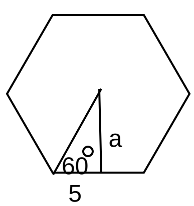
2022 MAΘ NATIONAL CONVENTION
THETA CPAV ANSWERS and SOLUTIONS

ANSWERS:

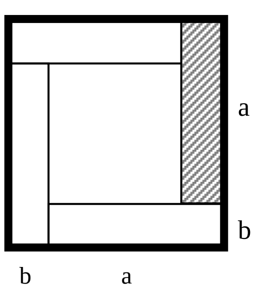
1. B.	11. B	21. A
2. B	12. C	22. D
3. A	13. C	23. C
4. A	14. A	24. D
5. C	15. C	25. B
6. B	16. D	26. D
7. D	17. D	27. A
8. B	18. B	28. C
9. D	19. A	29. A
10. C	20. C	30. D

SOLUTIONS:

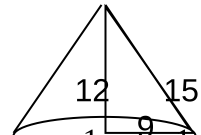
1. **B.** $4\pi r^2 = \frac{4}{3}\pi r^3$. $r \neq 0$ and so divide by $4\pi r^2$. Get r=3.
2. **B.** Add the expressions: $12n+21=261$. $12n=240$. n=20. Sides are 92, 70 and 99 and the longest side is 99 cm.
3. **A.** $\frac{3}{2} side^2\sqrt{3} = 150\sqrt{3}$. side = 10.
   Apothem is $5\sqrt{3}$



4. **A.** Diagonals are perpendicular and bisect each other so two diagonals create 4 congruent right triangles in the interior of the rhombus. Each is a 6-8-10 triple. The second diagonal is 16 cm.
5. **C.** We are told $2a+2b=28$. So $a+b=14$. And since the large square has side length $(a+b)$ the area of the square is $(a+b)^2$ = 14(14)=196.

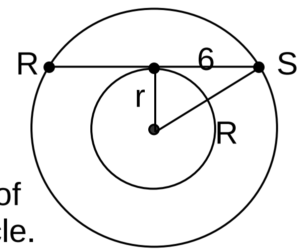


6. **B.** Use the Pythagorean Theorem to get radius 9.

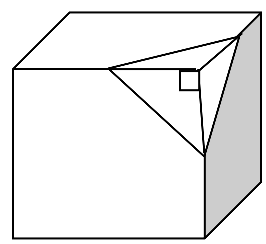


V = $\frac{1}{3}Bh = \frac{1}{3}(81\pi)(12) = 324\pi$

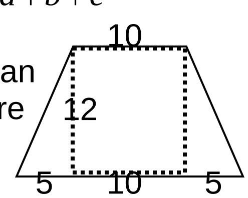
7. **D.** $4\pi r^2 = 40\pi$ and the great circle of the sphere has area πr^2 for the same radius. So divide by 4. Answer is 10π



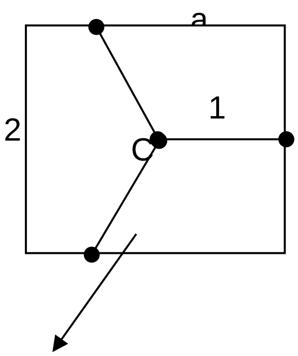
8. **B.** Let R be the radius of the large circle and r of the small circle.
   $R^2 - r^2 = 36$ by the Pythagorean Th.
   Since area of the annulus is $\left(R^2 - r^2\right)\pi$ the answer is 36π
9. **D.** The old surface was 6(20)(20) = 2400. Now we subtract the 3 right triangles each with area $\frac{1}{2}(10)(10) = 50$. Then add the additional equilateral triangle face of area $\frac{\left(10\sqrt{2}\right)^2}{4}\sqrt{3}$. $2400 - 150 + 50\sqrt{3} =$ $2250 + 50\sqrt{3}$.= $a + b\sqrt{c}$. $a+b+c$ = 2250+50+3=2303

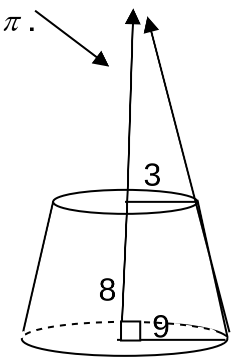


10. **C.** Use the Pythagorean Theorem to get legs are each 13. Then P = 10+13+20+13=56



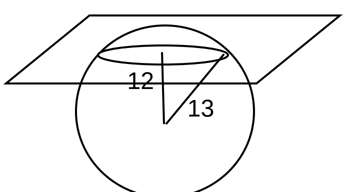
11. **B.** Extend the slant height in the diagram and the center axis and use similar triangles. $\frac{h}{3}=\frac{h+8}{9}$. Solve to get h = 4. The surface area is The "large cone's" LA, minus the small cone's LA and add the two base areas.
LA= $\frac{1}{2}(sl)(C)$. Since h=4, the top slant height is 5 and the large cone slant height is 15. SA=
$\frac{1}{2}(15)(18\pi)-\frac{1}{2}(5)(6\pi)+9\pi+81\pi$ =
$135\pi-15\pi+90\pi$ = 210π.





12. **C.** In one trapezoid, $\frac{1}{2}(1)(a+1)=\frac{4}{3}$ since the trapezoid is 1/3 of the area of the square. So a = $\frac{5}{3}$.

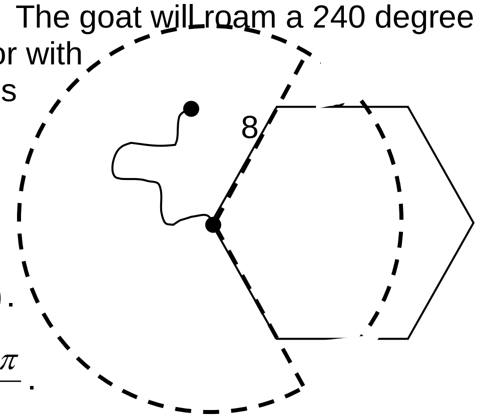
13. **C.** Use the Pythagorean Th to get r=5.
Area = 25π



14. **A.** The sector arc length will become the circumference of the cone. $\frac{288}{360}(20\pi)=2\pi r$ gives r=8 for the cone. The sector radius will become the slant height of the cone. So the cone has a slant height of 10 and r=8 for h=6. $V=\frac{1}{3}(64\pi)(6)$ = 128π.

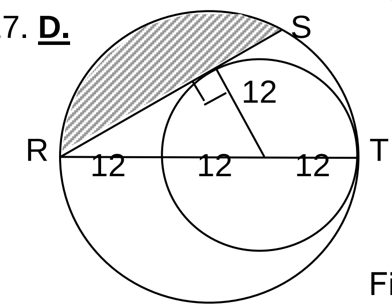
15. **C.** Area = $\frac{1}{2}ap=\frac{1}{2}(k)(60)=30k$.

16. **D.** The goat will roam a 240 degree sector with radius 8. This is area $\frac{2}{3}(64\pi)$. Or $\frac{128\pi}{3}$.

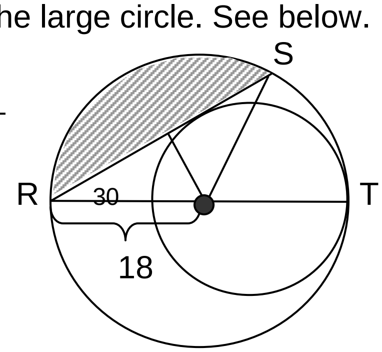


With the extra 2 ft of leash, the goat can wrap a corner to get $\frac{60}{360}(4\pi)$ on each side. This makes $\frac{128\pi}{3}+2\left(\frac{2}{3}\pi\right)$ = 44π

17. **D.**



Draw the radius of the small circle to the point of tangency of the chord. Fill in three "12" values. Now the large circle has a diameter 36, so subtracting the two radii values on RT, we see that from R to the small circle is also 12. We have then a right triangle with leg 12, and hypotenuse 24 so angle R must be 30 degrees. And the arc labeled TS above must be 60^{o}. The area shaded is that of a 120 degree segment of the large circle. See below.
We want
$\frac{120}{360}(18)(18)\pi-$
$\frac{1}{2}(9)(18\sqrt{3})$
= $108\pi-81\sqrt{3}$



18. **B.** The ratio of the areas is !:4 so the ratio of the sides is 1:2. $\frac{1}{2}=\frac{20}{YZ}$ and YZ=40.

19. **A.** Radii are 1/3 and 1/6. So volume is $\pi\left(\frac{1}{9}\right)(3)$ $-\pi\left(\frac{1}{36}\right)(3)$ = $\frac{\pi}{4}$.

20. **C.** If the original area is $\frac{1}{2}bh$ then the new area is $(1.10b)(0.90h)$ = $\frac{1}{2}(0.99bh)$ and the change is from 0.5bh to 0.495bh. This is a decrease of 0.005bh and the percent decrease is 0.005bh/0.5bh which is 0.01 = 1%.

21. **A.** Let the area be 1. $s^2 = 1$ and $\pi r^2 = 1$. $r^2 = \frac{1}{\pi}$. Approximate this as 1/3 and so the radius is about $\sqrt{\frac{1}{3}}$ This is smaller than 1. So $r < s$.

22. **D.** Dimensions are x and 4x so fencing is 10x feet. $f = 10x$ and $x = \frac{1}{10}f$ so now dimensions are $\frac{1}{10}f$ and $\frac{2}{5}f$. That means area is $\frac{1}{25}f^2$

23. **C.** Volume of the pyramid is $\frac{1}{3}(100)(12) = 400$. If 90% is below water then 10% is above water. This is 40 cubic km.

24. **D.** One exterior angle is 180-179.5=0.5 degrees so there are 360/0.5=720 sides. Perimeter is 5(720)=3600

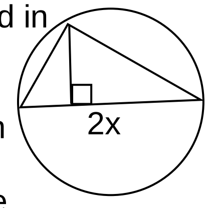
25. **B.** $(4k+8):(50+25k)$ = $\frac{4(k+2)}{25(k+2)} = \frac{4}{25}$.

The perimeter ratio is 2:5 so $\frac{2}{5} = \frac{k+2}{50}$.

100= 5k+10 so k = 18.

26. **D.** The semi-perimeter of ΔRST is 16, so using Heron's Formula, we have area is $\sqrt{16(16-8)(16-10)(16-14)}$ = $\sqrt{16(8)(6)(2)} = 16\sqrt{6}$

27. **A.** A triangle inscribed in a semicircle is a right triangle. The hypotenuse has length 2x. The height to that hypotenuse is from the circle to the diameter (2x). The greatest height will be from the horizontal diameter shown to the max point of the circle, which will be a radius, x.



So the greatest area is $\frac{1}{2}(2x)(x) = x^2$.

28. **C.** The side of the cube will be $\frac{2\sqrt{6}}{\sqrt{3}}$ = $2\sqrt{2}$. Volume will be $\left(2\sqrt{2}\right)\left(2\sqrt{2}\right)\left(2\sqrt{2}\right) = 16\sqrt{2}$

29. **A.** Hypotenuse is $12\sqrt{6}$ cm so legs are each $12\sqrt{3}$. Area is then $\frac{1}{2}(12\sqrt{3})(12\sqrt{3}) = 216$ sq. cm.

30. **D.** Circle 1 will have diameter which is the diagonal of the square, $\sqrt{2}$ and circle 2 will have radius half the side of the square, which is ½. Areas are $\frac{1}{2}\pi$ to $\frac{1}{4}\pi$. Ratio is $2:1$.