1. B: Factor: $f(x) = (2x-3)(3x+1)$ and $g(x) = (2x-3)(4x+1)$ so they have a common root at $x = \frac{3}{2}$. Thus, $p + q = 3 + 2 = 5$.

2. D: There are 6 letters, but there is a duplicate of I. There are $\frac{6!}{2!} = \frac{720}{2} = 360$ arrangements of the letters.

3. D: Factor $N = 2025 = (45)^2 = (3^2 \cdot 5)^2 = 3^4 \cdot 5^2$. There are $(4+1)(2+1) = 15$ positive integer factors. Since $N = (3^2 \cdot 5)^2$, any perfect square factor is of the form $(3^a \cdot 5^b)$ where $a = 0, 1, 2$ and $b = 0, 1$. Thus, there are $(3)(2) = 6$ perfect square factors, resulting in a probability of $\frac{6}{15} = \frac{2}{5}$.

4. B: Multiply by the LCD, $(x+4)(x-4)$, to get $2(x+4) + 2x = 0$ so $4x + 8 = 0$ and $x = -2$.

5. B: $\det(A) = 2(3) - x(x+1) = 6 - x^2 - x = 0$ so $x^2 + x - 6 = (x+3)(x-2) = 0$ and $x = -3, x = 2$. When $x = -3, A = \begin{pmatrix} 2 & 2 \\ 3 & 3 \end{pmatrix}$ and $AX = \begin{pmatrix} 2 & 2 \\ 3 & 3 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ 9 \end{pmatrix}$ whose sum of entries is 15. When $x = -3, A = \begin{pmatrix} 2 & -3 \\ -2 & 3 \end{pmatrix}$ and $AX = \begin{pmatrix} 2 & -3 \\ -2 & 3 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ whose sum of entries is 0. The greatest sum of entries is 15.

6. C: $p = 1 + \frac{2}{1+\frac{2}{1+\frac{2}{\cdots}}}$ so $\frac{p-1}{2} = \frac{1}{1+\frac{2}{1+\frac{2}{\cdots}}}$. Reciprocating both sides, $\frac{2}{p-1} = 1 + \frac{2}{1+\frac{2}{1+\frac{2}{\cdots}}} = p$. Thus, $2 = p(p-1)$ and $p^2 - p - 2 = 0$. Factoring, $(p-2)(p+1) = 0$ so $p = 2, -1$. Since all the terms in the continued fraction are positive, $p = 2$. Similarly, $m = 2 + \frac{1}{2+\frac{1}{2+\frac{1}{\cdots}}}$ so $m - 2 = \frac{1}{2+\frac{1}{2+\frac{1}{\cdots}}}$ and $\frac{1}{m-2} = m$. Thus, $1 = m(m-2)$ or $m^2 - 2m - 1 = 0$. Using the Quadratic Formula, $m = \frac{2\pm\sqrt{8}}{2} = 1 \pm \sqrt{2}$. Since all the terms are positive, $m = 1 + \sqrt{2}$. Thus, $p - m = 2 - (1 + \sqrt{2}) = 1 - \sqrt{2}$.

7. E: Find the mutual points of intersection of each pair of lines to be $A(-4,4), B(2,1), C(-1,-5)$. There are many ways to find the area of the triangle. The easiest is probably the shoelace method:
Area = $\frac{1}{2}\left(abs\left(\begin{vmatrix} -4 & 2 \\ 4 & 1 \end{vmatrix}\right) + abs\left(\begin{vmatrix} 2 & -1 \\ 1 & -5 \end{vmatrix}\right) + abs\left(\begin{vmatrix} -1 & -4 \\ -5 & 4 \end{vmatrix}\right)\right) = \frac{1}{2}(12 + 9 + 24) = \frac{1}{2}(45) = 22.5$.

8. B: $lcm(2,3,4,5,6,7,8,9,10) = 2^3 \cdot 3^2 \cdot 5 \cdot 7 = 2520$. The sum of the digits is $2 + 5 + 2 + 0 = 9$.

9. B: Factor each polynomial with the help of the Rational Root Theorem. We get:

$a(x) = (x-2)^3(x+2)$: 4 real roots
$b(x) = (x-2)^2(x^2+4)$: 2 real roots, 2 non-real roots
$c(x) = (x^2+4)(x^2-2x+2)$: 4 non-real roots
$d(x) = (x+2)(x-2)(x+4)(x-4)$: 4 real roots

Thus, $b(x)$ is the only polynomial with exactly 2 non-real roots.

10. D: Note that $f_1(x) = 2^x, f_2(x) = (2^x)^2 = 2^{2x}, f_3(x) = 2^x \cdot 2^{2x} = 2^{3x}$, and so on. One can prove that $f_n(x) = 2^{nx}$ by induction. Then $g_1(x) = 2^x, g_2(x) = 2^x \cdot 2^{2x} = 2^{3x}, g_3(x) = 2^{3x} \cdot 2^{3x} = 2^{6x}$, etc. In general, $g_n(x) = 2^{x+2x+3x+\cdots+nx} = 2^{(x+nx)\cdot\frac{n}{2}} = 2^{\frac{n(n+1)}{2}x}$. Now $g_8(x) = 2^{36\,x} = 4^{18x} = 16^{9x}$. So $16^{9x} = 16^{27}$ and $x = 3$.

11. D: Exponentiate both sides with base 64: $64^{\log_{64}(\log_2 x)} = 64^{\log_8(\log_4 x)}$. The left side becomes $\log_2 x$. Since $64 = 8^2$, the right sand becomes $8^{2\log_8(\log_4 x)} = 8^{\log_8((\log_4 x)^2)} = (\log_4 x)^2$. By Change of Base, $\log_4 x = \frac{\log_2 x}{\log_2 4} = \frac{\log_2 x}{2}$. Thus, $\log_2 x = \left(\frac{\log_2 x}{2}\right)^2 = \frac{1}{4}(\log_2 x)^2$. Let $m = \log_2 x$. Then $m = \frac{1}{4}m^2$. So $0 = m(\frac{1}{4}m - 1)$ and $m = 0, m = 4$. Then $x = 2^0 = 1$ or $x = 2^4 = 16$. But $x = 1$ is extraneous in the original equation since $\log_2 1 = 0$ and $\log_{64} 0$ is undefined. Then $\log_4 16 = 2$.

12. D: Solve $\frac{4}{3}\pi r^3 = 288\pi$ to get $r = 6$ so the diameter of the sphere is $2r = 12$. By symmetry, the maximum volume occurs when the prism is a cube. The space diagonal of the cube is $s\sqrt{3} = 12$, where s is the side length of the cube. Then $s = \frac{12}{\sqrt{3}} = 4\sqrt{3}$. Therefore, the maximum volume is $V = s^3 = 192\sqrt{3}$.

13. A: Let $m = x^2 - 5$ so that $m^2 - 3m - 4 = (m-4)(m+1) = 0$ and $m = 4, -1$. Therefore, $x^2 - 5 = 4$ or $x^2 - 5 = -1$. Thus, $x^2 = 9$ or $x^2 = 4$. Therefore, $x = \pm 3, \pm 2$. The product of the solutions is $(3)(-3)(2)(-2) = 36$.

14. C: $\sqrt{\sqrt[3]{\sqrt{\sqrt[3]{x}}}} = \left(\left(\left(x^{\frac{1}{3}}\right)^{\frac{1}{2}}\right)^{\frac{1}{3}}\right)^{\frac{1}{2}} = x^{\frac{1}{3}\cdot\frac{1}{2}\cdot\frac{1}{3}\cdot\frac{1}{2}} = x^{\frac{1}{36}}$. $\sqrt[4]{\sqrt[n]{x}} = \left(x^{\frac{1}{n}}\right)^{\frac{1}{4}} = x^{\frac{1}{n}\cdot\frac{1}{4}} = x^{\frac{1}{4n}}$. $\frac{1}{36} = \frac{1}{4n}$, so $n = 9$.

15. B: Here, $a^2 = (3+k)^2, b^2 = 3^2$ and $c^2 = a^2 + b^2 = (3+k)^2 - 3^2 = 6k + k^2$. Thus, $c = \sqrt{k^2 + 6k}$ must be a positive integer. Then $k^2 + 6k - c^2 = 0$ so $k = -3 + \sqrt{3^2 + c^2}$. It is sufficient for $3^2 + c^2$ to be a perfect square. Since $3^2 + 4^2 = 5^2$ is a familiar triple, the least c is 4. The distance between the foci is $2c = 8$.

16. A: To get x^{11}, we could have $x^5 \cdot x^6$ or $1 \cdot x^{11}$. The x^6 term in the binomial expansion is $\binom{8}{2}(2x^2)^6\left(-\frac{1}{x^3}\right)^2 = 28(2^6)x^6 = 1792\,x^6$. The x^{11} term is $\binom{8}{1}(2x^2)^7\left(-\frac{1}{x^3}\right)^1 = -8(128)x^{11} = -1024x^{11}$. Thus, the x^{11} term is $(1792 - 1024)x^{11} = 768x^{11}$.

17. D: There are $\binom{5}{2}\cdot\binom{4}{2}\cdot\binom{6}{3} = 10 \cdot 6 \cdot 20 = 1200$ different bowls that can be made.

18. B: We compute $f(s)$ for each $s = 1, 2, \ldots, 10$ to see if it is an integer:
$f(1) = \frac{1}{1} = 1$ ✅
$f(2) = \frac{1}{1} + \frac{1}{2} = \frac{3}{2}$ ❌
$f(3) = \frac{1}{1} + \frac{1}{3} = \frac{4}{3}$ ❌
$f(4) = \frac{1}{1} + \frac{1}{2} + \frac{1}{4} = \frac{7}{4}$ ❌
$f(5) = \frac{1}{1} + \frac{1}{5} = \frac{6}{5}$ ❌
$f(6) = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{6} = 2$ ✅
$f(7) = \frac{1}{1} + \frac{1}{7} = \frac{8}{7}$ ❌
$f(8) = \frac{1}{1} + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} = \frac{15}{8}$ ❌
$f(9) = \frac{1}{1} + \frac{1}{3} + \frac{1}{9} = \frac{13}{9}$ ❌
$f(10) = \frac{1}{1} + \frac{1}{2} + \frac{1}{5} + \frac{1}{10} = \frac{9}{5}$ ❌
Thus, there are two cases that work out of 10 for a probability of $\frac{2}{10} = \frac{1}{5}$.

19: B: Write as $(2^x)^4 - 2(2^x)^2 - 8 = 0$ or $(4^x)^2 - 2(4^x) - 8 = 0$. Let $m = 4^x$. Factor $m^2 - 2m - 8 = 0$ as $(m-4)(m+2) = 0$ so $m = 4, m = -2$. Thus, $4^x = 4$ when $x = 1$ and $4^x = -2$ is impossible. There is only one real solution.

20. B: Consider the first few powers of z, remembering $i^2 = -1$:

$z^0 = 1$

$z^1 = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$

$z^2 = \left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)^2 = \frac{1}{2} - i - \frac{1}{2} = -i$

$z^3 = \left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)^3 = -i\left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$

$z^4 = \left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)^4 = (z^2)^2 = (-i)^2 = -1$

Since $z^4 = -1$, then $z^8 = 1$ showing that powers of z cycle with a period of 8. Note by symmetry, $\sum_{k=0}^{7} z^k = 0$. Since 2025 leaves a remainder of 1 upon dividing by 8, then $s(2025) = z^0 + z^1$. This gives $1 - \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$. So $a + b = 1 - \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = 1$.

21. A: The LCD is $(x-y)(y-z)(z-x)$. Thus, $f(x,y,z) = \frac{(y-z)(z-x)}{LCD} + \frac{(x-y)(z-x)}{LCD} + \frac{(x-y)(y-z)}{LCD}$. Expand the numerator to get $yz - z^2 - xy + xz + xz - yz - x^2 + xy + xy - y^2 - xz + yz$. This simplifies to $xy + yz + xz - (x^2 + y^2 + z^2)$. If we instead make the LCD $(x-y)(y-z)(x-z)$ then we multiply the numerator by -1. This gives $\frac{x^2 - xy + y^2 - yz + z^2 - xz}{(x-y)(y-z)(x-z)}$.

22. C: Given $A = 3B - 20$ and $A + B = 180$. Substitute to find $4B - 20 = 180$ so $B = 50$. The complement measures $90 - 50 = 40$.

23. A: Note $y = \frac{1}{4}x$ and $x = 8$ intersect at $y = 2$. The solid will be a cylinder of radius 8 and height 2 with a cone removed of the same dimensions. The volume is then $\pi(8)^2(2) - \frac{1}{3}\pi(8)^2(2) = \frac{2}{3}\pi(64)(2) = \frac{256\pi}{3}$.

24. A: Either factor could be zero. Complete the square for $x^2 + y^2 - 2x - 2y - 34 = 0$ to write $(x-1)^2 + (y-1)^2 = 36$. This is a circle centered at $(1,1)$ of radius 6. Complete the square for $x^2 + y^2 + 8x + 8y - 54 + 60\sqrt{2} = 0$ to write $(x+4)^2 + (y+4)^2 = 86 - 60\sqrt{2}$. This is a circle centered at $(-4,-4)$ of radius $\sqrt{86 - 60\sqrt{2}}$. Note both centers are contained on the line $y = x$. Substitute $y = x$ into the first equation to find $2y^2 - 4y - 34 = 0$ or $y^2 - 2y - 17 = 0$. By the Quadratic Formula, $y = \frac{2 \pm \sqrt{72}}{2} = 1 \pm 3\sqrt{2}$. Thus, $(1 + 3\sqrt{2}, 1 + 3\sqrt{2})$ and $(1 - 3\sqrt{2}, 1 - 3\sqrt{2})$ lie on the diameter of the first circle. Substitute $y = x$ into the second equation to find $2y^2 + 16y - 54 + 60\sqrt{2} = 0$. When $y = 1 + 3\sqrt{2}$, $2\left(1 + 3\sqrt{2}\right)^2 + 16\left(1 + 3\sqrt{2}\right) - 54 + 60\sqrt{2}$ expands to $2 + 12\sqrt{2} + 36 + 16 + 48\sqrt{2} - 54 + 60\sqrt{2} = 120\sqrt{2} \neq 0$. But when $y = 1 - 3\sqrt{2}$, $2\left(1 - 3\sqrt{2}\right)^2 + 16\left(1 - 3\sqrt{2}\right) - 54 + 60\sqrt{2} = 2 - 12\sqrt{2} + 36 + 16 - 48\sqrt{2} - 54 + 60\sqrt{2} = 0$. This shows that $(1 - 3\sqrt{2}, 1 - 3\sqrt{2})$ is the sole point of intersection making the circles tangent. The area contained is just the union of the two circle's areas: $36\pi + \left(86 - 60\sqrt{2}\right)\pi = \left(122 - 60\sqrt{2}\right)\pi$. Thus, $p + q = 122 + 60 = 182$.

25. D: Consider the following slopes: $MA = 2, AT = -\frac{1}{2}, TH = 2$. Due to opposite reciprocal slopes, there are right angles at A and T. By the Distance Formula: $MA = 2\sqrt{5}, AT = 3\sqrt{5}, TH = 4\sqrt{5}$. The area of the trapezoid is $\frac{1}{2}\left(2\sqrt{5} + 4\sqrt{5}\right)\left(3\sqrt{5}\right) = \left(3\sqrt{5}\right)^2 = 45$.

26. A: Note $2\phi - 1 = \sqrt{5}$ so $4\phi^2 - 4\phi + 1 = 5$ or $4\phi^2 - 4\phi - 4 = 0$. Divide by 4 to get $\phi^2 - \phi - 1$.

27. C: Setup a proportion comparing corresponding sides: $\frac{3x}{5x+2} = \frac{4x+7}{15x}$. Cross-multiply to get $45x^2 = 20x^2 + 43x + 14$ or $25x^2 - 43x - 14 = 0$. Factor as $(25x + 7)(x - 2) = 0$ so $x = -\frac{7}{25}$ or $x = 2$. Since side lengths are positive, we take $x = 2$. Thus, the three side lengths of ΔABC are $6, 10,$ and 12 for a perimeter of 28. Since $DE = 15$, the scale factor is $\frac{15}{6} = 2.5$ so the perimeter of ΔDEF is $2.5(28) = 70$.

28. B: Let θ be the measure of the interior angle opposite the long diagonal. By the Law of Cosines: $8^2 = 4^2 + 6^2 - 2 \cdot 4 \cdot 6\cos(\theta)$. Simplifying, $12 = -48\cos(\theta)$ so $\cos(\theta) = -\frac{1}{4}$. Since consecutive angles in a parallelogram are supplementary, the angle opposite the short angle measures $180° - \theta$. Note that $\cos(180° - \theta) = -\cos(\theta)$. Letting d be the short diagonal length, $d^2 = 4^2 + 6^2 - 2 \cdot 4 \cdot 6\left(\frac{1}{4}\right)$. Simplifying, $d^2 = 40$ so $d = \sqrt{40} = 2\sqrt{10}$.

29. E: The number of diagonals is $n - \binom{n}{2} = n - \frac{n(n-1)}{2} = \frac{n(n-3)}{2}$. Set $\frac{n(n-3)}{2} = 100$ to get $n^2 - 3n - 200 = 0$. By the Quadratic Formula, $n = \frac{3+\sqrt{809}}{2}$. Note $28 = \sqrt{784} < \sqrt{809} < \sqrt{841} = 29$ so $n \geq \frac{3+29}{2} = 16$ to have more than 100 diagonals. Each exterior angle measures $\frac{360}{16} = 22.5$.

30. B: The area is $\frac{\sqrt{3}}{4}s^2$ and the perimeter is $3s$. Thus, $\frac{\sqrt{3}}{4}s^2 = 2\sqrt{3}(3s) = 6\sqrt{3}s$. Since $s \neq 0$, $\frac{\sqrt{3}}{4}s = 6\sqrt{3}$ so $s = 24$. Then an altitude of the equilateral triangle is $\frac{s}{2}\sqrt{3} = 12\sqrt{3}$ and the apothem is $\frac{1}{3}$ the altitude, or $\frac{1}{3}\left(12\sqrt{3}\right) = 4\sqrt{3}$.