1. **B**.
   The series adds even numbers as the series progresses, defined as each term $a_n = a_{n-1}+2(n-1)$. Therefore, the next term is 43+2(8-1) = 57.
2. **A**.
   If we take the given series and find the differences between each terms, we get a new series 2, 4, 6, 8, 10, 12…. If we repeat this step one more time, we find that each of the terms differ by 2. This is 2a, where a is the coefficient in a quadratic expression an^2+bn+c. This gives us n^2+bn+c. Plugging in n=0 tells us a_0= c = 1. Now the expression yields $n^2+bn+1= a_n$. Checking with the original series: 1, 3, 7, 13, 21, 31, 43, we can find the value for b. Using a_1=3, we know that $(1)^2+b(1)+1 = 3$. Solving for b reveals b = 1. Thus the equation is $a_n = n^2+n+1$. The vieta's formula for sum of the squares of the roots gives $(1)^2-2(1) = -1$.
3. **A**.
   Using star and bars, the formula for the number of terms in an expanded polynomial is $\binom{n+k-1}{n}$ such that n is the power and k is the number of terms within the parentheses. In this case, n = 12 and k = 4, so our desired number is $\binom{12+4-1}{12} = \binom{15}{12} = 455$
4. **D**.
   Let the expression equal X.
   We can rewrite this equation as
   $X = \sqrt{182+X}$
   This creates the equation
   $X^2 - X - 182 = 0$
   Solving gives us x=14.
5. **C**.
   Spelling out each term: five, seven, eleven twelve, seventeen.
   Notice that the terms share a "v" and that no other number up to 17 besides those terms have a "v" in its spelling. The next term in the sequence would thus have to be 25 (twenty-five).
6. **D**.
   The formula for sum of consecutive cubes starting with 1 is equal to $\frac{n^2(n+1)^2}{4}$, where n is the number of terms. We recognize this as the square of the sum of consecutive numbers starting with 1: $\frac{n(n+1)}{2}$. Using this formula, we know that the sum of the cubes from 1^3 to 25^3 is equal to the square of the sum of the numbers from 1 to 25. However, since the problem asks for P, we only need to find $\frac{25(26)}{2} = 325$. 3+2+5= 10
7. **C**
   (-686/-2)^(1/3) = 7
8. **B**.
   The formula is the first term plus the last term divided by 2, times the number of terms.
   1+(1+3x9)/2 x 10 = 145
9. **D**

The number “n” appears n times, so we can set up an expression to find the last appearing repetition for any value of “n”.
n(n+1)/2
Plugging in n = 63, we get 2016, so 63 is the 2016th term and the last appearing 63 in the entire sequence. It follows that the 2017th and thus 2018th term must be 64.

10. **C**
    12*(-1+i)^(23-7)=3072
11. **A**.
    The orientation of each numbered door solely depends on the number of factors the number on the door has, specifically the parity the number of factors. In other words, after all 100 classmates have closed or opened their respective multiples, the only doors that would be open would be doors that have an odd number of factors. These are the perfect squares, so the desired answer is the number of perfect squares between 1 and 100, inclusive, which is 10.
12. A
    Prime numbers: 2, 3, 5
    Composite numbers: 4, 6
    Jerry’s probability of success is (3/6)^2 = ¼ and failure is 1-1/4 = ¾ Beth’s chance of success is (2/6)^2 = 1/9 and thus failure is 1-1/9 = 8/9.
    The infinite number of cases in which Beth wins form an infinite geometric pattern that can be observed in the first few cases.
    Jerry loses, Beth wins: 3/4 x 1/9
    Jerry loses, Beth loses, Jerry loses, Beth wins: 3/4 x 8/9 x 3/4 x 1/9
    Jerry loese, Beth loses, Jerry loses, Beth loses, Jerry loses, Beth wins: 3/4 x 8/9 x 3/4 x 8/9 x 3/4 x 1/9
    First term is 3/4 x 1/9 = 1/12 with common ratio of 3/4 x 8/9 = 2/3
    (1/12)/(1-2/3) = 1/4
13. **A**
    You can break the fraction into $\frac{1}{x+4} - \frac{1}{x+7}$ so the series become
    $(\frac{1}{5} - \frac{1}{8}) + (\frac{1}{6} - \frac{1}{9}) + (\frac{1}{7} - \frac{1}{10}) + (\frac{1}{8} - \frac{1}{11})\ldots$
    All terms cancel out except the 1/5 , 1/6 , and 1/7, so the sum is 107/210.
14. **A**
    Sum of the even squares can be factors by 2^2. So that we have
    2^2(1^2+2^2+3^2…+17^2) = 4(17)(2x17+1)(17+1)/6 = 7140
15. **B**
    a_1=1, a_2=1, a_3=1, a_4=2, a_5=23
    (1+2^2)+(1+2^3)+(2+2^4)+(23+2^5) =87
16. **B**
    An enneagon has 9 angles so you can denote the angles that form an arithmetic sequence as a-4d, a-3d, a-2d, a-d, a, a+d, a+2d, a+3d, and a+4d. Their sum of 9a adds to 180(7) so a = 140. Maximizing d enables one to find the smallest angle and since a+4d (the largest angle) cannot exceed 180 and must be an integer, d at most can be 9. This means a-4d (the smallest angle) is 104 degrees.

17. **A**

The first 5 odd numbers only have 1 digit. The next 45 odd numbers have 2 digits and the last 271 odd numbers to complete the 321 needed have 3 digits so intuitively one gets the sum 5(1) + 45(2) + 271(3) = 908

18. **D**.

Take the series S and multiply it by$\frac{1}{6}$. This yields a new series:

$$\frac{1}{6}S = \frac{1}{6^1} + \frac{1}{6^2} + \frac{2}{6^3} + \frac{3}{6^4} + \frac{5}{6^5} + \frac{8}{6^6} + \frac{13}{6^7} + \ldots$$

Subtracting this from the original series creates:

$$\frac{5}{6}S = \frac{1}{6^0} + \frac{1}{6^2} + \frac{1}{6^3} + \frac{2}{6^4} + \frac{3}{6^5} + \frac{5}{6^6} + \frac{8}{6^7} + \frac{13}{6^8} + \ldots$$

Looking at this series, we find that

$$\frac{5}{6}S = 1 + \frac{1}{6^2}S$$

Solving for S results in S = $\frac{36}{29}$

19. **C**.

Before solving the actual question, we can use the equation
$f(1) = 0(-1)^0 - 2f(0) = f(2019)$
to get $2f(0) = -f(2019)$.
Since we have an f(2019) in the given equation, we should solve for
$f(1) + f(2) + f(3) + f(4) + \ldots + f(2019)$
$= 0(-1)^0 - 2f(0) + 1(-1)^1 - 2f(1) + \ldots + 2018(-1)^{2018} - 2f(2018)$
$= 0 - 1 + 2 - 3 + \ldots - 2017 + 2018 - 2(f(0) + f(1) + f(3) + \ldots + f(2018))$
$= 1009 - 2(f(0) + f(1) + f(3) + \ldots + f(2018))$

moving functions to the other side yields
$2f(0) + 3(f(1) + f(2) + f(3)\ldots f(2018)) + f(2019) = 1009$.
Since $2f(0) = -f(2019)$, they cancel out and we get
$3(f(1) + f(2) + f(3)\ldots f(2018)) = 1009$
Our solution is $\frac{1009}{3}$

20. **B**

4.192 (2 repeating) = 4 + (192-19)/900
173/900 + 4 = 3773 / 900 which is simplified
3773 + 900 = 4673

21. **A**

This question can be bashed or the formula (n^2+n+2)/2 can be used because the nth cut creates n new pieces.

$$\begin{aligned} f(n) &= n + [(n-1) + f(n-2)] \\ &= n + (n-1) + \ldots + 2 + f(1) \\ &= f(1) + \sum_{k=2}^{n} k \\ &= 2 + \tfrac{1}{2}(n+2)(n-1) \\ &= \tfrac{1}{2}\left(n^2 + n + 2\right). \end{aligned}$$

Plugging in 10 and 11 we get (10^2+10+2)/2+(11^2+11+2)/2 = 123

22. **B**

As given, the first and last terms of the sequence are 13 and 1003. The two common differences are 6 and 5, so the two sequences overlap every 6*5=30. Essentially we're counting the number of terms in a sequence that begins with 13, ends with 1003 and has common difference of 30.

(1003-13)/30+1 = 34

23. **E**

Split the sum into two difference sequences $\sum_{m=0}^{\infty} \left(\frac{8m}{2^m} - \frac{3}{2^m}\right)$

Half the first sum of

S= 8(1/2 + 2/4 + 3/8 + 4/16…)

To get

1/2S = 8(1/4 + 2/8 + 3/16…)

Subtract the two sums to get

1/2S = 8(1/2 +1/4 + 1/8 +1/16…) so S = 16(1) = 16

The second sum is simply

3(1+ 1/2 + 1/4 +1/8 +1/16…) = 3(2)=6

16-6 = 10

24. **A**

$x + 1 = 4 + \frac{1}{x+1}$and manipulating you get $x^2 - 2x - 4 = 0$ and $x = 1 + \sqrt{5}$

[can't use negative conjugate]

25. **D**

The first term is 2^0 and the last term is 2^10. Dividing the last term by the first term gives us 2^10 = r^n where r is the common ratio and both r and n have to be integers. The factors of 10 (values for n) are 1, 2, 5, and 10 which make the r = 1024, 512, 4, or 2 respectively. However, for even values of n, we can have negative values of r, so r = -512 or -2 also. The number of values or r determines the number of sequences so 4+2 = 6 sequences.

26. **E**

The meatball drops 20 ft. and bounces 10 ft. up for the first bounce. Since the first bounce's ratio was 1/2, the second bounce has a ratio of 1/3, the third has a ratio of 1/4, up until the sixth which has a ratio of 1/7. To find the total vertical distance, we add:

$$20 + 2\left(20 \cdot \frac{1}{2} + 20 \cdot \frac{1}{2} \cdot \frac{1}{3} + \cdots + 20 \cdot \frac{1}{6!}\right) = \frac{877}{18}$$

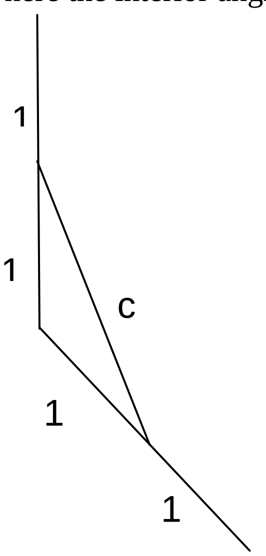
27. **B**

The formula for the area of an octagon is

$A = 2(1+\sqrt{2})a^2$

This formula is derived by removing a triangle from each corner of a square to form an octagon. The area of the first octagon is thus 8(1+√2)

The areas of the subsequent octagons will have a common ratio equal to the square of the common ratio of their side lengths. The square of the side length of the first octagon is 2^2=4. The square sidelength of the second octagon can be found using the law of cosine where the interior angle of an octagon is 135°.



c^2=1^2+1^2 - 2(1)(1)cos135=2+√2

Now we have an infinite geometric sequence with initial value 8(1+√2) and common ratio (2+√2)/4. Our answer is [8(1+√2)]/[1-(2+√2)/4]=64 + 48√2

28. A

Factor the expression to (x-17)^3. The summation then simplifies down to $\sum_{n=1}^{13} x^3$. Plug 13 into the formula n^2(n+1)^2/4. (13)^2(14)^2/4=8281

29. E

We want to find the minimum value "n" number of days that satisfies the inequality below.

(200)+(200+50)+(200+50*2)...+ (200+50*(n-1)) >= 14115

Or

200n + 50(n-1)(n)/2 >= 14115

Plugging in n = 20 results in 13500 but plugging in n = 21 results in 14700 which satisfies the inequality.

21 days beginning on a Monday will end on Sunday.

30. C

Formula for triangular numbers is n(n+1)/2 and for hexagonal is n(2n-1).

20(20+1)/2 + 18(2*18-1) = 840