Answer Key:

1. A
2. D
3. B
4. D
5. B
6. A
7. E
8. D
9. C
10. B
11. C
12. C
13. C
14. B
15. B
16. C
17. B
18. D
19. A
20. D
21. A
22. D
23. C
24. D
25. C
26. C
27. B
28. A
29. B
30. D

Solutions:

1. $\boxed{\text{A}}$: The area is $AC^2/2 = 200$

2. $\boxed{\text{D}}$: Since rectangles have congruent diagonals, so the area is maximized when two diagonals are perpendicular to each other, and it is identical to question.

3. $\boxed{\text{B}}$: Since all the sides of a rhombus are the same, so ABC is equilateral. The area is $2[ABC] = 2 \cdot \frac{\sqrt{3}}{4} \cdot 6^2 = 18\sqrt{3}$

4. $\boxed{\text{D}}$: Since $AB = BC = BE$, $\angle BEC = (180 - \angle EBC)/2 = 75$ degrees.

5. $\boxed{\text{B}}$: Since AC, CF are diagonals of the squares, so they are perpendicular to each other. The desire area is $2\sqrt{2} \cdot 5\sqrt{2}/2 = 10$

6. $\boxed{\text{A}}$: The perpendicular statement indicates the shape is a rhombus. Since $BD = \sqrt{3}AC, \angle BAD = 60^\circ$. The area is $\frac{25\sqrt{3}}{2}$

7. $\boxed{\text{E}}$: Notice $\angle C = 180^\circ - \angle A/2$, so B, C, D lie on the circle with center A, the answer is just 25.

8. $\boxed{\text{D}}$: $EFGH$ is a square, the distance from each vertex to the center of the square is $\frac{1}{2} + \frac{\sqrt{3}}{2}$, so the square of the side length is $2(\frac{1+\sqrt{3}}{2})^2 = 2 + \sqrt{3}$

9. $\boxed{\text{C}}$: $IJKL$ is a square, The distance from I to AB is $\frac{1}{2\sqrt{3}}$, the distance from the center of $ABCD$ to AB is $\frac{1}{2}$ so $IJ^2 = 2(\frac{1}{2\sqrt{3}} + \frac{1}{2})^2 = \frac{2+\sqrt{3}}{3}$

10. $\boxed{\text{B}}$: Since N is the midpoint of minor arc BC, so AF bisects $\angle BAC$. Thus $\angle BAF = \angle AFC = \angle CAF, AC = CF = 5\sqrt{2}$

11. $\boxed{\text{C}}$: The distances from O to AD and AB are both r, $OC = r$, so we have $\sqrt{2}(4 - r) = r, r = 8 - 4\sqrt{2}$

12. $\boxed{\text{C}}$:Since M, N are the midpoints, so $BC = 2MN$, so we have $AM + AN + MN = 20, BM + CN + MN + BC = 25$, since $AM = BM, AN = CN, BC = 5, MN = 2.5, AM + AN = 20 - 2.5 = 17.5$, the desired is $2(AM + AN) = 35$

13. $\boxed{\text{C}}$: Denote $\angle DCE = \alpha$. Since $AE = AB$, drop the foot from A to BE at F. We have $BF = EF; \angle ABE = (180 - 2\alpha)/2 = 90 - \alpha, \angle CBE = \alpha = \angle FAB, CE \perp BE$. Since $AB = BC$, we conclude $\triangle AFB \cong \triangle BEC$, so we have $BE = 4$. So the desired value is $2^2 + 4^2 = 20$

14. $\boxed{\text{B}}$: $[ABM] + [CNB]$ equals half the area of the whole quadrilateral, $[DMN] = \frac{1}{4}[ADC] = \frac{1}{4}(50 - 30) = 5$, The desired value equals to $50 - 5 - 25 = 20$

15. $\boxed{\text{B}}$: The distance from B to the foot of A-altitude to BC is $\sqrt{100 - 64} = 6$. Since it's an isosceles trapezoid, the distance from C to the foot of M-altitude is 6. By pitot, $AM + BC = 20 = 2AM + 12, AM = 4, BC = 4 + 12 = 16$.

16. $\boxed{\text{C}}$: By pitot, $DM = 5 + 10 - 8 = 7$ Then by law of cosines, $\angle C = 60^\circ, r = 8 \cdot \sin 60^\circ = 4\sqrt{3}, d^2 = 48$

17. $\boxed{\text{B}}$: A, C are not necessarily right and it is easy to give an example. D is not right as well, you could just extend some segments of a cyclic hexagon outside the circle.

18. $\boxed{\text{D}}$: As $AC = BC, \angle ABC = 60^\circ$. $\frac{BS}{2}$ is the height from S to base BC. The desired value is attained when QS is perpendicular to BC, which leads to $3\sqrt{3}$ as the minimum value.

19. $\boxed{\text{A}}$: Notice that $AM = AB = 8$ and $\angle AMB = 60^\circ$ since the condition implies AMD is equilateral. Thus $BM = 4, AB = 4\sqrt{3}$, the area is $(8+12) \cdot 4\sqrt{3}/2 = 40\sqrt{3}$

20. $\boxed{\text{D}}$: Since ω is tangent to AB, AC, we can conclude $AB = AC$. We can set $BG = 3x, DG = 4x, DB = 5x$. By symmetry, $BG = CF = 3x, GE = 12 - 6x$. By similar triangles, $DG^2 = BG \cdot GE, x = \frac{18}{17}$. The desired answer is $(10 - 5x) \cdot \frac{3}{4} = \frac{60}{17}$

21. $\boxed{\text{A}}$: Drop the feet from B to CD at H. Equal tangents from a point to a circle : $AB = BE$, $CD = DE$ Pythagoras implies $BD^2 = BH^2 + DH^2$. As $AC = BH$, hence area is $(13^2 - (9-4)^2)\pi/4 = 36\pi$

22. $\boxed{\text{D}}$: The inradius is 4, the altitude from A to $PQ = 12 - 8 = 4$. Similar triangle yields $\frac{4}{12} = \frac{PQ}{14}, PQ = \frac{14}{3}$.

23. $\boxed{\text{C}}$: Drop the height from C, D to AB, denote the feet as X, Y. We could see $BX = AY = (38-34)/2 = 2, PY = 9, PX = 25$. Since $\angle CPD = 90^\circ$, so $\triangle PDY \sim \triangle CPX, PY/DY = CX/PX, DY = CX = \sqrt{9 \cdot 25} = 15$, the area is $(34+38) \cdot 15/2 = 540$

24. $\boxed{\text{D}}$: By triangle inequality in $\triangle ABC, \triangle ACD$, we have $4 < AC < 10; 1 < AC < 17$. So the possible lengths are $5, 6, 7, 8, 9$ and that's 35.

25. $\boxed{\text{C}}$: Note by rotation, we have $\triangle ABB' \sim \triangle ADD', DD' = \frac{4}{3}, C'D = \frac{5}{3}$.

Since $\triangle DEC' \sim \triangle B'EC, \frac{C'D}{CB'} = \frac{5}{9}$. Assume $CE = x, C'E = \frac{5x}{9}$. So we have $\frac{3-x}{4-5x/9} = \frac{5}{9}$, we have $x = \frac{9}{8}$

26. $\boxed{\text{C}}$: $AB = AQ = BQ = 1, \angle ABQ = 60^\circ, \angle CBQ = 48^\circ; BC = BQ, \angle BQC = (180-48)/2 = 66^\circ$

27. $\boxed{\text{B}}$: Denote the side length of the square as l. Translate $\triangle AED$ to $\triangle BFC$, $EF = BC = l$. Since $\angle AED + \angle BEC = 180^\circ \implies BECF$ is a cyclic quadrilateral. Thus, $BFCE$ is an isosceles trapezoid due to congruent diagonals. We have $BE = CF = 6$.

By British Flag theorem, we have $AE^2 + 64 = DE^2 + 36; DE = CF = 6, BE = 2\sqrt{2}$. By Ptolemy, $l^2 = BE \cdot CF + CE \cdot BF = 16\sqrt{2} + 36$

Second case: when $BE = DE = 8, AE = 2\sqrt{23} \implies l^2 = 12\sqrt{23} + 64$. Sum up to attain $\boxed{100 + 12\sqrt{23} + 16\sqrt{2}}$

28. $\boxed{\text{A}}$: Select point E on segment BC such that $AB = AE$. Since $\arccos(B) = \frac{1}{4}, BD = 4, CE = 10 = AD$. As $\angle B = \angle C, \angle AEB = \angle B$, we conclude AE is parallel to CD and $ADCE$ is an isosceles trapezoid. We could thus have $CD = 10 \cdot \frac{1}{4} \cdot 2 + 8 = \boxed{13}$. The other case is when $ADCE$ is a parallelogram, which yields 8, the answer is $\boxed{21}$.

29. $\boxed{\text{B}}$: From $\angle ABD = \angle CBD = \angle CAD, A, B, C, D$ are concyclic and $AD = CD$. Let AC, BD meet at E. Angle bisector theorem implies $AE = 2, CE = 3$, by Stewart, $BE^2 = (48+72)/5 - 6 = 18, BE = 3\sqrt{2}$. Then, by the power of point, $DE \cdot BE = AE \cdot CE, DE = \sqrt{2}$.

We finish with Ptolemy, $6AD + 4CD = 5 \cdot 4\sqrt{2}, AD = CD = 2\sqrt{2}, AD^2 = 8$

30. **D** : $x^2 = 4x, x = 4$