\frac{1}{8}$, so the focus is to the left of the vertex, and hence, the parabola opens to the left. So, the equation must be of the form $x=-\frac{1}{4p}(y-k)^2+h$. Then, since $p=\frac{1}{8} \longrightarrow \frac{1}{4p}=2$ and the vertex is $(7,3)$, the equation is $x=-2(y-3)^2+7$.

18. C: The eccentricity is the distance from the point to the focus divided by the distance from the point to the directrix, which is $\frac{\sqrt{1^2+7^2}}{4}=\frac{5\sqrt{2}}{4}$.

19. B: The quadrilateral is a rhombus with area $A=\frac{1}{2}d_1d_2=\frac{1}{2}(2a)(2b)=\frac{1}{2}(8)(18)=72$.

20. A: Substituting, $x^2-3x-7=-x^2-5x+5 \longrightarrow 2x^2+2x-12=0 \longrightarrow 2(x-2)(x+3)=0$, so the intersection points have x-coordinates 2 and -3. Substituting these values into one of the two parabola equations yields the intersection points $(2,-9)$ and $(-3,11)$. Thus, $a+b+c+d=1$.

21. E: Since the hyperbola has perpendicular asymptotes, the lengths of its transverse and conjugate axes must be equal. Thus, since the vertex is 3 units above the center, the covertices are 3 units to the left and right of the center, at $(0,-2)$ and $(6,-2)$. Neither point is provided as an answer.

22. A: The graph of $h(t)$ is a parabola with vertex $(2,33)$, so $h(t)=a(t-2)^2+33$. Since the height at $t=0$ is 1, $h(0)=a(0-2)^2+33=1 \longrightarrow 4a+33=1 \longrightarrow a=-8$. So, $h(t)=-8(t-2)^2+33$ and hence, $h(3)=-8(3-2)^2+33=25$. Thus, the height after 3 seconds is 25 meters.

23. B: Since the ball moves horizontally at 5 meters per second, its horizontal displacement in meters after t seconds is $x=5t$. So, the time when the displacement is x is $t=\frac{1}{5}x$. Substituting into the above equation, $h(x)=-8\left(\frac{1}{5}x-2\right)^2+33=-8\left(\frac{1}{25}x^2-\frac{4}{5}x+4\right)+33=-\frac{8}{25}x^2+\frac{32}{5}x+1$.

24. B: The discriminant of the conic is $8^2 - 4(6)(2) = 16$, which is greater than 0. This implies that the graph is a hyperbola if the conic is not degenerate. The determinant of the associated matrix is

$$\begin{vmatrix} 6 & 8/2 & -2/2 \\ 8/2 & 2 & 4/2 \\ -2/2 & 4/2 & -4 \end{vmatrix} = \begin{vmatrix} 6 & 4 & -1 \\ 4 & 2 & 2 \\ -1 & 2 & -4 \end{vmatrix} = -26$$ which is not zero. Thus, the conic is not degenerate.

25. B: $-2x^2 - 10x + 3y^2 - \frac{4}{3}y - 16 = 0 \longrightarrow -2\left(x + \frac{5}{2}\right)^2 + 3\left(y - \frac{2}{9}\right)^2 = \frac{197}{54}$. Thus, the center is $\left(-\frac{5}{2}, \frac{2}{9}\right)$.

26. C: Let $f(x) = ax^2 + bx + c$ and let $g(x) = f(x) - 2$. So, $g(x)$ goes through the points $(-1, 0)$, $(4, 0)$, and $(6, 8)$. Since the first two points are x-intercepts, $g(x) = a(x + 1)(x - 4)$. Substituting the third point, $8 = a(6 + 1)(6 - 4) = 14a \longrightarrow a = \frac{4}{7}$. So, $g(x) = \frac{4}{7}(x + 1)(x - 4) = \frac{4}{7}x^2 - \frac{12}{7}x - \frac{16}{7}$ and hence, $f(x) = g(x) + 2 = \frac{4}{7}x^2 - \frac{12}{7}x - \frac{2}{7}$. Thus, $7(a + b + c) = 7\left(\frac{-10}{7}\right) = -10$

27. C: By constructing two special right triangles, we can see that the angle of the arc between the two points is $60° + 45° = 105°$. Then, since the circumference of the circle is 4π, the length of the arc is $\frac{105}{360} \cdot 4\pi = \frac{7\pi}{6}$. Since this is the distance the ant travels, the number of seconds it takes is $\frac{7\pi/6}{\pi/6} = 7$.

28. A: The line will be tangent to the parabola when they intersect exactly once. $y = x + k \longrightarrow x = y - k$, which can be substituted: $y - k = \frac{1}{2}(y - 6)^2 + 7 \longrightarrow y - k = \frac{1}{2}y^2 - 6y + 25 \longrightarrow \frac{1}{2}y^2 - 7y + (25 + k) = 0$. Setting the discriminant equal to 0, $7^2 - 4\left(\frac{1}{2}\right)(25 + k) = 0 \longrightarrow k = -\frac{1}{2}$.

29. D: $4x^2 - 32x + 16y^2 - 96y + 144 = 0 \longrightarrow \frac{(x-4)^2}{16} + \frac{(y-3)^2}{4} = 1$, so the center is $(4, 3)$ and the major axis is horizontal with radius length 4. Thus, the vertices are $(0, 3)$ and choice D, $(8, 3)$.

30. B: The equation of the parabola will be the locus of points equidistant to $(-1, 1)$ and $y = x$. The distance between an arbitrary point (x, y) and $(-1, 1)$ is $\sqrt{(x + 1)^2 + (y - 1)^2}$, and the distance between an arbitrary point (x, y) and the line $y = x \leftrightarrow x - y = 0$ is $\frac{|x-y+0|}{\sqrt{1^2+1^2}} = \frac{|x-y|}{\sqrt{2}}$. Setting these distances equal to each other and squaring both sides, $(x + 1)^2 + (y - 1)^2 = \frac{(x-y)^2}{2} \longrightarrow 2(x^2 + 2x + 1 + y^2 - 2y + 1) = (x - y)^2 \longrightarrow 2x^2 + 2y^2 + 4x - 4y + 4 = x^2 - 2xy + y^2 \longrightarrow x^2 + 2xy + y^2 + 4x - 4y + 4 = 0$.