

For the following questions the term “NOTA” stands for none of the above answers is correct. The test is written in the order of Geometry, Algebra, Pre-Calculus, Calculus, and Statistics so feel free to skip around the test to these different topics you are best at. Good luck and have fun!

GEOMETRY

1.) AB and CD are chords on a circle, and are perpendicular to each other. Their intersection occurs at point E . $AE = 4$, $BE = 6$, and $CE = 8$. Find the area of the quadrilateral formed by points A, B, C , and D .

- A) 48 B) 55 C) 64 D) 72 E) NOTA

2.) The side length of a regular tetrahedron whose surface area is equal to its volume can be written in the form $m\sqrt{n}$. Find $m + n$.

- A) 5 B) 8 C) 9 D) 12 E) NOTA

3.) The area of the equilateral triangle that is inscribed within a circle whose radius is 4 can be written in the form $m\sqrt{n}$. Find $m + n$.

- A) 6 B) 9 C) 10 D) 15 E) NOTA

4.) A right triangle has side lengths of 20, 21, and x . The possible values of x are \sqrt{m} and n where $m > n$ are integers, and n is not divisible by the square of any prime. Find $m + n$.

- A) 70 B) 105 C) 140 D) 144 E) NOTA

5.) A rectangle has base length $x - 1$ and height length $x + 1$. If the area of this rectangle is 63, compute the value of x^2 .

- A) 56 B) 60 C) 64 D) 72 E) NOTA

6.) Which of the following are true?

I. A central angle of a circle is equal to the arc it intercepts.

II. Vertical angles are congruent.

III. Adjacent angles are congruent.

- A) I only B) III only C) I and II D) II and III E) NOTA

ALGEBRA

7.) Find the sum of the entries of the resulting product of $\begin{bmatrix} 2 & -1 & 0 \\ 6 & 2 & 5 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 6 & -4 \\ -2 & -1 \end{bmatrix}$.

- A) -5 B) -2 C) 6 D) 10 E) NOTA

8.) The following simplification can be written in the form $\frac{-x^4+ax^3+bx^2+cx+d}{fx^4+gx^3+hx^2+jx+k}$.

Find $a + b + c + d + f + g + h + j + k$ given that they are relatively prime positive integers.

$$\frac{\left(\frac{x}{x+1} - x\right) - \left(\frac{1}{x} + \frac{x+1}{x}\right)}{\left(\frac{x-1}{x+1} + \frac{x+1}{x+1}\right) + \left(\frac{1}{x-1} - 1\right)}$$

- A) 2 B) 3 C) 4 D) 5 E) NOTA

9.) What is the y-intercept of $f(x) = 3(x + 2)^2 - 9$?

- A) -3 B) 3 C) $\sqrt{3} - 2$ D) $\sqrt{3} + 2$ E) NOTA

The following information will be used for questions 10 and 11. In physics, the formula to find kinetic energy the formula is $K = \frac{1}{2}mv^2$ where K is kinetic energy in joules, m is mass in kilograms, and v is velocity in meters per second. To find potential energy the formula is $P = mgh$ in joules where P = potential energy, m = mass in kilograms, g = gravity, h = height in meters.

10.) The kinetic energy of a car is known to be 40,000 joules and is going 20 meters per second. What is its mass in kilograms?

- A) 200 B) 400 C) 2000 D) 4000 E) NOTA

11.) Suppose a toy car's kinetic energy is equal to its potential energy when sliding down a ramp. Let the force of gravity equal 10 meters per second and the height of the ramp is 10 meters. In meters per second, what is the velocity of the toy car?

- A) 5 B) $5\sqrt{2}$ C) 10 D) $10\sqrt{2}$ E) NOTA

12.) Find the length of the minor axis of the following ellipse: $x^2 + 2y^2 - 4x + 12y - 14 = 0$

- A) 6 B) 12 C) $3\sqrt{2}$ D) $6\sqrt{2}$ E) NOTA

PRE-CALCULUS

13.) Given that $\sin x = \frac{1}{2021}$ where $\frac{-\pi}{2} \leq x < \frac{\pi}{2}$ what is $\sin^2 x + \cos^2 x$?

- A) 2021 B) -2021 C) $\frac{1}{2021}$ D) $\frac{-1}{2021}$ E) NOTA

14.) Given that $\sin x = \frac{3}{5}$ where $\frac{-\pi}{2} \leq x < \frac{\pi}{2}$ then $\tan \frac{x}{2}$ can be written in the form $\frac{m}{n}$ where both n and m are relatively prime positive integers. What is $m + n$?

- A) 3 B) 4 C) 7 D) 11 E) NOTA

15.) Classify the following: $r = 3 + 2 \cos \theta$

- A) Limacon with dimpled loop B) Limacon with inner loop
C) Lemniscate D) Cardioid E) NOTA

16.) Which of the following is a solution for the following equation?

$$|3 + xi| = 4$$

- A) $-\sqrt{2}$ B) -1 C) $\sqrt{7}$ D) 5 E) NOTA

17.) How many trailing zeroes exist in the number of distinct permutations of the word "COMPREHENSIVE"?

- A) 0 B) 1 C) 2 D) 3 E) NOTA

18.) The function $f(x) = \frac{x^2+5x}{x^2+10x+25}$ has a vertical asymptote at $x = n$ and a horizontal asymptote at $y = m$, Find the sum of $n + m$.

- A) -4 B) -1 C) 1 D) 4 E) NOTA

CALCULUS

19.) Evaluate: $\lim_{\theta \rightarrow 0} \frac{\cos \theta - 1}{\sin \theta}$

- A) -1 B) 0 C) 1 D) DNE E) NOTA

20.) Evaluate: $\int_{-3}^2 |x| dx$

- A) 2 B) 4 C) $\frac{7}{2}$ D) $\frac{13}{2}$ E) NOTA

21.) For a given continuous and integrable function, $f(x)$, assume that a, b, c , and d are within the domain of $f(x)$ and that $f(a), f(b), f(c)$, and $f(d)$ exist. Additionally, assume that $a < b < c < d$. It is known that $\int_a^d f(x) dx = 30$ and that $\int_a^b f(x) dx = \int_b^c f(x) dx$. Also, $\int_a^c f(x) dx = 3 \int_b^c f(x) dx$. Find the value of $\int_c^a f(x) dx + \int_c^b f(x) dx$.

- A) -60 B) -30 C) 30 D) 60 E) NOTA

22.) Evaluate:

$$\sum_{n=0}^{\infty} \frac{n}{2^n}$$

- A) 2 B) 3 C) 4 D) 6 E) NOTA

23.) Given that $f(x) = x^3 \ln x$, then $f'(2)$ can be written in the form $a(b + c \ln d)$ where a, b, c and d are relatively prime positive integers and where $a > 0$. What is the sum of $a + b + c + d$?

- A) 6 B) 10 C) 15 D) 16 E) NOTA

24.) For the following differential equation, it is known that $y(2) = 1$. What is $y(4)$?

$$y' = \frac{2y}{x}$$

- A) $\frac{1}{4}$ B) $\frac{1}{2}$ C) 4 D) 8 E) NOTA

STATISTICS

25.) Mr. Snow believes that in his statistics class the distribution of scores should be 20% A's, 30% B's, 20% C's, 20% D's, and 10% F's. The actual grades of his statistics scores are: 9 A's, 8 B's, 6 C's, 5 D's, and 2 F's. The χ^2 statistic of Mr. Snow's class can be written in the form $\frac{m}{n}$ when fully reduced. What is the sum of $m + n$?

- A) 16 B) 19 C) 23 D) 28 E) NOTA

26.) Given that $P(A) = .35$ and $P(B) = .6$ and that A and B are independent. Find $P(A \cup B)$ '.

- A) 0.21 B) 0.26 C) 0.53 D) 0.79 E) NOTA

27.) The least squares regression line $y = .5x - 16$ describes the data of 30 adult men where y measures their height in inches and x measures their weight in pounds. A random adult man is selected and is weighed to be 160 pounds and is a total of 70 inches in height. What is his residual?

- A) -6 B) -2 C) 6 D) 10 E) NOTA

28.) Which of the following is the reason to conduct a 2 Sample T-Test as opposed to a 2 Sample Z-Test?

- A) The sample size is not large enough and does not satisfy the central limit theorem.
B) The sample taken is not taken from 2 homogenous groups. Using a T-Test lowers sampling bias.
C) T-Tests only work for non-normal distributions.
D) The degrees of freedom are not known. Using a T-Test lowers sampling bias.
E) NOTA

29.) Given that X is a binomial distribution with $p =$ probability of success and $n =$ number of trials. Calculate $E(X^2)$ in terms of n and p .

- A) $np(1 - p + np)$ B) $np(1 + p - np)$ C) $p(1 - p + n)$ D) $p(1 + p - n)$ E) NOTA

30.) If Clownboy were to ask every 10th test taker of the difficulty of the previous problem, they would most likely say that it was pretty difficult, or they skipped it entirely, or they just guessed. What kind of sampling technique is this?

- A) Convenience B) Cluster C) Stratified D) Systematic E) NOTA