Good luck, have fun, and as always: "NOTA" stands for "None of These Answers is correct."

1. Consider the functions $f(x) = ax + 4$ and $g(x) = -x + 10$. If $f(g(x)) = g(f(x))$, determine the value of a.

(A) $\frac{2}{5}$ **(B)** -1 **(C)** $\frac{1}{5}$ **(D)** $\frac{5}{2}$ **(E)** NOTA

2. Determine the domain of the function $f(x) = \frac{\ln(x-e)}{x^2-e^2}$

(A) (e, ∞) **(B)** $(-e, \infty)$ **(C)** $(-e, e)$ **(D)** $(-\infty, \infty)$ **(E)** NOTA

3. Find the sum of x for the following equation: $\log_2(x-1) + \log_4(x-4) = 2$

(A) 20 **(B)** 9 **(C)** 6 **(D)** 5 **(E)** NOTA

4. Find the number of asymptotes the function $f(x) = \frac{x^3-8}{x^2-4}$ has.

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** NOTA

5. Let $Q_0(x) = -12x^2 + 8x + 7$. For integers $n \geq 1$, define $Q_n(x) = Q_{n-1}(x+n)$. What is the coefficient of x^2 in $Q_{15}(x)$?

(A) -14 **(B)** -12 **(C)** -10 **(D)** -8 **(E)** NOTA

6. Let $R(x)$ be a polynomial such that when $R(x)$ is divided by $x-7$, the remainder is 42, and when $R(x)$ is divided by $x-42$, the remainder is 7. What is the remainder when $R(x)$ is divided by $(x-7)(x-42)$?

(A) $-x + 49$ **(B)** $x + 49$ **(C)** $-x + 7$ **(D)** $x + 7$ **(E)** NOTA

7. Let $q(x) = x^2 + ax + b$, where a and b are integers. If $q(x)$ is a factor of both $x^4 + 5x^2 + 9$ and $2x^4 + 6x^2 - 4x + 6$, what is $q(2)$?

(A) 7 **(B)** 9 **(C)** 10 **(D)** 12 **(E)** NOTA

8. Consider the piecewise function defined by:

$$h(x) = \begin{cases} x^3 + 1, & \text{if } x < 0 \\ \frac{3}{x}, & \text{if } 0 < x \leq 4 \\ 6x - 8, & \text{if } x > 4 \end{cases}$$

What is the sum of all x-values where $h(x) = 7$?

(A) $\sqrt{6} + \frac{3}{7}$ **(B)** $\sqrt[3]{6} + \frac{3}{7}$ **(C)** $\frac{3}{7}$ **(D)** $\sqrt[3]{6}$ **(E)** NOTA

9. Let c and d be rational numbers such that when $x^2 + 2x + 3$ is divided into $cx^3 + dx^2 + 4$, there is no remainder. What is d?

(A) $\frac{7}{3}$ **(B)** $\frac{7}{9}$ **(C)** $\frac{13}{3}$ **(D)** $\frac{13}{9}$ **(E)** NOTA

10. A line passing through the point $(0,4)$ intersects the graph of $y = x^2 + 1$ in two distinct points. The positive difference in x-coordinates of these two points is 6. Compute the positive difference between the points' y-coordinates.

(A) $\sqrt{6}$ **(B)** $3\sqrt{6}$ **(C)** $6\sqrt{6}$ **(D)** $12\sqrt{6}$ **(E)** NOTA

11. Consider the region bounded by the graphs of the absolute value functions $f(x) = |x-2|$ and $g(x) = 4 - |x|$. Find the area of the region.

(A) 2 **(B)** 4 **(C)** 6 **(D)** 12 **(E)** NOTA

12. Suppose a and b are real numbers such that $\log_a(b) = 3$ and $\log_{3a}(3b) = 4$. What is $\log_{9a}(9b)$?

(A) 6 **(B)** 4 **(C)** 5 **(D)** 7 **(E)** NOTA

13. Which set of ordered pairs does **not** represent a function?

(A) $\{(1,2),(2,3),(4,5)\}$
(B) $\{(2,4),(3,6),(2,8)\}$
(C) $\{(0,1),(-1,2),(3,4)\}$
(D) $\{(5,6),(7,8),(9,10)\}$
(E) NOTA

14. The function $h(a,b)$ has a domain of positive integers for each argument, and gives the length of the hypotenuse of a right triangle with side lengths a and b. Find the number of ordered pairs (a,b) such that $h(a,b) = 65$.

(A) 1 **(B)** 2 **(C)** 4 **(D)** 8 **(E)** NOTA

15. For $x \neq 0$, the function is defined as:

$$g(x) = \begin{vmatrix} -x & 1 & x \\ (x+2) & 2x & 1 \\ 4 & 3 & \left(\frac{2}{x}\right) \end{vmatrix}$$

Find the sum of the values of x for which $g(x) = 0$.

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** NOTA

16. Given that the polynomials $2x^3 - 41x^2 + 125x - 102$ and $x^2 - 7x + 10$ share one root, what is the sum of all the roots of the following function?

$$f(x) = \left|2x^3 - 41x^2 + 125x - 102\right|^{(x^2-7x+10)} = 1$$

(A) 42 **(B)** 44 **(C)** 46 **(D)** 48 **(E)** NOTA

17. Which of the following functions is an example of a bijection from $f : (0,\infty) \to (0,1)$.

(A) $f(x) = 2^x$ **(B)** $f(x) = \ln(x)$ **(C)** $f(x) = \frac{x}{x+1}$ **(D)** $f(x) = \frac{x}{x-1}$ **(E)** NOTA

18. What is the sum of all integers for which the function $\ln(\sqrt{x^2-4x+3})$ is undefined?

(A) 6 **(B)** 1 **(C)** 3 **(D)** 5 **(E)** NOTA

19. Tim, living in fantasy football loser land. He has a infinitely large field with water in a straight line on one side, and is building a fence to create a rectangular dance studio so he can finally do his fantasy football punishment. If he has 20 meters of fencing to use, and one side of the dance studio must be unfenced along the water, what is the maximum dance area he can achieve?

(A) 200 **(B)** 100 **(C)** 25 **(D)** 50 **(E)** NOTA

20. A quadratic function g passes through the points $(1,3)$, $(4,7)$, and $(5,11)$. Find the value of $g(2)$.

(A) 1 **(B)** 3 **(C)** 5 **(D)** 7 **(E)** NOTA

21. If the function $f(x) = \frac{(x^2-4)(x^2-9)}{x^3-9x^2-24+26x}$ has a vertical asymptote at $x = a$, find the sum of all possible distinct values of a.

(A) 2 **(B)** 4 **(C)** 7 **(D)** 9 **(E)** NOTA

22. Find the coefficient of the x^2y term of the function $f(x,y) = (2x-5y)^3$.

(A) -60 **(B)** 60 **(C)** 20 **(D)** -20 **(E)** NOTA

23. For a set to be countably infinite, there must be a bijection between $\mathbb{N}$ and A, where A is the set you are trying to determine if it is countably infinite. How many of the following sets are countably infinite:

$$\mathbb{N}, \mathbb{Q}, \mathbb{R}, \mathbb{Z}$$

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(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** NOTA

24. Find the domain of the function $g(x) = \sqrt{2-\sqrt{2x^2+4x}}$.

(A) $(-2,2)$
(B) $[-1-\sqrt{3},-2] \cup [0,\sqrt{3}-1]$
(C) $(-2,-1+\sqrt{3})$
(D) $[-2,-1+\sqrt{3}]$
(E) NOTA

25. Find the range of the function $g(x) = \sqrt{2-\sqrt{2x^2+4x}}$.

(A) $[0,\sqrt{2}]$ **(B)** $[0,\sqrt{3}]$ **(C)** $[0,1+\sqrt{2}]$ **(D)** $[0,1-\sqrt{2}]$ **(E)** NOTA

26. What is the minimum value of the function $g(x) = x^2 + \frac{1}{x^2} + 5$ for all real numbers $x \neq 0$?

(A) $\sqrt{5}$ **(B)** $\sqrt{5}$ **(C)** $2\sqrt{5}$ **(D)** 7 **(E)** NOTA

27. What is the area formed by the shape made of the points: $(0,1), (5,25), (2,15), (2,3)$?

(A) 1 **(B)** 2 **(C)** 30 **(D)** 60 **(E)** NOTA

28. Let $f(x)$ be a function such that for all real numbers x and y,

$$f(x+y) = f(x) \cdot f(y)$$

and $f(0) = 1$ and $f(1) = 2$. What is $f(2025)$?

(A) 2^{2025} **(B)** 2^{2024} **(C)** 2^{2026} **(D)** 2^{2023} **(E)** NOTA

29. What is the sum of the real roots to $3x^2 - 4x - 5$

(A) 0 **(B)** $\frac{4}{3}$ **(C)** $-\frac{5}{3}$ **(D)** 4 **(E)** NOTA

30. If $f(x + 2025) = x^2 + 2x - 5$, find the product of the values of x for which $f(x) = 0$.

(A) 4100620 **(B)** 4096570 **(C)** 4092524 **(D)** -5 **(E)** NOTA