1) D
2) A
3) D
4) B
5) B

6) B
7) A
8) C
9) D
10) A

11) B
12) D
13) C
14) B
15) C

16) D
17) C
18) A
19) A
20) C

21) C
22) B
23) C
24) D
25) D

26) D
27) A
28) D
29) B
30) B

1) The correct application of De Morgan's Theorem is $\neg(p \wedge q) \equiv (\neg p \vee \neg q)$. The other laws are, respectively, Addition, Disjunctive Syllogism, and Material Implication. **[D]**

2) Recognizing 17 and 25 as hypotenuses of $8-15$ and $15-20$ right triangles, the common leg length is 15 and the base of the triangle is 28. $\frac{15\cdot 28}{2} = 210$. **[A]**

3) 5 is the hypotenuse of the $3-4-5$ right triangle, so the common leg length can be 3 or 4 and the possible third sides of the triangle are $2\cdot 3 = 6$ or $2\cdot 4 = 8$. $6+8=14$. **[D]**

4) Drawing a picture can help determine which angle is most likely to be relevant. Using the Law of Cosines on the middle angle, $\cos\theta = \frac{25+64-49}{2\cdot 5\cdot 8} = \frac{40}{80} = \frac{1}{2}$. $\theta = 60°$. **[B]**

5) IDR is a right triangle with $\angle D = 90°$, so $DI = \sqrt{2^2 - 1^2} = \sqrt{3}$. The other leg of the triangle is given as having length 1, so the area is $\frac{\sqrt{3}\cdot 1}{2} = \frac{\sqrt{3}}{2}$. **[B]**

6) Let $AB = x$ and $BC = y$. Since $\angle I = 90°$, $RDI \sim ICB$, giving $BC = \sqrt{3}CI$ or $y = (x-\sqrt{3})\sqrt{3} = x\sqrt{3} - 3$. From right triangle BAR, $x^2 + (y-1)^2 = 8$, so $x^2 + \left(x\sqrt{3} - 4\right)^2 = 8$. Simplifying, $x^2 + 3x^2 - 8x\sqrt{3} + 16 = 8$, or $x^2 - 2x\sqrt{3} - 2 = 0$. Using the quadratic formula, $x = \frac{2\sqrt{3}\pm\sqrt{12-8}}{2} = \sqrt{3} \pm 1$. The lower solution results in negative y, so $x = \sqrt{3}+1$, $y = 3+\sqrt{3} - 3 = \sqrt{3}$, and the area of $ABCD$ is $3+\sqrt{3}$. **[B]**

7) A diagram from the previous two problems can still be used! $\angle IBR = 45°$ and $\angle CBI = 30°$, so $\angle ABR = 15°$. Since sine is opposite divided by hypotenuse, $\sin\angle ABR = \frac{\sqrt{3}-1}{2\sqrt{2}} = \frac{\sqrt{6}-\sqrt{2}}{4}$. **[A]**

8) Let the triangle be ABC, and let the extra drawn lines (from top to bottom) be B_kC_k for k from 1 to 5. All AB_kC_k are similar to ABC and have height $\frac{k}{6}$ that of ABC and thus area ratio $\frac{k^2}{36}$. Subtracting and adding in back areas, the shaded region is $ABC - AB_5C_5 + AB_4C_4 - \cdots AB_1C_1 = 36 - 25 + 16 - \cdots - 1 = 21$. **[C]**

9) $\tan\angle A = \frac{BC}{AB} = \frac{8}{6} = \frac{4}{3}$. **[D]**

10) $ABC \sim AZY \sim YXC$. Let the square have side length ℓ. Then $AZ = 6 - \ell$ by length subtraction and $AZ = \frac{3\ell}{4}$ by similar triangles. Setting these equal gives $\ell = \frac{24}{7}$. **[A]**

11) The inradius and semiperimeter of a triangle multiply to give the area. Here, that gives $24 = 12r$, so $r = 2$. **[B]**

12) Similar to in Question 10, let $AY' = x$ and $BY' = y$. Then by similar triangles, $CZ' = \frac{4y}{3}$ and $BZ' = \frac{4x}{3}$; the former is the side length of the square, so $\frac{4x}{3} + y = \frac{4y}{3}$ and $y = 4x$. $AY'B$ is a right triangle with leg lengths x and y and hypotenuse 6, so $x^2 + 16x^2 = 36$ and $x = \frac{6}{\sqrt{17}}$. $\frac{4y}{3} = \frac{32}{\sqrt{17}}$. **[D]**

13) The sum of the radii of any pair of circles gives a side of ABC, so the system is $r_1 + r_2 = 6$, $r_2 + r_3 = 8$, $r_3 + r_1 = 10$. Adding all these together and dividing by 2 gives $r_1 + r_2 + r_3 = 12$, so the radii are 6, 4, and 2. The sum of the areas of these circles is $36\pi + 16\pi + 4\pi = 56\pi$. **[C]**

14) $\angle A = \angle AOB = \frac{180^\circ - 40^\circ}{2} = 70^\circ$. By vertical angles, $\angle BOC = 70^\circ$. $\angle C = \angle D = \frac{180^\circ - 70^\circ}{2} = 55^\circ$. **[B]**

15) Geometric probability can be used. The universe is the triangle bounded by the origin, $(180,0)$, and $(0,180)$. A point's x-coordinate represents a, and its y-coordinate represents b; these sum to less than 180, so the difference is c. If either $x > 90$ or $y > 90$, then the triangle is obtuse because of that respective angle. If $x + y < 90$, then $c > 90$ and the triangle is obtuse again. These are 3 of 4 congruent triangles that make up the universe, so the probability a triangle is obtuse is $\frac{3}{4}$. **[C]**

16) The external angle is $\frac{360^\circ}{N}$ and is an integer iff the internal angle is an integer. Only $N = 7$ produces a non-integer, so the probability is $\frac{7}{8}$. **[D]**

17) The center of the coin must be a distance of $\frac{1}{4}$ from the nearest gridline. This puts it in a square of side length $\frac{1}{2}$ and area $\frac{1}{4}$. **[C]**

18) Two lines can be selected to produce an intersection. The number of ways to do this is $\binom{102}{2} = \frac{102 \cdot 101}{2} = 101 \cdot 51 = 5151$. **[A]**

19) Draw a circle that contains all of the intersections. Each drawn line intersects this circle twice, for a total of 204 intersections of the lines with the circle. The arc between any two consecutive of these points on the circle represents the closure of a previously unbounded region. Thus, there are 204 unbounded regions, and the number of regions with finite area is $5051 - 204 = 4847$. **[A]**

20) Noticing that the first two points give a vertical segment, the base of the first triangle is 7 and its height is 6, so its area is 21. Its centroid is the average of its coordinates, $(-6,10)$. For the second triangle, the first two points give a horizontal segment, so its base is 8 and its height is 7 for an area of 28. Its centroid is $(8,-4)$. The centroid of the barbell as a whole is the area-weighted average of these two centroids. $\frac{21\cdot(-6,10)+28\cdot(8,-4)}{49} = \frac{(-18,30)+(32,-16)}{7} = \frac{(14,14)}{7} = (2,2)$. **[C]**

21) ℓ_3 and ℓ_4 are perpendicular. For any pair of perpendicular lines not parallel to the axes, the product of their slopes is -1. **[C]**

22) Line ℓ_1 represents a line with a 60-degree angle to the origin, and line ℓ_2 respresents a line with a 30-degree angle to the origin. Their bisector would have a 45-degree angle to the origin and thus slope 1. **[B]**

23) A hexagon consists of six equilateral triangles with the same side length. $\frac{3\cdot 6^2\sqrt{3}}{2} = 54\sqrt{3}$. **[C]**

24) Let the centers of the circles be O_x, O_y, and O_z. The altitude from O_z to O_xO_y forms a right triangle with hypotenuse $2r$. The height of the right triangle is $2-2r$. O_xO_y is also $2-2r$, and since it is also twice the base of the right triangle, the other leg has length $1-r$. $(1-r)^2 + (2-2r)^2 = (2r)^2$. Simplifying, $5(1-r)^2 = (2r)^2$, so $(1-r)\sqrt{5} = 2r$ and $r = 5-2\sqrt{5}$. **[D]**

25) An equation for the plane is $x + \frac{y}{2} + \frac{z}{8} - 1 = 0$. The distance from the origin to this is $\frac{|0+0+0-1|}{\sqrt{1+\frac{1}{4}+\frac{1}{64}}} = \frac{1}{\sqrt{\frac{81}{64}}} = \frac{8}{9}$. **[D]**

26) A dodecahedron has $\binom{20}{2} = 190$ connections between vertices. 30 of these are edges. Each of the faces contains $\frac{5\cdot 2}{2} = 5$ face diagonals. The number of space diagonals is $190 - 30 - 12\cdot 5 = 100$. **[D]**

27) The base is an equilateral triangle with side length 6 and thus area $\frac{6^2\sqrt{3}}{4} = 9\sqrt{3}$. The distance from the midpoint of the side of an equilateral triangle to its center is $\frac{s}{2\sqrt{3}}$, which here is $\sqrt{3}$. The altitude of an equilateral triangle is $\frac{s\sqrt{3}}{2}$, which here is $3\sqrt{3}$. The height of the tetrahedron is the other leg of a right triangle that has one leg as the distance to the center and the hypotenuse as the altitude. Solving $\left(\sqrt{3}\right)^2 + h^2 = \left(3\sqrt{3}\right)^2$ gives $h^2 = 27 - 3 = 24$ and $h = 2\sqrt{6}$. The volume of the pyramid is $\frac{9\sqrt{3}\cdot 2\sqrt{6}}{3} = 18\sqrt{2}$. **[A]**

28) The radius is 4. Using similar triangles, $h = 3r$, or $r = \frac{h}{3}$. The volume of the water doesn't matter. Substituting these into $V = \frac{\pi r^2 h}{3}$ gives $V = \pi r^3 = \frac{\pi h^3}{27}$. $k_1 = \frac{1}{3}$, $k_2 = 1$, and $k_3 = \frac{1}{27}$. **[D]**

29) The circumcenter, which is the intersection of the perpendicular bisectors of the sides, is outside the triangle if the triangle is obtuse. **[B]**

30) There are eight letters in GEOMETRY with one repeated E so $\frac{8!}{2!} = 20160$. **[B]**