For all questions below, the answer E) NOTA means "None of these answers". Good luck and have fun!

1. The functions $f(x) = 6x^2 - 7x - 3$ and $g(x) = 8x^2 - 10x - 3$ have one common rational root, $x = \frac{p}{q}$, where p and q are positive coprime integers. What is $p + q$?

A: 4 B: 5 C: 6 D: 7 E: NOTA

2. How many arrangements of the letters of the word "GEMINI" are there?

A: 5040 B: 2520 C: 720 D: 360 E: NOTA

3. The year N =2025 is a perfect square. If a positive integer factor of N is selected, what is the probability that it too is a perfect square?

A: $\frac{1}{5}$ B: $\frac{4}{15}$ C: $\frac{1}{3}$ D: $\frac{2}{5}$ E: NOTA

4. What is the sum of all real solutions to $\frac{2}{x-4} + \frac{2x}{x^2-16} = 0$?

A: -6 B: -2 C: 2 D: 6 E: NOTA

5. Let $A = \begin{pmatrix} 2 & x \\ x+1 & 3 \end{pmatrix}$ and $X = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$. If $\det(A) = 0$, what is the greatest possible sum of the entries of AX?

A: 21 B: 15 C: 12 D: 0 E: NOTA

6. Peter makes the continued fraction $p = 1 + \cfrac{2}{1+\cfrac{2}{1+\cfrac{2}{\ddots}}}$ and Mary makes the continued fraction $m = 2 + \cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{\ddots}}}$. What is the value of $p - m$?

A: $1 + \sqrt{2}$ B: $2 - \sqrt{2}$ C: $1 - \sqrt{2}$ D: $2 + \sqrt{2}$ E: NOTA

7. Triangle ABC has its vertices as the intersection points of the lines whose equations are:

$$2x - y = 3, \quad x + 2y = 4, \quad 3x + y = -8$$

What is the area of ΔABC?

A: 19.5 B: 21.5 C: 25.5 D: 27.5 E: NOTA

8. Let M be the least positive integer such that $\frac{M}{2} - \frac{M}{3} + \frac{M}{4} - \frac{M}{5} + \frac{M}{6} - \frac{M}{7} + \frac{M}{8} - \frac{M}{9} + \frac{M}{10}$ is an integer. What is the sum of the digits of M?

A: 7 B: 9 C: 11 D: 13 E: NOTA

9. Which of the following polynomials has exactly 2 non-real roots?

A: $a(x) = x^4 - 4x^3 + 16x - 16$ B: $b(x) = x^4 - 4x^3 + 8x^2 - 16x + 16$

C: $c(x) = x^4 - 2x^3 + 6x^2 - 8x + 8$ D: $d(x) = x^4 - 20x^2 + 64$ E: NOTA

10. Let $f(x) = 2^x$ and define $f_n(x) = (f \cdot f_{n-1})(x)$ for $n \geq 2$ and $f_1(x) = 2^x$. Define $g_n(x) = \prod_{k=1}^{n} f_k(x)$ for $n \geq 1$. If $g_8(x) = 16^{27}$, find x.

A: 1 B: $\frac{3}{2}$ C: 2 D: 3 E: NOTA

11. If $\log_{64}(\log_2 x) = \log_8(\log_4 x)$, find $\log_4 x$.

A: $\frac{1}{2}$ B: 1 C: $\frac{3}{2}$ D: 2 E: NOTA

12. A right rectangular prism is inscribed in a sphere. The volume of the sphere is 288π. What is the maximum volume of the prism?

A: $108\sqrt{2}$ B: $164\sqrt{2}$ C: $164\sqrt{3}$ D: $192\sqrt{3}$ E: NOTA

13. What is the product of the real solutions to the equation $(x^2 - 5)^2 = 3(x^2 - 5) + 4$?

A: 36 B: -36 C: 9 D: 6 E: NOTA

14. If $\sqrt{\sqrt[3]{\sqrt{\sqrt[3]{x}}}} = \sqrt[4]{\sqrt[n]{x}}$ for all $x \geq 0$, what is the value of n?

A: 3 B: 6 C: 9 D: 81 E: NOTA

15. Consider ellipses of the form $\left(\frac{x-k}{3}\right)^2 + \left(\frac{y+k}{3+k}\right)^2 = 1$ where k is a positive integer. What is the distance between the foci for the <u>least k</u> that makes the foci have integer coordinates?

A: 4 B: 8 C: 16 D: 32 E: NOTA

16. When $(x^5 + 1)\left(2x^2 - \frac{1}{x^3}\right)^8$ is expanded, what is the coefficient of the x^{11} term?

A: 768 B: 824 C: 1024 D: 1792 E: NOTA

17. At a restaurant, there are five options for meat, 4 options for salsa, and 6 options for toppings for a burrito bowl. Jack makes his bowl by selecting two different meats, 2 different salsas, and 3 different toppings. How many different bowls are possible if he mixes together all the ingredients and doesn't care about the order in which they are selected?

A: 86,400 B: 28,800 C: 1260 D: 1200 E: NOTA

18. Let $f(n)$ be the sum of the reciprocals of the positive integer factors of n, where $n \geq 1$. Let $S = \{1, 2, 3, \ldots, 10\}$. An element s is randomly selected from S. What is the probability $f(s)$ is an integer?

A: $\frac{1}{10}$ B: $\frac{1}{5}$ C: $\frac{3}{10}$ D: $\frac{2}{5}$ E: NOTA

19. How many real solutions does $16^x = 2^{2x+1} + 8$ have?

A: 0 B: 1 C: 2 D: 3 E: NOTA

20. Define $s(n) = \sum_{k=0}^{n} z^k$, where $z = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$. If $s(2025) = a + bi$ for real a, b, find $a + b$.

A: 0 B: 1 C: $\sqrt{2}$ D: $\sqrt{2} - 1$ E: NOTA

21. Let $f(x, y, z)$ be defined as $\frac{1}{x-y} + \frac{1}{y-z} + \frac{1}{z-x}$ where x, y, z are distinct real numbers. Which of the following is equivalent to $f(x, y, z)$?

A: $\frac{x^2-xy+y^2-yz+z^2-xz}{(x-y)(y-z)(x-z)}$ B: $\frac{x^2-xy+y^2-yz+z^2-xz}{(x-y)(y-z)(z-x)}$ C: $\frac{x^2-2xy+y^2-2yz+z^2-2xz}{(x-y)(y-z)(x-z)}$

D: $\frac{x^2-2xy+y^2-2yz+z^2-2xz}{(x-y)(y-z)(z-x)}$ E: NOTA

22. Angles $\angle A$ and $\angle B$ are supplementary where $m\angle A$ is 20° less than 3 times $m\angle B$. What is the measure of the complement of $\angle B$?

A: 20° B: 30° C: 40° D: 50° E: NOTA

23. Define the region R bounded by $y = \frac{1}{4}x, x = 8, y = 0$. What is the volume of the solid when R is revolved about the y-axis?

A: $\frac{256\pi}{3}$ B: $\frac{128\pi}{3}$ C: $\frac{64\pi}{3}$ D: $\frac{32\pi}{3}$ E: NOTA

24. The area interior to the graph of $(x^2 + y^2 - 2x - 2y - 34)\left(x^2 + y^2 + 8x + 8y - 54 + 60\sqrt{2}\right) = 0$ can be written in the form $\left(p - q\sqrt{2}\right)\pi$ for positive integers p, q Find $p + q$.

A: 182 B: 162 C: 142 D: 122 E: NOTA

25. Find the area interior to the quadrilateral $MATH$ with vertices $M(0,0), A(-2,-4), T(4,-7), H(8,1)$.

A; 30 B: 35 C: 40 D: 45 E: NOTA

26. The famous Golden Ratio is denoted ϕ, where $\phi = \frac{1+\sqrt{5}}{2}$. It has many applications in architecture, biology, and music, among others! Which expression evaluates to 0?

A: $\phi^2 - \phi - 1$ B: $\phi^2 + \phi + 1$ C: $\phi^2 + \phi - 1$

D: $\phi^2 - \phi + 1$ E: NOTA

27. Given $\Delta ABC \sim \Delta DEF$ where $AB = 3x, BC = 6x - 2, AC = 5x + 2, DE = 4x + 7$, and $FD = 15x$. What is the perimeter of ΔDEF?

A: 28 B: 42 C: 70 D: 84 E: NOTA

28. A parallelogram has side lengths 4 and 6. If the length of its longer diagonal is 8, what is the length of the shorter diagonal?

A: $2\sqrt{5}$ B: $2\sqrt{10}$ C: $3\sqrt{5}$ D: $2\sqrt{7}$ E: NOTA

29. Let n be the smallest positive integer such that number of diagonals of a regular n-gon exceeds 100. What is the measure of each exterior angle of this n-gon in degrees?

A: 36 B: 30 C: 24 D: 20 E: NOTA

30. The area of an equilateral triangle is numerically equal to $2\sqrt{3}$ times its perimeter. What is the length of the apothem of the equilateral triangle?

A: 4 B: $4\sqrt{3}$ C: 6 D: $6\sqrt{3}$ E: NOTA