

The term "NOTA" stands for none of the above answers is correct. Good luck and have fun!

1.) Find the altitude of an equilateral triangle whose area is  $16\sqrt{3}$ .

- A) 4                      B) 8                      C)  $4\sqrt{3}$                       D)  $8\sqrt{3}$                       E) NOTA

2.) Find the remainder when  $x^5 - 4x^4 + 6x^2 - 8x + 12$  is divided by  $(x - 2)$ .

- A) -44                      B) -12                      C) 12                      D) 44                      E) NOTA

The following information will be needed to solve questions 3-4. Alexander has a bag containing 4 red marbles, 6 white marbles, and 5 blue marbles.

3.) Suppose he picks out a marble, notes the color, then puts it back into the bag. The probability that out of 3 tries he notes a different color for each time can be written in the form  $\frac{m}{n}$  when fully reduced. What is the sum of  $m + n$ ? (Note: he is selecting some variant of red, white, and blue in any order).

- A)75                      B)91                      C)233                      D)241                      E) NOTA

4.) Suppose he picks out a marble, notes the color, then put it back into the bag. The probability that out of 3 tries he notes the same color each time can be written in the form  $\frac{m}{n}$  when fully reduced. What is the sum of  $m + n$ ?

- A) 28                      B) 41                      C) 60                      D) 83                      E) NOTA

5.)  $\triangle ABC$  has  $m\angle ABC = (2x + 21)^\circ$ ,  $m\angle CAB = (3x + 2)^\circ$ , and  $m\angle ACB = (5x - 13)^\circ$ . What is the measure of the smallest angle in  $\triangle ABC$ ?

- A)  $49^\circ$                       B)  $53^\circ$                       C)  $55^\circ$                       D)  $59^\circ$                       E) NOTA

6.) The expression  $\sqrt{18 - 8\sqrt{2}}$  can be written in the form  $a - \sqrt{b}$  where  $a$  and  $b$  are positive integers. Find the sum of  $a + b$ .

- A) 2                      B) 4                      C) 5                      D) 6                      E) NOTA

7.) An annulus is constructed by two concentric circles with radii 4 and 6 respectively. Find the area of the annulus.

- A)  $10\pi$                       B)  $12\pi$                       C)  $20\pi$                       D)  $24\pi$                       E) NOTA

8.) Jonathan is running a small trading card game company and he is trying to maximize his profits per day. The cost for making a pack of cards can be modeled by the function  $f(x) = 6x^2 - 3x - 1$ . The amount of money that Jonathan earns can be modeled by the function  $g(x) = 5x^2 + 5x + 11$ . Find the sum of how many packs of cards Jonathan should sell per day in order to maximize his profits and the amount he earns through profits by selling  $x$  card packs that maximizes profit per  $x$  card packs. Assume that Jonathan is a good salesman and that every card pack he makes will sell.

- A) 112                      B) 116                      C) 128                      D) 144                      E) NOTA

9.) A triangle has side lengths 12, 21 and  $x$ . Find the sum of the possible integral values of  $x$ .

- A) 483                      B) 516                      C) 559                      D) 612                      E) NOTA

10.) It takes 6 students 28 hours to create 4 replicas of the Washington Monument. How many hours will it take for 18 students to make 9 replicas? Assume that the rate of the students remains constant.

- A) 21                      B) 24                      C) 30                      D) 32                      E) NOTA

11.) There exists a complex number  $z$  which is written in the form  $a + bi$ . Let  $\bar{z}$  be the conjugate of  $z$ .

$z + \bar{z} = -18$ . Given that  $(z)(\bar{z}) = 82$ . Find  $a + b$  where  $b > 0$ .

- A) -10                      B) -9                      C) -8                      D) -1                      E) NOTA

12.) Find the sum of the following sequence:  $e^2\pi, e\pi, \pi, \frac{\pi}{e}, \frac{\pi}{e^2}, \frac{\pi}{e^3} \dots$

- A)  $e\pi(e - 1)$                       B)  $e^3\pi(e - 1)$                       C)  $\frac{e\pi}{e - \pi}$                       D)  $\frac{e^3\pi}{e - 1}$                       E) NOTA

13.) Given that  $f(x) = \frac{2x^3 - 3x^2 - 2x}{2x^2 + x}$ . Its holes (removable discontinuities) occur at  $x = m$  and  $x = n$ . Find  $|m + n|$ .

- A) 1                      B) 2                      C) 4                      D) 5                      E) NOTA

14.) Find the volume of the cylinder that is circumscribed about a sphere whose surface area is  $144\pi$ .

- A)  $288\pi$                       B)  $360\pi$                       C)  $384\pi$                       D)  $432\pi$                       E) NOTA

15.) In 4 years, Chad will be twice as old as Chris. Six years ago, Chad was thrice as old as Chris. Find the units digit of Chad's current age.

- A) 0                      B) 2                      C) 4                      D) 6                      E) NOTA

16.) Find the product of the solutions of  $f(x) = 4x^6 - 32x^5 + x^4 - 12x^3 + 12x^2 - 24x + 64$ .

- A) 16                      B) 8                      C) -8                      D) -16                      E) NOTA

17.) The solutions in the following equation can be written in the form  $a \pm b\sqrt{c}$ . Find the sum of  $a + b + c$ .  
 $\log_2(x + 1) - \log_4(x) = 2$

- A) 16                      B) 14                      C) 7                      D) 3                      E) NOTA

18.) Consider the data set 5, 7, 9, 9, 11,  $x$ . It is known that the range of the data set is 6 and that its median is 9. Given that  $x$  must be an integer, find the sum of the possible values for  $x$ .

- A) 10                      B) 19                      C) 21                      D) 30                      E) NOTA

19.) Find the value of  $k$  such that the slope between the points  $(4, 6)$  and  $(2, k)$  is equal to  $k$ .

- A)  $\frac{1}{4}$                       B)  $\frac{1}{2}$                       C) 2                      D) 4                      E) NOTA

20.) Which of the following is true?

I. If two different circles have the same radii, then their areas are the same.

II. If two different circles have different radii, then their eccentricities are the same.

III. If two different circles have the same center, then their areas are the same.

- A) I only                      B) II only                      C) I and II                      D) II and III                      E) NOTA

21.) Which of the following is the line equation that is perpendicular to  $y = 3x - 6$  and passes through the point  $(9, 2)$ ?

- A)  $y = \frac{-1}{3}x + 5$                       B)  $y = \frac{-1}{3}x + 2$                       C)  $y = 3x - 16$                       D)  $y = 3x + 2$                       E) NOTA

22.) A circle whose area is  $16\pi$  is inscribed within a square. That square is inscribed within a circle. Find the area of this circle.

- A)  $16\pi$                       B)  $32\pi$                       C)  $64\pi$                       D)  $128\pi$                       E) NOTA

23.) Calculate the distinct permutations exist in the word "GEMINI."

- A) 720                      B) 540                      C) 360                      D) 180                      E) NOTA

24.) How many roots does  $f(x)$  have given that  $f(x) = x^6 - 3x^5 + 4x^4 + 7x^7 - x^2 + 7x - 24$

- A) 2                      B) 4                      C) 6                      D) 7                      E) NOTA

25.) Find the area of the segment of a circle where the radius of the circle is 6 and the central angle is  $\frac{\pi}{3}$ .

- A)  $9\pi + 6\sqrt{3}$               B)  $9\pi - 6\sqrt{3}$               C)  $12\pi + 9\sqrt{3}$               D)  $12\pi - 9\sqrt{3}$               E) NOTA

26.) Solve:  $\begin{vmatrix} 2021 & 2020 \\ 2022 & 2021 \end{vmatrix}$

- A) 2                      B) 1                      C) 0                      D) -1                      E) NOTA

27.) At a party there is a total of 40 people. Thirty-three of these people know each other and the other 7 do not know anybody. At this party, people who know each other will fist bump each other when they greet each other and people who do not know each other will high five each other when they greet each other. Assume that everyone greets everybody only once. Find the positive difference between fist bumps and high fives at this party.

- A) 231                      B) 252                      C) 276                      D) 300                      E) NOTA

28.) Which of the following is true?

If two parallel lines are cut by a transversal, then:

I. Alternate exterior angles are supplementary

II. Alternate interior angles are congruent

III. Consecutive interior angles are congruent

- A) II only                      B) III only                      C) I and II                      D) II and III                      E) NOTA

29.) What is the constant term of the expansion of  $\left(x^2 - \frac{2}{x}\right)^6$ ? Note :  $x \neq 0$ .

- A) 360                      B) 480                      C) 525                      D) 560                      E) NOTA

30.) Matthew and Daniel are playing with a random integer machine in which they generate a random integer over a closed interval and the person who gets the higher integer wins. Matthew gets a random integer over the closed interval  $[0,2021]$  while Daniel gets a random integer over the closed interval  $[0,4042]$ . The probability that Matthew wins the game can be written in the form  $\frac{m}{n}$  when fully reduced. Find the sum of  $m + n$ . Assume Matthew and Daniel always get different integers.

- A) 5                      B) 7                      C) 11                      D) 13                      E) NOTA