Greetings adventurer. I am the mysterious Mr. E. You may not realize this, but you are the most high-IQ individual in your homeland, Application Nation. The wicked mathemagicians are plotting an attack from their headquarters in Mathemagica. You are our only hope. So, I bequeath to you some knowledge you may need for your perilous journey: NOTA means "none of the above," and never trust the scaling on the pictures. **Disregard any unmentioned dimensions; for example, in problem 21, the size of the dog is negligible.** I wish you Godspeed for this battle of endurance and wits.

1) A huge dragon descends upon your village, but you see something on its stomach: $2, 5, 5, 10$? Using your high-IQ, you realize that it's a 24 game. Your mind racing, you shout the only operation that is **not** used to solve this 24 problem. What is that operation? The objective of a 24 game is to create 24 from 4 numbers using the 4 arithmetic operations: $+, -, \times, \div$. The numbers are not always given in order. For example, for $2, 3, 6, 12$, we could answer $2 \times 3 + 6 + 12 = 24$ or $3 \times (12 - 2) - 6 = 24$.

A) $+$ B) $-$ C) $\times$ D) $\div$ E) NOTA

2) "Outstanding work, young hero." The king of Application Nation, Hector the Vector, says. "But I need to make sure your victory wasn't simply luck. Answer me this: On a Cartesian plane, we are standing at $(3, 5)$, my castle is at $(9, 10)$, and there's a river represented by the line $y = 3x + 6$. What is the minimum distance I can ride to reach my castle if my horse must stop at the river for a drink along the way?"

A) $\sqrt{61}$ B) $3\sqrt{17}$ C) $2\sqrt{5} + \sqrt{65}$ D) $\sqrt{10} + \sqrt{97}$ E) NOTA

3) "We need a hero like you to defeat the mathemagicians who are giving us much trouble. Will you go?" King Hector requests (with both direction and magnitude). Of course, you say yes. Taking just a rope, a map, and marbles with you, you depart. Your map is a 3×5 grid of fifteen 1×1 squares. You are at the bottom left corner and Mathemagica is at the top right corner. To conserve energy, you must take the shortest route possible, but you can only move along the gridlines. How many ways can this be done?

A) 54   B) 58   C) 206   D) 210   E) NOTA

4) You finally arrive at Mathemagica and approach cautiously, knowing there must be many traps involving both math and magic. To your surprise, the door is guarded only by a heroic couplet. It reads, "To unlock the door and see what is behind, the sum of the squares of roots you must find: $2x^4 + 8x^3 - 5x^2 - 17x + 33$." (In other words, find the sum of the squares of the roots of the given fourth degree polynomial.)

A) 11   B) 16   C) $\frac{37}{2}$ D) 21   E) NOTA

5) *Too easy!* you think as you crack the lock. Suddenly, you see an arrow flying towards you and it's too late to dodge. Luckily, you have on a chestplate in the shape of a regular hexagon, but only the area enclosed by the largest square you can inscribe in the hexagon can block the arrow. If the arrow will hit somewhere randomly on your chestplate, what is the probability that it hits a spot that cannot block it?

A) $\frac{4\sqrt{3}-6}{3}$ 　　　 B) $\frac{8\sqrt{3}-12}{3}$ 　　　 C) $\sqrt{3}-1$ 　　　 D) $12-6\sqrt{3}$ 　　　 E) NOTA

6) *Ow!* With some wincing, you pull the arrow out of your chest and inspect it. "Why does it have to be poison?" you say to no one in particular. "And why didn't I bring my Super-Duper-All-Curing-Antidote-2000?" Luckily, you know that licky-snakes secrete the antidote in a lick-able form from their necks. Only the females secrete the antidote, and the chance of any given snake being female is exactly $\frac{1}{2}$. You hear some hissing noises and notice three licky-snakes. You know that only males can hiss, but you can't tell which one(s) hissed. Male and female licky-snakes are not visually distinguishable. If you lick all three snakes, what is the chance you survive? The antidote is 100% effective.

A) $\frac{1}{2}$ 　　　 B) $\frac{3}{4}$ 　　　 C) $\frac{6}{7}$ 　　　 D) $\frac{7}{8}$ 　　　 E) NOTA

7) After being cured, you move deeper in the castle. You notice that each door has an interesting design. The design features a semicircle inscribed in a right triangle ABC with hypotenuse AB. The diameter of the semicircle lies along leg AC. Side AB has length 15 units and side AC has length 12 units. What is the length of the semicircle's diameter in units?

A) 4 　　　 B) $\frac{9}{2}$ 　　　 C) 8 　　　 D) 9 　　　 E) NOTA

8) You move onward. In a dimly lit hallway, you find a pair of guards. "Halt! I'm Ms. Taken, she is Ms. Guided, and you shall not pass." one declares. You exclaim, "Alright, if you're Ms. Taken, then give me your problem. If I solve it correctly, you shall let me pass." "Very well." she responds. "I have an equation: $2^{x+1} = 4^{x-1} \cdot 8^x$. What is x?"

A) $\frac{2}{3}$ 　　　 B) $\frac{3}{4}$ 　　　 C) $\frac{3}{2}$ 　　　 D) 2 　　　 E) NOTA

9) Ms. Guided still doesn't let you pass, so you demand a problem from her too!
Quick, answer it: What is the domain of $\log_5(\log_4(\log_3(\log_2(x))))$?

A) $(8, \infty)$ 　　　 B) $[8, \infty)$ 　　　 C) $(2^{81}, \infty)$ 　　　 D) $[2^{81}, \infty)$ 　　　 E) NOTA

10) Shocked by your high-IQ-ity, the guards let you pass. You are met by an 8-step staircase with the 1st stair defined as the bottom-most stair. The rest of the staircase is numbered upwards with the 8th stair at the top. Given the length of your legs, you can take the steps one, two, or three at a time. Your sixth sense, infinite IQ, tells you that there is a pit on the 5th stair which would lead to your demise, so you must avoid it. Starting at ground level, how many ways can you safely reach the top stair?

A) 29   B) 32   C) 68   D) 81   E) NOTA

11) At the top of the stairs, there is a suspicious room with a cauldron. *Ouch!* Another poison arrow sticks out of your chest. "Dang it!" you sigh. You're fresh out of lick-able snakes, so you turn to the cauldron. It contains 20 liters of 20% magic and 80% health, but the correct proportions for the Super-Duper-All-Curing-Antidote-2000 are 45% magic and 55% health. Looking around, you find two bottles with infinite quantities of their respective mixture: one with 80% magic and 20% health and one with 35% magic and 65% health. What is the least number of liters of liquids you must add to make your Super-Duper-All-Curing-Antidote-2000?

A) 5   B) $\frac{11}{2}$ C) 6   D) $\frac{13}{2}$ E) NOTA

12) "Going on quests isn't good for my health." you say as you finish up the potion. "Maybe I should try gardening." You continue infiltrating the castle, but trip over a small crack in the ground and stumble into a kitchen. A cook stares at you blankly. "This place is Stew Pit and I am a Stew Pit chef," he says. "Okay. . ." you say, thinking about that name. "If you are a stupi- I mean *Stew Pit* chef then give me your problem. If I solve it correctly, you shall let me pass... and give me some soup." He agrees. "Making stew takes steps and so does math. What is the value of $2 \times 18 \div 12 - 216 \div 4(7 + 2)$?"

A) -483 B) $\frac{1917}{4}$ C) $-\frac{71}{12}$ D) -3 E) NOTA

13) A little math and a bowl of soup later, you're back on your journey. You are met with another door surrounded by two oddly shaped arches. Stepping back, you notice that the decorations vaguely resemble a product. *Okay, nice! This must be the key.*
Evaluate the following:
$$\prod_{n=2}^{127} \log_n(n + 1)$$

A) $\frac{1}{7}$ B) $\log_2 127$ C) 7   D) 8   E) NOTA

14) A key pops out, and a wild imp appears! The imp uses Snatch! The imp snatches the key and runs in circles around a column. By the time you realize what happened, the imp has already run halfway around the column! You use Chase immediately and run in the same circle the imp was running, but in the opposite direction. You meet the imp for the first time after 10 seconds. You couldn't catch him that time, so you both keep running in the same directions around the column. How many seconds after the first meeting do you meet the imp for the second time? Assume nobody changes direction and you and the imp run at constant speeds.

A) 2   B) 5   C) 10   D) 20   E) NOTA

15) "Whew," you exclaim as you grab the key. You walk into the dreaded second layer, which is a circle with the mathemagic generator in the center. You also know that the the circle can be plotted on a Cartesian plane. The points $(12, 14), (2, -10)$, and $(12, -10)$ lie on the circle. If each unit is 30 yards and you stand at the point $(12, 14)$, how many <u>feet</u> are you from the generator?

A) 13   B) 390   C) 780   D) 1170   E) NOTA

16) Heading towards the generator, you are met with more doors (This must be a fire hazard or something). The first door is shaped like a giant urn with 10 blue marbles and 20 red marbles inside of it. In order to pass, you must draw a blue marble on your first try. Because you are a cautious person, you want the probability of drawing a blue marble to be at least 85%. You whip out the infinite supply of red and blue marbles you brought and start adding marbles to the urn. What is the least number of marbles you can add to the urn to reach your desired probability?

A) 16   B) 50   C)   D)   E) NOTA

17) You stand at the door, but it doesn't open! You finally notice a small hidden inscription that reads: "Use the answer to number 7 to solve this lock. Take the answer to number 7 (which will be called n), add 4, multiply by 3, subtract 1, divide by 3, and subtract n in that order." What is your final answer? (Hint: don't worry if you weren't able to solve number 7!)

A) $-\frac{2}{3}$ B) 1   C) $\frac{11}{3}$ D) 4   E) NOTA

18) "I never want to see another door in my life," you state, crHINGEing. All this math-ing has been tiring, so you take a break before checking out the next door (WOOD you please stop with the doors?!). Your rate of breathing is 40 breaths per minute. With each breath, you take in 500 milliliters of air. If oxygen is 20% of the volume of air and your breathing rate is constant, for how many hours would you have to breathe to take in 360 liters of oxygen?

A) $\frac{3}{2}$ B) 3   C) $\frac{9}{2}$ D) 90   E) NOTA

19) While breathing, you stare at the cool critters on the ground. You spot some blue beetles, with 6 legs and 1 head each. You also spot some mini red giraffes, with 4 legs and 1 head each. Finally, you see some friendly red spiders waving at you, with 8 legs and 1 head each. You are so tired that when you try to calculate the total number of heads, you add the numbers of heads of the spiders and beetles, but you accidentally subtract the number of giraffe heads! You make the same mistake when you calculate the total number of legs (you add the legs of the spiders and beetles, but subtract the legs of the giraffes). The results of your incorrect calculations are 8 total heads and 78 total legs. How many red critters are there?

A) 5   B) 15   C) 24   D) 38   E) NOTA

20) "Man, that was refreshing," you say, moving to the next door. Your high-IQ carbon dating skills tell you that this blue door is half as old as that red door over there. In 100 years, the blue door will be two-thirds as old as that same red door. How old, in years, is the blue door?

A) 25   B) 100   C) 200   D) 300   E) NOTA

21) The floor of your next room is in the shape of a square. One end of a 10-foot long leash is attached to one corner of the floor, and a three-headed dog is attached to the other! Your extraordinary mathematical instinct tells you that the floor's area is 100 square feet. What is the total area in square feet that the dog can roam? The dog cannot break the leash or leave the floor.

A) $\frac{25}{2}\pi$ B) 25π C) 50π D) 100π E) NOTA

22) The scary-looking dog is actually very nice and lets you pass. You find a random steel rectangular prism magically suspended in the air. Its dimensions are $2 \times 10 \times 5$ and an ant is crawling on it. "I'm lazy, so can you help me find the length of the shortest path I can take to get from vertex A to vertex B?" the ant asks. As an incredibly generous individual, you oblige. Refer to the diagram below. The ant can only travel along the surface of the prism, NOT through the prism or on air. All measurements are in units, including the answers.
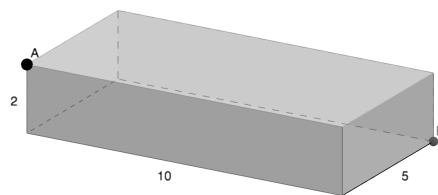
A) $2 + 5\sqrt{5}$ B) $\sqrt{149}$ C) 13   D) $\sqrt{229}$ E) NOTA

23) Leaving the ant, you're met with a broken bridge. "Finally! Not a door!" you exclaim. The bridge is 1 mile long. You begin fixing it with your rope you brought, but bridge-building is hard and you can only build a foot in the first hour. Improving, you build the next foot in half an hour, and the third foot in 15 minutes. Because you have infinite IQ, this trend continues where each foot takes you half as long to build as the previous foot. Which answer is closest to the time it will take you to repair the whole bridge in hours?

A) $\frac{2^{5280}-1}{2^{5281}}$ 　　　　 B) $\frac{2^{5280}-1}{2^{5280}}$ 　　　　 C) 1 　　　　 D) 2 　　　　 E) NOTA

24) Crossing the bridge, you are met with a huge door. "This must be the boss room." you remark. This door seems weirdly simple. It reads: "What is x given that $2x + y = 10$ and $3x - y = 15$?"

A) 0 　　　　 B) 3 　　　　 C) 5 　　　　 D) 6 　　　　 E) NOTA

25) You easily crack the code and open the door. The Mathemagical King, Mr. Lu the Wiz, appears. "I've been waiting," The Wiz says in a deep, intimidating voice. "Here's my first question: I have here a triangle ABC inscribed in a circle with center O. Refer to the lengths in the diagram below. What is $\frac{CD}{BD}$?"
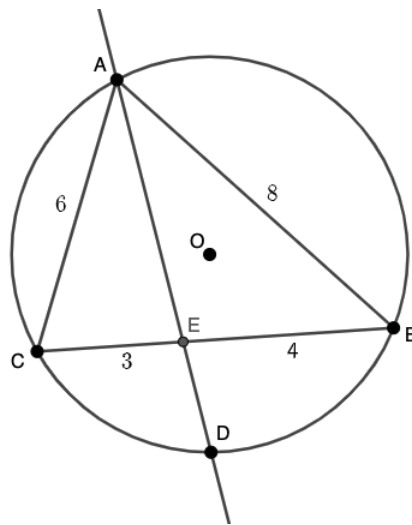
A) $\frac{3}{4}$ 　　　　 B) $\frac{8}{9}$ 　　　　 C) $\frac{4\sqrt{2}}{5}$ 　　　　 D) 1 　　　　 E) NOTA

26) "I have 3 doors labeled A, B, and C." Mr. Lu says. "Behind one of them is a prize. You pick one of the doors at random. I then open a door (not the one you picked) that doesn't have the prize behind it. However, the door I open will be as close to the beginning of the alphabet as possible. You can then choose to switch doors or stick with your original. What is the probability that you win the prize if you use an optimal strategy?"

A) $\frac{1}{3}$ 　　　　 B) $\frac{1}{2}$ 　　　　 C) $\frac{2}{3}$ 　　　　 D) $\frac{7}{9}$ 　　　　 E) NOTA

27) Mr. Lu looks shocked as you answer it correctly. "Ah, this wouldn't be fair if only you asked questions now would it?" you ask slyly. "It is currently 6:30 PM. After how many minutes will the two hands coincide on my analog watch for the first time after 6:30 PM?" While Mr. Lu ponders the question, you also solve it. What is the correct answer?

A) $2\frac{8}{11}$ B) 3   C) $68\frac{2}{11}$ D) $163\frac{7}{11}$ E) NOTA

28) "Ha, you thought you could stump me." the Wiz gloats. Meanwhile, his pet birb flies around. The birb's magic allows it to fly in any direction, just as easily above ground as below. It travels at a constant speed on a path denoted by $f(x) = x^4 - 7x^3 + 16x^2 - 12x$ for $x \geq 0$ where $f(x)$ gives the birb's height above ground level (ground level is $y = 0$) at horizontal distance x from the starting point at $x = 0$. A "transition" is an instance where the birb goes from traveling above ground to traveling below ground, or vice versa. What is the sum of the cubes of each distinct value of x where a transition occurs? Note that simply touching the border between above/below ground is not a transition.

A) 4   B) 9   C) 13   D) 35   E) NOTA

29) Racking your brain, you decide to go all out. "You've activated my trap card!" you scream at the top of your lungs. "I've proven that $1 + 1 = 0$ and debunked math!" "W-what?" The Wiz replies, shaking.

$$
\begin{aligned}
1 + 1 &= 1 + \sqrt{1} & (1)\\
&= 1 + \sqrt{(-1)(-1)} & (2)\\
&= 1 + \sqrt{(-1)^2} & (3)\\
&= 1 + (-1) & (4)\\
&= 0 & (5)
\end{aligned}
$$

"NOOOOOOOOO!!!!!" The Wiz screams as he melts into a puddle of mathemagically enhanced Kool-Aid. The numbers on the right side in parentheses denote the step number in the proof. Of course, math hasn't really been debunked, so at what step is the proof flawed? In other words, which is the first step that is not a logical progression from the previous step?

A) 2   B) 3   C) 4   D) 5   E) NOTA

30) 7 years later, you are reclining on your front lawn contemplating life. *Hey, wait a minute. Didn't I say I was going to try gardening all the way back in question 12?* You plant a new garden which is in the shape of a regular octagon with area $16 + 16\sqrt{2}$ square yards and center O. Because you are high-IQ, you labeled the vertices of your garden $ABCDEFGH$. You want to plant potatoes in regions AHO and $OCDE$. What is the area of the planting region in square yards?

A) $2\sqrt{2} + 2$ B) $6\sqrt{2} + 6$ C) $\frac{16}{3}\sqrt{2} + \frac{16}{3}$ D) $8\sqrt{2} + 8$ E) NOTA