Important Instructions For This Test: ”In simplest form” means the numerator and denominator are relatively prime to each other, and ”square-free” means the maximum power of a divisor of that integer is 1, for example: 14 is square-free but not 12.

Good luck, have fun, and as always: “NOTA” stands for “None of These Answers is correct.”

1. Consider square $ABCD$, $AC = 20$, compute the area of $ABCD$.

   (A) 200 **(B)** 250 **(C)** $200\sqrt{2}$ **(D)** 400 **(E)** NOTA

2. Consider rectangle $ABCD$, let AC and BD meet at K, given $AK = 10$, find the maximum possible area of $ABCD$.

   (A) 50 **(B)** 100 **(C)** 150 **(D)** 200 **(E)** NOTA

3. Consider rhombus $ABCD$ with $AB = AC = 6$, find the area of $ABCD$.

   (A) $9\sqrt{3}$ **(B)** $18\sqrt{3}$ **(C)** 36 **(D)** 18 **(E)** NOTA

4. Consider square $ABCD$, point E lies inside the square such that ABE is equilateral. Find the measure of $\angle BEC$ in degrees.

   (A) 30 **(B)** 45 **(C)** 60 **(D)** 75 **(E)** NOTA

5. Consider squares $ABCD$ and $CEFG$ with $AB = 2, EF = 5$. (The squares don't share a common region) Given that B, C, G lie on the same line and C, D, E lie on the same line, compute the area of triangle ACF.

   (A) 5 **(B)** 10 **(C)** $2\sqrt{5}$ **(D)** $5\sqrt{2}$ **(E)** NOTA

6. Consider the parallelogram $ABCD$ with $AC \perp BD$. If $BD = \sqrt{3}AC, AB = 5$. Find the area of the parallelogram; the answer is in the simplest form of $\frac{a\sqrt{b}}{c}$, calculate $a + b + c$.

   (A) 30 **(B)** 28 **(C)** 27 **(D)** 25 **(E)** NOTA

7. Consider a convex quadrilateral $ABCD$ such that $\angle A = 130°, \angle C = 115°, AB = AD = 25$. Find the length of AC.

   (A) $25\sqrt{3}$ **(B)** $25\sqrt{2}$ **(C)** $\frac{25}{2}$ **(D)** $\frac{25}{\sqrt{2}}$ **(E)** NOTA

Please Use the Following Information to Answer Questions 8 to 9:

Consider a unit square $ABCD$, the points E, F, G, H outside the square satisfy that$\triangle ABE, \triangle BCF, \triangle CDG, \triangle DAH$ are equilateral triangles. Denote the incenters of the equilateral triangles above as I_1, I_2, I_3, I_4.

8. Compute the area of $EFGH$.

(A) 3 **(B)** $4 + 2\sqrt{3}$ **(C)** $\sqrt{3} + 1$ **(D)** $\sqrt{3} + 2$ **(E)** NOTA

9. Compute the area of $I_1 I_2 I_3 I_4$, the answer is in the simplest form of $\frac{a+b\sqrt{c}}{d}$, c is square-free. Compute $a + b + c + d$.

(A) 12 **(B)** 10 **(C)** 8 **(D)** 11 **(E)** NOTA

10. Consider square $ABCD$ with side length 5. Denote the circumcircle of the square as Γ. Choose the midpoint N of the minor arc BC, connect AN and extend to meet the line DC at point F, find the length of CF.

(A) 5 **(B)** $5\sqrt{2}$ **(C)** $5(\sqrt{2} - 1)$ **(D)** 10 **(E)** NOTA

11. Consider square $ABCD$ such that $AB = 4$, circle ω centered at O is tangent to AB and AD, it also passes through C. The radius of ω is in the form of $a - b\sqrt{c}$, c is square-free. Compute $a + b + c$.

(A) 8 **(B)** 12 **(C)** 14 **(D)** 16 **(E)** NOTA

12. Consider triangle ABC, denote M, N as the midpoints of AB, AC. If the perimeter of $\triangle AMN$ is 20 and the perimeter of quadrilateral $BCNM$ is 25, find $AB + AC$.

(A) 25 **(B)** 30 **(C)** 35 **(D)** 50 **(E)** NOTA

13. Consider square $ABCD$ and a quarter circle inside $ABCD$ centered at A with radius AB. Point E is selected on the quarter circle satisfying $\angle EAB = 2\angle ECD$, compute AB^2 if $CE = 2$.

(A) 5 **(B)** 15 **(C)** 20 **(D)** 40 **(E)** NOTA

14. In a quadrilateral $ABCD$ with $\angle ABC = 90^\circ$, points M, N are the midpoints of AD, CD respectively. Given that $AB = 6, BC = 10$, the area of $ABCD$ is 50, find the area of $\triangle BMN$.

(A) 15 **(B)** 20 **(C)** 25 **(D)** 22.5 **(E)** NOTA

15. Consider a parallelogram $ABCD$, circle ω with a radius of 4 is tangent to AB, BC, AD. The tangent from C (other than BC) to ω meets AD at M. Given $AB = MC$, find the length of BC when $AB = 10$.

(A) 14 **(B)** 16 **(C)** 18 **(D)** 20 **(E)** NOTA

16. In parallelogram $ABCD, AB = 8, AD = 10$. Circle ω is tangent to AB, BC, AD. Denote M as the midpoint of BC, DM is tangent to ω. Denote d as the length of the diameter of ω, find d^2.

(A) 12 **(B)** 24 **(C)** 48 **(D)** 54 **(E)** NOTA

17. Which of the following must have a circumcircle?

(A) rhombus **(B)** isosceles trapezoid **(C)** kite **(D)** hexagon **(E)** NOTA

18. Given rhombus $ABCD$ with $AC = BC$. E is the intersection of AC, BD. Denote the midpoint of AE as Q, and define a moving point S on segment BD. Find the minimum value of $QS + \frac{BS}{2}$ if $AC = 8$. The answer is in the radical form of $\sqrt{p}$, find p.

(A) 12 **(B)** 18 **(C)** 20 **(D)** 27 **(E)** NOTA

19. Consider trapezoid $ABCD$ with $\angle A = \angle B = 90°$. Point M is selected on segment BC such that $AMCD$ is a rhombus with $AD = MD = 8$. Find the area of $ABCD$.

(A) $40\sqrt{3}$ **(B)** $36\sqrt{3}$ **(C)** $50\sqrt{3}$ **(D)** $64\sqrt{3}$ **(E)** NOTA

20. Consider triangle ABC with $AB = 10, BC = 12$. Point O lies inside the triangle and circle ω centered at O is tangent to AB, AC at points D, E and ω meets BC at points F, G. The radius of ω when $DEFG$ is a rectangle can be written in the simplest form of $\frac{p}{q}$, find $p + q$.

(A) 97 **(B)** 35 **(C)** 117 **(D)** 77 **(E)** NOTA

21. Let $\overline{AB}$ and $\overline{CD}$ be two parallel external tangents to circle ω , such that $\overline{AB}$ is tangent to ω at A, $\overline{CD}$ is tangent to ω at C, and $\overline{AC}$ does not intersect $\overline{BD}$. If $\overline{BD}$ is also tangent to ω at a point E. Given $AB = 4$, and $CD = 9$, then compute the area of ω.

(A) 36π **(B)** 18π **(C)** 54π **(D)** 72π **(E)** NOTA

22. Consider $\triangle ABC$ with $AB = 13, BC = 14, AC = 15$ and incircle ω. $P \neq B, Q \neq C$ on AB, AC such that PQ is tangent to ω and $PQ||BC$. The length of PQ can be written in the simplest form of $\frac{m}{n}$, compute $m + n$.

(A) 19 **(B)** 20 **(C)** 29 **(D)** 17 **(E)** NOTA

23. Consider isosceles trapezoid $ABCD$ with $AB = 38, CD = 34$. Point P is selected on segment AB such that $\angle CPD = 90^\circ, AP = 11$. Compute the area of the trapezoid.

(A) 270 **(B)** 360 **(C)** 540 **(D)** 600 **(E)** NOTA

24. Consider quadrilateral $ABCD$ with $AB = 3, BC = 7, CD = 9, AD = 8$. Given the length of AC is an integer, find the sum of the possible lengths of AC.

(A) 21 **(B)** 49 **(C)** 28 **(D)** 35 **(E)** NOTA

25. Consider parallelogram $ABCD(\angle B < 90^\circ)$ with $AB = 3, BC = 4$. Rotate the parallelogram about A to $AB'C'D'$, B' lies on segment BC and $C'DD'$ lie on the same line. If $BB' = 1$ and $B'C'$ meets CD at E, find CE.

(A) $\frac{5}{3}$ **(B)** $\frac{3}{2}$ **(C)** $\frac{9}{8}$ **(D)** $\frac{7}{4}$ **(E)** NOTA

26. Consider an unit regular pentagon $ABCDE$. Unit circles centered at A, B intersect at point Q inside $ABCDE$. Find $\angle BQC$ in degrees.

(A) 54 **(B)** 60 **(C)** 66 **(D)** 72 **(E)** NOTA

27. Point E lies inside square $ABCD$ such that $CE = 6, DE = 8, \angle AEB + \angle CED = 180^\circ$. The sum of all possible areas of $ABCD$ can be written in the form of $a\sqrt{b} + c\sqrt{d} + e$ for integers a, b, c, d, e, and b, d are square-free. Compute $a + b + c + d + e$.

(A) 147 **(B)** 153 **(C)** 165 **(D)** 173 **(E)** NOTA

28. Consider convex quadrialteral $ABCD$ with $AB = 8, AD = 10, BC = 14, \angle B = \angle C = \arccos(\frac{1}{4})$. Find the the sum of the possible lengths of CD.

(A) 21 **(B)** 13 **(C)** 19 **(D)** 23 **(E)** NOTA

29. In convex quadrilateral $ABCD$, $AB = 4, BC = 6, AC = 5$, BD bisects $\angle ABC$ and $\angle ABD = \angle CAD$, find the value of AD^2.

(A) 6 **(B)** 8 **(C)** 12 **(D)** 15 **(E)** NOTA

30. If the perimeter of the square and the area of the square are numerically the same, find the side-length of the square.

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** NOTA