ANSWERS and SOLUTIONS

1. C. \(-2 = \frac{a+1}{2}\) and \(3 = \frac{-8+b}{2}\) so \((a,b) = (-5,14)\). \(a+b = 9\).

2. D. Height to the longest side is \(\sqrt{16-9} = \sqrt{7}\) so
   \[A = \frac{1}{2}(6)(\sqrt{7}) = 3\sqrt{7}\] . Squared area is \(9(7) = 63\).

3. A. One exterior angle is \(360/720 = \frac{1}{2}\) of a degree. So the interior angles are
   \[180-0.5 = 179.5^\circ\]

4. B. The circle radius is the same as the side of the hexagon in length. So area is
   \[\frac{3}{2} \text{side}^2 \sqrt{3} = \frac{3}{2}(36)(\sqrt{3}) = 54\sqrt{3}\] .

5. C. \(\sqrt{22^2 + 30^2} = \sqrt{1384}\). The third side has length \(\sqrt{1440}\) which is greater. Obtuse!

6. B. Since this is a multiple of a 8-15-17 Pythagorean Triple, this is a right triangle.
   So the hypotenuse is the diameter of the circle. And \(C = \pi d = 34\pi\)

7. B. Area of \(\Delta TRU\) is 120 so \(\frac{1}{2}(12)h = 120\) and \(h = 20\), so the distance between the
   parallel lines is 20. The area of \(\Delta TSV\) is then \(\frac{1}{2}(18)(20) = 180\).

8. A. In triangle PLN, hypotenuse \(\overline{NL}\) has length
   \(30\) (a 3-4-5 triple, times 6). In triangle NYG, NY = 13.
   So YL = 30-13 = 17. Perimeter then is 18+24+17+5+12 = 76

9. D. Chord \(\overline{RS}\) has length twice \(\sqrt{225-144}\) or 18.
   Chord \(\overline{PQ}\) has length twice \(\sqrt{225-81}\) or 24.
   The positive difference is 6.

10. B. \(4x - y + x = 180\) and \(4x - y = 7y\) . This gives \(x = 2y\) and \(5x - y = 180\). \(10y-y=180\) and \(y=20\), \(x=40\). \(x + y = 60\).
11. **A.** Area = \( \frac{1}{2} (d_1)(d_2) = \frac{1}{2} (10)(24) = 120 \). Since area of a parallelogram is \( bh \) and side length is 13 (see diagram), \( 13h = 120 \) and height is \( \frac{120}{13} \).

12. **C.** Using the Geometric Mean ratios/formulas,

\[
 n^2 = (3n-6) \left( \frac{3n}{n-2} \right), \quad n^2 = 3(n-2) \left( \frac{3n}{n-2} \right), \quad n^2 = 9n.
\]

\( n=0 \) or \( n=9 \). For lengths, \( n=9 \). That makes the missing leg of the largest right triangle \( \sqrt{21^2 - 9^2} = \sqrt{360} = 6\sqrt{10} \). \( a + b = 6 + 10 = 16 \).

13. **A.** Look at \( \triangle GHN \). Fill in the side lengths for the 30-60-90 triangles and we see \( HG = 6\sqrt{3} \) which is equal to MP. Now look at \( \triangle MPG \).

\[
 MG = \sqrt{\left(6\sqrt{3}\right)^2 + 3^2} = \sqrt{108 + 9} = \sqrt{117} = 3\sqrt{13}
\]

14. **A.** Draw from \( P \) to \( S \) and from \( P \) to \( T \). This shows \( \triangle PST \) is equilateral since \( PS \) and \( PT \) are both radii, which is equal to 10. So from \( P \) to \( ST \), length is \( 5\sqrt{3} \). That makes from \( P \) to \( RU \) equal to \( 10 - 5\sqrt{3} \).

15. **C.** Let \( RS = x \). Since \( X \) divides \( x \) into lengths in the ratio of 2:3, \( RX = \frac{2}{5}x \) and \( XS = \frac{3}{5}x \). Since \( Y \) divides \( x \) into the ratio of 3:4, \( RY = \frac{3}{7}x \) and \( XY = \frac{3}{7}x - \frac{2}{5}x = \frac{1}{35}x \) and this is equal to 4. So \( RS = x = 140 \).

16. **D.** Consider the triangle \( PCQ \) in the diagram to the right.

\[
 \frac{2}{8} = \frac{r}{6} \quad \text{and} \quad r = 3/2.
\]

The small cone has slant height

\[
 \sqrt{2^2 + \left(\frac{3}{2}\right)^2} = \sqrt{4 + \frac{9}{4}} = \sqrt{\frac{25}{4}} = \frac{5}{2}.
\]

\[
 SA = \frac{1}{2} (C)(slant) + \pi r^2
\]

\[
 = \frac{1}{2} \left(3\pi\right) \left(\frac{5}{2}\right) + \frac{9}{4} \pi = \frac{24}{4} \pi = 6\pi.
\]

17. **D.** \( R(-5,4) \) and \( S(4,-2) \). The x-coordinates are 9 units apart. 9/3 means the trisection points will occur at 3 unit intervals: \( x = -5 + 3 = -2 \) and \( x = -2 + 3 = 1 \). The y-values are 6 units apart, so 6/3=2 units distance for trisection points. That is \( y = 4 - 2 = 2 \) and \( y = 2 - 2 = 0 \). So points are \( T(-2,2) \) and \( U(1,0) \).
18. **D.** Looking at the points from #17 above, we have the triangle vertices at $T(-2, 2)$ and $U(1, 0)$ and $P(2, 3)$. So $TP = \sqrt{4^2 + 1^2} = \sqrt{17}$. $TU = \sqrt{3^2 + 2^2} = \sqrt{13}$. $PU = \sqrt{1^2 + 3^2} = \sqrt{10}$ so $17 + 13 + 10 = 40$.

19. **C.** Area of the first square is $8 \times (8)$ and the area of the second is $(8)^2 + (8)^2$. Reduce the common eights and the ratio of 1: 2.

20. **B.** If the base edge is 120 then the radius of the square is $60 \sqrt{2}$. The lateral edge is 100 and so with the height we have a right triangle, shown to the right. $h = \sqrt{100^2 - (60 \sqrt{2})^2} = 10 \sqrt{100 - 72} = 10 \sqrt{28} = 20 \sqrt{7}$. $V = \frac{1}{3} (120)(120)(20 \sqrt{7}) = 96000 \sqrt{7}$

21. **D.** $2x + 10 = \frac{1}{2} (7x + 10 - (4y + 2))$. $4x + 20 = 7x - 4y + 8$. $3x - 4y = 12$ and we are told $x + y = 32$. $3(32 - y) - 4y = 12$. $y = 84$. $y = 12$ and $x = 20$. $x - y = 8$.

22. **B.** See the diagram to the right. Let the radius be $x$. Tangent segments which intersect are congruent. Now let the hypotenuse $25 = 7 - x + 24 - x$ which solves to $x=3$.

23. **A.** $\frac{1}{2} (2 \pi r) + 2r = 12$. $\pi r + 2r = 12$. $r(\pi + 2) = 12$. $r = \frac{12}{\pi + 2}$

The sector then has area $\frac{1}{6} \pi \left( \frac{12}{\pi + 2} \right)^2 = \frac{24 \pi}{\pi^2 + 4 \pi + 4} = \frac{a \pi}{\pi^2 + b \pi + c}$. $a + b + c = 24 + 4 + 4 = 32$.

24. **D.** Extend $PR$ to $ST$ and let the point of intersection be $U$. Now look at $\triangle RUS$. Using the remote interior angles to the exterior angle that is 90, we have angle $RST$ has measure 30 degrees.

25. **A.** See diagram to the right. $10(6) = x(19 - x)$.

$x^2 - 19x + 60 = 0$. $(x - 4)(x - 15) = 0$. $x$ is either 4 or 15. So since $AP < PB$, $AP = 4$ and $PB = 15$.

26. **C.** $S$ is the vertex angle. $5n + 20 + 2(2n + 35) = 180$. $9n = 90$. $n = 10$. Angle $R$ has the measure $2n + 35$, as a base angle, so the answer is 55.

27. **C.** $\frac{4}{3} \pi r^3 = 36 \pi$. $r = 3$. The diameter of the sphere is 6, which is also the space diagonal of the cube, which is $(\text{side} \sqrt{3})$. $\text{side} \sqrt{3} = 6$ means the side is $2 \sqrt{3}$.

One face has area 12, so the total surface area is $6(12) = 72$.

28. **D.** Opposite angles of a cyclic quadrilateral are supplementary, so $6x + 30 + 180 - (2x + 50) = 180$. $x = 5$. 

\[ x = \sqrt{3} \]
29. C. Pardon the misshapen arcs. The leash will go 180 degrees in a full radius of 12 ft. That is $72\pi$ sq. ft. Then at the top, it will go around the vertex, for a 120 degree sector of radius 4. That adds $\frac{1}{3}(16\pi)$ in area. At the bottom right, the leash allows 120 degrees with a radius of 8. That gives $\frac{1}{3}(64\pi)$. That gives a total of $\frac{296}{3}\pi$.

30. Let P be the center of the circle. PQ = r. Consider the right triangle drawn with hypotenuse PQ. The horizontal leg is (r-4) and since MP=r, from M, past P to the square side is r+r-4 = 2r-4 so the square side is 2r-4 and half the side is r-2. So $(r-4)^2 + (r-2)^2 = r^2$. r = 10 or 2. It cannot be 2, so r = 10 and side of the square is 16. Area of the square is 256. Answer B.