

Where applicable, “E) NOTA” indicates that none of the above answers is correct. Throughout this test, let a_n be the n th term of a sequence, and let S_n be the sum of the first n terms of a series.

1. Find a_{13} in the arithmetic sequence $\frac{5}{18}, \frac{13}{36}, \dots$

- A) $\frac{23}{18}$ B) $\frac{29}{6}$ C) $\frac{127}{36}$ D) $\frac{10}{27}$ E) NOTA

2. Find S_{13} for the arithmetic series $4 + 7 + 10 + \dots$

- A) 40 B) 286 C) 44 D) 147 E) NOTA

3. In an arithmetic sequence, $a_8 = 31$ and $a_{53} = 400$. Find the value of a_{51} .

- A) $\frac{1939}{5}$ B) $\frac{17242}{45}$ C) $\frac{15602}{45}$ D) $\frac{1918}{5}$ E) NOTA

4. Find the sum of the infinite geometric series $64\sqrt{7} + 48\sqrt{7} + 36\sqrt{7} + \dots$

- A) $256\sqrt{7}$ B) $249\sqrt{7}$ C) $242\sqrt{7}$ D) $235\sqrt{7}$ E) NOTA

5. Suppose that $-3, a, b, c, -243$ is a geometric sequence with negative common ratio. Find the value of $a + b$

- A) 100 B) 36 C) -18 D) -36 E) NOTA

6. Evaluate: $\sum_{j=6}^{45} (6j - 20)$

- A) 5290 B) 5760 C) 4938 D) 5320 E) NOTA

7. Find S_{15} for the geometric series $-3 + 6 - 12 + \dots$

- A) -32769 B) -65537 C) -32767 D) -65535 E) NOTA

8. Evaluate: $\sum_{n=5}^{\infty} 7\left(\frac{1}{8}\right)^{n-1}$

- A) 1 B) $\frac{1}{512}$ C) 8 D) $\frac{1}{1024}$ E) NOTA

9. A square has a side length of 4. An infinite sequence of squares is inscribed in it by joining the midpoints of the sides of each preceding square. Find the total length of all the sides of the segments in this figure.

- A) $32 + 16\sqrt{2}$ B) $16 + 16\sqrt{2}$ C) $16 + 8\sqrt{2}$ D) $32 + 8\sqrt{2}$ E) NOTA

10. Find the second term of the geometric sequence with $a_6 = 128x^{17}y^5$ and common ratio $r = 2x^3y^2$.

- A) $4x^2y^5$ B) $\frac{4x^2}{y^2}$ C) $\frac{8x^8}{y}$ D) $\frac{8x^5}{y^3}$ E) NOTA

11. In an arithmetic sequence, $a_k = 17$ and $a_{k-3} = 44$. What is the value of a_{k-5} ?

- A) -19 B) -1 C) 62 D) -28 E) NOTA

12. A sequence satisfies $a_n = a_{n-1} - a_{n-3}$ for integers $n \geq 4$. If $a_2 = 3$ and $a_5 = 30$, find a_9 .

- A) 128 B) -72 C) -63 D) 59 E) NOTA

13. Find the sum of the values of x such that $5x - 8$, $3x$, $2x + 2$ forms a geometric sequence.

- A) 6 B) 0 C) 10 D) -3 E) NOTA

14. A sequence $\{t_n\}_{n=1}^{\infty}$ is defined as follows: $t_1 = 14$, and $t_k = 24 - 5t_{k-1}$ for all integers $k \geq 2$. If $t_n = p \cdot q^n + r$, where p , q , and r are real numbers, find the value of $p + q - r$.

- A) -5 B) -11 C) 17 D) 31 E) NOTA

15. Robert C. Byrd holds the record for the longest consecutively held U.S. Senate seat at 51 years, 5 months, and 26 days. He was initially elected to the Senate (as opposed to taking over midterm for someone else). If his last day of service was June 28, 2010, which equation represents the years in which Sen. Byrd began a new term? Let Y_n = the year in which Sen. Byrd began his n th term. Remember that a term in the U.S. Senate is 6 years.

- A) $Y_n = 1959 + 6(n-1)$ B) $Y_n = 1959 + 6n$ C) $Y_n = 2010 - 6(n-1)$ D) $Y_n = 2010 - 6n$ E) NOTA

16. Sally saves \$1 on the first day of November, \$4 on the second day of November, \$9 on the third day of November, and she continues this trend, saving $\$n^2$ on November n . How much money will Sally save by the end of November, assuming she began the month with no money?

- A) \$9211 B) \$10416 C) \$9455 D) \$11731 E) NOTA

17. The arithmetic sequence 3, 10, ..., 108 consists of how many terms?

- A) 10 B) 11 C) 13 D) 15 E) NOTA

18. Which explicit formula gives the n th term of the sequence defined recursively as $a_{n+1} = 5a_n$, $a_5 = 5000$?

- A) $a_n = 1.6(5)^{n-1}$ B) $a_n = 8(5)^{n+1}$ C) $a_n = 1.6(5)^{n+1}$ D) $a_n = 8(5)^{n-1}$ E) NOTA

19. Solve for n : $\frac{1}{1+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{7}} + \dots + \frac{1}{\sqrt{2n-1}+\sqrt{2n+1}} = 100$

- A) 20200 B) 20201 C) 450 D) 451 E) NOTA

20. Find the 15th term in the sequence defined recursively as $a_n = 3a_{n-1} - 2a_{n-2}$ for all integers $n \geq 3$, $a_1 = 1$, and $a_2 = 3$.

- A) 32767 B) 16385 C) 33754 D) 24762 E) NOTA

21. Find a_5 for a geometric sequence with common ratio 3, where $a_1 = 5 + 2x$.

- A) $1215 + 486x^5$ B) $125 + 8x^3$ C) $405 + 162x$ D) $1215 + 486x$ E) NOTA

22. An arithmetic sequence has first term 5, last term 57, and the sum of the terms of the sequence is 837. What is the common difference of this sequence?

- A) 1 B) 1.5 C) 2 D) 2.5 E) NOTA

23. Suzy writes down a sequence of 101 distinct positive integers so that the sum of any two consecutive integers in the sequence is divisible by 5. Let N be the smallest possible sum of all 101 integers. Find the value of N .

- A) 12756 B) 12751 C) 12761 D) 12746 E) NOTA

24. What is the greatest possible sum for an infinite geometric series with first term 12005 and fifth term 5?

- A) 13720 B) $\frac{84035}{9}$ C) 15435 D) $\frac{81835}{8}$ E) NOTA

25. Elizabeth writes the perfect square sequence 1, 4, 9, 16, After the number 1, she then alternates by making two terms negative, followed by leaving two terms positive, then repeating; in other words, her new sequence is 1, -4, -9, 16, 25, -36, -49, 64, 81, What is the sum of the first 2016 terms of this new sequence?

- A) -4062240 B) 0 C) -4046132 D) 2016 E) NOTA

26. Which one of the following series has the greatest sum?

I) $\sum_{i=0}^6 \left(\frac{5}{3}i + 9 \right)$

II) $\sum_{i=0}^6 \left(19 - \frac{5}{3}i \right)$

III) $\sum_{i=0}^6 2^i$

- A) I B) II C) III D) they all have the same sum
E) NOTA

27. The first term of an infinite geometric sequence is one greater than the 16th smallest prime number. The sum of all the terms in the sequence is 36. Find the fifth term of the sequence.

- A) $\frac{27}{8}$ B) $\frac{328}{27}$ C) $\frac{176630}{6561}$ D) $\frac{16}{81}$ E) NOTA

28. Evaluate: $\sum_{n=10}^{23} n^3$

- A) 72050 B) 72150 C) 74051 D) 74151 E) NOTA

29. Evaluate: $\sum_{n=1}^{45} \log_2 \left(1 + \frac{1}{n+2} \right)$

- A) $\log_2 \left(\frac{136}{91} \right)$ B) $\log_2 \left(\frac{2}{3} \right)$ C) 4 D) 3 E) NOTA

30. In an arithmetic sequence, the r th term is s while the s th term is r . Given that $r \neq s$, find the $(r+s)$ th term of the sequence.

- A) 0 B) -1 C) $-r$ D) $-s$ E) NOTA