All uppercase letter variables are positive integers unless otherwise stated. All fractions containing uppercase letter variables are in lowest terms. NOTA means "None of the Above."

~~~~~~ Good Luck, and have fun! ~~~~~~

1) Which of the following is not a semantically valid form of classical logic? You may take $\vDash$ to mean "therefore" and $\equiv$ to mean "is equivalent to".
   A. $p \vDash (p \vee q)$
   B. $\big((p \vee q) \wedge \neg p\big) \vDash q$
   C. $(p \rightarrow q) \equiv (\neg p \vee q)$
   D. $\neg(p \wedge q) \equiv (\neg p \wedge \neg q)$
   E. NOTA

For questions $2-3$, you may use the following information.
A Heronian triangle has integer side lengths and integer area. Define a "double-right triangle" as a Heronian triangle containing an altitude that splits a side into two pieces, both with positive integer length.

2) Find the area of the $17-25-28$ double-right triangle.
   A. 210 B. 221 C. 250 D. 252 E. NOTA

3) Two sides of a double-right triangle both have length 5. Let L be the length of the third side. Find the sum of all possible values of L.
   A. 6 B. 8 C. 12 D. 14 E. NOTA

4) Triangle ABC has side lengths 5, 7, and 8. Which of the following could be the measure of angle A?
   A. $30°$ B. $60°$ C. $90°$ D. $120°$ E. NOTA

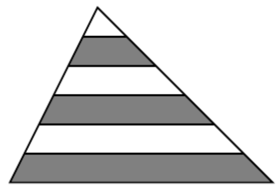
For questions $5-6$, you may use the following information.
Rectangle $ABCD$ has right triangle BIR inscribed in it such that R is on $\overline{AD}$, I is on $\overline{CD}$, $BI = IR = 2$, and $DR = 1$.

5) Find the area of triangle IDR.
   A. $\frac{\sqrt{2}}{2}$ B. $\frac{\sqrt{3}}{2}$ C. $\sqrt{2}$ D. $\sqrt{3}$ E. NOTA

6) Find the area of rectangle $ABCD$.
   A. $1+\sqrt{3}$ B. $3+\sqrt{3}$ C. $1+3\sqrt{3}$ D. $3+3\sqrt{3}$ E. NOTA

7) Find $\sin 15°$

A. $\frac{\sqrt{6}-\sqrt{2}}{4}$ B. $\frac{\sqrt{6}}{4}$ C. $\frac{\sqrt{3}+1}{4}$ D. $\frac{\sqrt{6}+\sqrt{2}}{4}$ E. NOTA



8) A triangle with area 36 is divided into six regions by equally spaced, parallel lines, as shown above. Find the area of the shaded region.

A. 19 B. 20 C. 21 D. 22 E. NOTA

For questions $9-13$, you may use the following information.
Right triangle ABC has $AB = 6$, $BC = 8$, and a right angle at B.

9) Find $\tan \angle A$.

A. $\frac{3}{5}$ B. $\frac{3}{4}$ C. $\frac{4}{5}$ D. $\frac{4}{3}$ E. NOTA

10) Square $BXYZ$ is inscribed in ABC. Find BX.

A. $\frac{24}{7}$ B. 4 C. $\frac{24}{5}$ D. 5 E. NOTA

11) Circle O is inscribed in ABC. Find its radius.

A. $\frac{12}{7}$ B. 2 C. $\frac{12}{5}$ D. $\frac{5}{2}$ E. NOTA

12) ABC is inscribed in square $CX'Y'Z'$. Find CX'.

A. $\frac{18}{\sqrt{17}}$ B. $\frac{24}{\sqrt{17}}$ C. $\frac{28}{\sqrt{17}}$ D. $\frac{32}{\sqrt{17}}$ E. NOTA

13) Mutually tangent circles with centers A, B, and C are drawn. Find the total area of the 3 circles.

A. 48π B. 50π C. 56π D. 64π E. NOTA

14) Segments $\overline{AD}$ and $\overline{BC}$ intersect at O. If $AB = BO$, $CO = DO$, and $\angle ABO = 40°$, find $\angle DCO$.

A. $40°$ B. $55°$ C. $70°$ D. $80°$ E. NOTA

15) a is a uniformly randomly selected real in the range $(0,180)$. b is a uniformly randomly selected real in the range $(0,180-a)$. Let $c = 180 - a - b$. Find the probability that a triangle with angle measures $a°$, $b°$, and $c°$ is obtuse.

A. $\frac{1}{4}$ B. $\frac{1}{2}$ C. $\frac{3}{4}$ D. $\frac{7}{8}$ E. NOTA

16) Find the probability that a regular N-sided polygon (N from 3 to 10, inclusive) has internal angles whose measures are an integer number of degrees.

A. $\frac{3}{4}$ B. $\frac{5}{6}$ C. $\frac{6}{7}$ D. $\frac{7}{8}$ E. NOTA

17) A floor is tiled infinitely by squares with side length 1. A coin with diameter $\frac{1}{2}$ is tossed onto the floor. Find the probability that the coin lands entirely within one of the squares.

A. 0 B. $\frac{1}{8}$ C. $\frac{1}{4}$ D. $\frac{1}{2}$ E. NOTA

For questions $18-19$, you may use the following information.
102 distinct pairwise non-parallel lines are drawn in the plane, dividing it into 5051 distinct regions.

18) Find the total number of (not necessarily distinct) points of intersection between two lines.

A. 5151 B. 5202 C. 10201 D. 10302 E. NOTA

19) Find the number of regions that are fully enclosed (have finite area).

A. 4847 B. 4898 C. 4949 D. 5000 E. NOTA

20) The barbell Ryuko bench-presses can be modeled as two triangles in the coordinate plane. One weight can be represented by a triangle with vertices at $(-8,8)$, $(-8,15)$, and $(-2,7)$, and the other weight can be represented by a triangle with vertices at $\left(3,-\frac{19}{3}\right)$, $\left(11,-\frac{19}{3}\right)$, and $\left(10,\frac{2}{3}\right)$. Assuming the weights have uniform density and the bar is massless, find the center of mass of the barbell.

A. $(1,2)$ B. $(1,3)$ C. $(2,2)$ D. $(2,3)$ E. NOTA

For questions $21-22$, you may use the following information.
Line ℓ_1 has slope $\sqrt{3}$, and line ℓ_2 has slope $\frac{1}{\sqrt{3}}$. The acute angles formed by their intersection is bisected by ℓ_3, and the obtuse angles formed by their intersection is bisected by ℓ_4.

21) Find the product of the slopes of lines ℓ_3 and ℓ_4.

A. -3 B. $-\sqrt{3}$ C. -1 D. $-\frac{1}{\sqrt{3}}$ E. NOTA

22) Find the slope of line ℓ_3.

A. $\frac{3}{2\sqrt{3}}$ B. 1 C. $\frac{2}{\sqrt{3}}$ D. $\frac{5}{2\sqrt{3}}$ E. NOTA

23) Find the area of a regular hexagon with side length 6.

A. $36\sqrt{3}$ B. $48\sqrt{3}$ C. $54\sqrt{3}$ D. $72\sqrt{3}$ E. NOTA

24) Square $ABCD$ has side length 2. Congruent circles X, Y, and Z are drawn such that Z is tangent to $\overline{CD}$, X is tangent to Z and sides $\overline{AB}$ and $\overline{AD}$, and Y is tangent to Z and sides $\overline{AB}$ and $\overline{BC}$. Find the common radii of the circles.

A. $\frac{1}{2}$ B. $9 - 6\sqrt{2}$ C. $6 - \sqrt{30}$ D. $5 - 2\sqrt{5}$ E. NOTA

25) Find the minimum possible distance between the origin and the plane passing through the points $(1,0,0)$, $(0,2,0)$, and $(0,0,8)$.

A. $\frac{1}{9}$ B. $\frac{2}{9}$ C. $\frac{4}{9}$ D. $\frac{8}{9}$ E. NOTA

26) Find the number of space diagonals in a regular dodecahedron, which contains 12 pentagonal faces, 20 vertices, and 30 edges.

A. 60 B. 80 C. 90 D. 100 E. NOTA

27) Find the volume of a regular tetrahedron with side length 6.

A. $18\sqrt{2}$ B. $18\sqrt{3}$ C. $36\sqrt{2}$ D. $36\sqrt{3}$ E. NOTA

28) An inverted cone has a height of 12 and a base diameter of 8. 47π cubic units of water are poured into the cone. The height of the water is h, and the radius of the surface of the water is r. It can be said that $k_1 r^2 h\pi = k_2 r^3 \pi = k_3 h^3 \pi = 47\pi$. Find $\frac{k_2}{k_1 k_3}$.

A. 192 B. 144 C. 96 D. 81 E. NOTA

29) Which of the following triangle centers can lie outside the triangle?

A. Centroid
B. Circumcenter
C. Incenter
D. Lemoine point
E. NOTA

30) Find the number of distinct arrangements of the word GEOMETRY.

A. 40320 B. 20160 C. 10080 D. 6720 E. NOTA