**THETA TEAM BOWL CONDENSED VERSION**
**2022 MAO NATIONAL CONVENTION**

**Question 0**

Let $A$ = the smallest positive prime divisor of 2019
Let $B$ = the largest positive prime divisor of 2020
Let $C$ = the largest positive divisor of 2021
Let $D$ = the sum of the first 100 positive integers
Find $A+B+C+D$.

**Question 1**

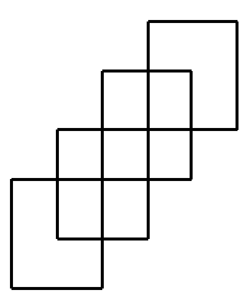
Let $A$ = the number of ways 4 gold, 3 silver, and 2 bronze medals can be placed in a row if all the medals of the same color are indistinguishable.
Let $B$ = the number of different 8-letter arrangements that can be made from the letters in the word DAUGHTER if the vowels must be together
Let $C$ = the number of different 8-letter arrangements that can be made from the letters in the word DAUGHTER that begin with D and end with R
Let $D$= how many numbers strictly between 100 and 1000 that can be formed using 0, 1, 2, 3, 4, and 5 if each digit may only be used once
Find the least common multiple of $\frac{A-D}{10}$ and $\frac{B-C}{100}$.

**Question 2**

Let $A\pi+B=$ the area of the region bounded by $y=|x|-6$ and $y=\sqrt{36-x^2}$.
Let $\frac{C\pi\sqrt{D}}{D}=$ the volume of the solid formed by rotating the triangle with vertices (−4, 1), (−2, 4), and (−1, 1) about the line $y=-3x-2$.
Find $\frac{AD}{B-C}$.

**Question 3**

Let $A$= the number of unique rectangles with integral side lengths that have an area of 210 sq. units
Let $B$ = the average perimeter of all the rectangles in $A$
Let $C$ = the total number of rectangles of any dimension in the figure below
Let $D$ = the original perimeter of a rectangle such that when the width of the rectangle is increased by 2 and its length is cut in half, its area remains unchanged. The perimeter of the original rectangle is 40% greater than the perimeter of the altered rectangle.
Find $A+B+C+D$.



**Question 4**

Let $A$ = the square of the difference in the legs of a right triangle with hypotenuse 13 and area 20.

Let $B$ = the larger value of $h$ for which $x^3 - 13x^2 + 15x + h = 0$ has one root 2 greater than another.

Find $B - A$.

**Question 5**

Andrew is riding a moped, traveling at 54 mph. Nate starts after the moped, traveling at 81 mph in his car. If the Andrew left 2.5 hours before Nate began his pursuit, find $A$, the number of hours it takes Nate to catch Andrew.

Forrest traveled to Martin's house at a rate of 10 mph. He stayed there for 30 minutes, then returned home at 5 mph. The whole trip took one hour. Let $B$ = the distance, in feet, between the two houses.

A tank contains a brine solution which is 2% salt. After 100 pounds of water have evaporated from the tank, the remaining solution is 3% salt. Let $C$ = the number of pounds of salt originally in the tank.

A 12 in. by 18 in. piece of cardboard has four squares of side length $x$ cut out of its corners. When the flaps are folded up, the open-top box thus formed has volume 160 sq. in. Find $D$, the sum of all possible $x$, in inches.

Find $\frac{BC}{AD}$.

**Question 6**

There are four numbers in geometric progression in which the third term is greater than the first term by 9, and the second term is greater than the fourth term by 18. Let $A$ = the common ratio.

The sum of the first four terms of an arithmetic progression is 56, and the sum of the last four terms of the same progression is 112. If the first term is 11, find the total number of terms, $B$.

$X$, $Y$, $Z$ is a geometric progression, and $X + Y + Z = 56$. $X - 1, Y - 7, Z - 21$ is an arithmetic progression. Find $C$, the smallest possible number in the arithmetic progression.

Find $B^{-A} + C$.

**Question 7**

The function $f(x) = 2x^2 - 24x + c$ has two real zeros that differ by 18. Find $c$, then let $A$ = $f(x)$.

Let $B$ = the equation of the parabola (with horizontal axis of symmetry) that passes through (–1, 1), (11, –2), and (5, –1).

Find the sum of the quadratic and constant terms of both $A$ and $B$.

**Question 8**

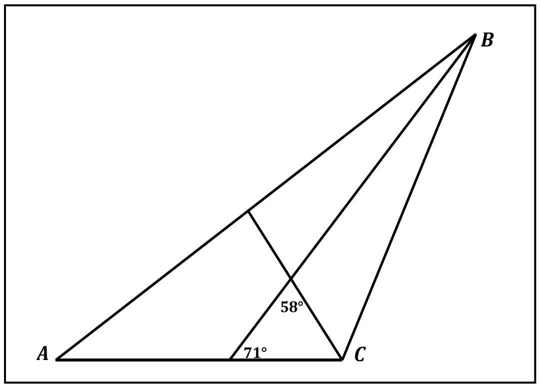
Let $A$ = the number of ordered pairs of integers $(x, y)$ that satisfy $y = \frac{2}{x}$.

Let $B$ = the number of ordered pairs of integers $(x, y)$ that satisfy $y = \frac{2x-2}{x-2}$.

Let $C$ = the number of ordered pairs of positive integers $(x, y)$ that satisfy $20x + 21y = 2021$.

Let $D$ = the degree measure of the smallest angle of $\triangle ABC$ when one angle bisector makes an angle of $71°$ with the opposite side and intersects another angle bisector at $58°$, as shown below.

Find $A+B+C+D$.



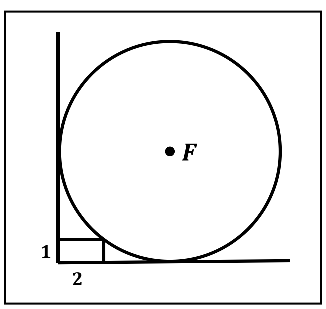

**Question 9**

A convex $n$-gon has five times as many diagonals as sides. Let $A = n$.

Let $f(x) = \ln(6-x)$ and $g(x) = \left|x^2 - 10x + 15\right|$. The domain of $f \circ g$ can be written in interval notation as $(a, b) \cup (c, d)$. Let $B = a+b+c+d$.

Two perpendicular lines form one vertex of a $1 \times 2$ rectangle. Circle $F$, that is tangent to the two lines, passes through the opposite vertex of the rectangle, as shown. If the interiors of the circle and rectangle do not overlap, find $C$, the radius of the circle.

Find $A + BC$.



## Question 10

Let $(A, B)$ be the real-valued $(x, y)$ ordered pair for which $(x-iy)(3+5i)$ is the conjugate of $-6-24i$, where $i=\sqrt{-1}$.

Let $C$ be the modulus of $\frac{1+i}{1-i}-\frac{1-i}{1+i}$, where $i=\sqrt{-1}$.

Let $D$ be the simplified value of $\frac{(\log_8 3)(\log_7 125)}{(\log_{49} 25)(\log_4 9)}$.

Find $\det\begin{bmatrix} A & B \\ C & D \end{bmatrix}$.

## Question 11

Let $A$ = the coefficient of the 13th term in the expansion of $\left(9x-\frac{1}{3\sqrt{x}}\right)^{18}$.

Let $B$ = the value of $a$ so that the 17th and 18th terms of the expansion of $(2+a)^{50}$ are equal.

Let $C$ = the coefficient of $x^3$ in the expansion of $\left(1+x+x^2\right)^3$.

Find $\frac{A}{BC}$.

## Question 12

Given $12y^2-4x^2+72y+16x+44=0$.
Let $(A, B)$ = center
Let $(C, D)$ and $(E, F)$ = vertices
Let $(G, H)$ and $(I, J)$ = foci
Let K = eccentricity
Let $L$ = the square of the length of conjugate axis
Let $M$ = length of transverse axis
Let $N$ = focal width
Let $O$ and $P$ = the constants in the equations of the directrices
Let $Q$ = the square of the reciprocal of the product of the slopes of the asymptotes
Find $A+B+C+D+E+F+G+H+I+J+K+L+M+N+O+P+Q$.

## Question 13

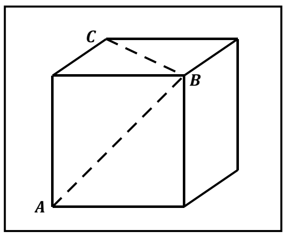
Let $A$ = the length of the hypotenuse of a right triangle such that the sum of the squares of the lengths of the three sides is 200.

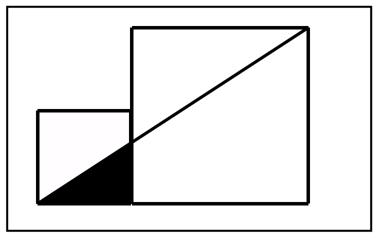
Equilateral triangle $ABC$ has side length 12. Points $X$ and $Y$ trisect $\overline{AC}$ so that $\triangle ABX$, $\triangle XBY$, and $\triangle YBC$ all have equal area; however, these three triangles do not have the same perimeter. If $a-b\sqrt{c}$ (when written in simplest form) is the positive difference between the perimeter of $\triangle ABX$ and $\triangle XBY$, find $B=a+b+c$.

Let $C$ = the degree measure of $\angle ABC$ for the cube shown.

Let $D$ = the area of the shaded triangle if the squares have side lengths 2 and 5.

Find $A+B+C$.



## Question 14

Part I: Find the $(x, y)$ point of intersection of $2x+3y=11$ and $2x-4y=-24$, then use those values to find $m$ so that $y=mx+3$.

Part II: Using the value of $m$ from above, find the two points of intersection of $y=mx+3$ with $(x-4)^2+(y+2)^2=5$.

Part III: Using the two points found in Part II, find the point on the $y$-axis that is equidistant to those two points.

Part IV: Find the square of the distance between the point you found in Part III and the point of intersection that you found in Part I.

Your final answer is the answer to Part IV.