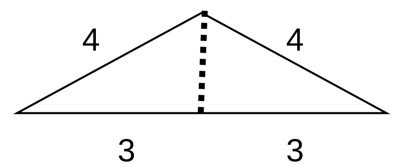
ANSWERS and SOLUTIONS

1. C	2. D	3. A	4. B	5. C	6. B	7. B	8. A	9. D	10. B
11. A	12. C	13. A	14. A	15. C	16. D	17. D	18. D	19. C	20. B
21. D	22. B	23. A	24. D	25. A	26. C	27. C	28. D	29. C	30. B

1. **C.** $-2=\frac{a+1}{2}$ and $3=\frac{-8+b}{2}$ so (a,b) = $(-5,14)$. $a+b$ = 9.

2. **D.** Height to the longest side is $\sqrt{16-9}=\sqrt{7}$ so A = $\frac{1}{2}(6)(\sqrt{7})=3\sqrt{7}$. Squared area is $9(7)=63$.



3. **A.** One exterior angle is $360\div 720=\frac{1}{2}$ of a degree. So the interior angles are 180-0.5=179.5^{o}

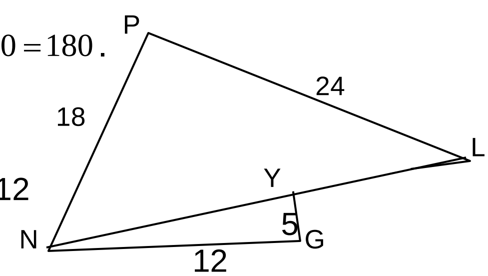
4. **B.** The circle radius is the same as the side of the hexagon in length. So area is $\frac{3}{2}side^2\sqrt{3}=\frac{3}{2}(36)\sqrt{3}=54\sqrt{3}$.

5. **C.** $\sqrt{22^2+30^2}=\sqrt{1384}$. The third side has length $\sqrt{1440}$ which is greater. Obtuse!

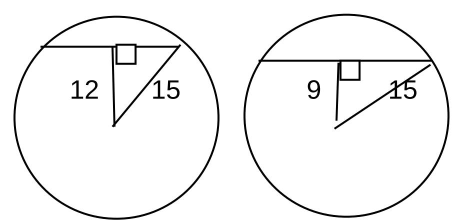
6. **B.** Since this is a multiple of a 8-15-17 Pythagorean Triple, this is a right triangle. So the hypotenuse is the diameter of the circle. And C = $\pi d=34\pi$

7. **B.** Area of ΔTRU is 120 so $\frac{1}{2}(12)h=120$ and h = 20, so the distance between the parallel lines is 20. The area of ΔTSV is then $\frac{1}{2}(18)20=180$.

8. **A.** In triangle PLN, hypotenuse $\overline{NL}$ has length 30 (a 3-4-5 triple, times 6). In triangle NYG, NY=13. So YL = 30-13=17. Perimeter then is 18+24+17+5+12 = 76

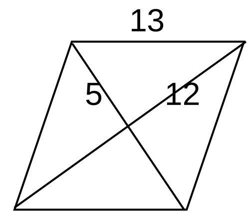


9. **D.** Chord $\overline{RS}$ has length twice $\sqrt{225-144}$ or 18. Chord $\overline{PQ}$ has length twice $\sqrt{225-81}$ or 24. The positive difference is 6.



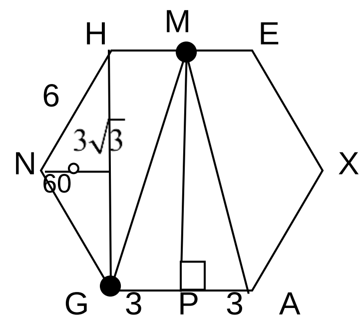
10. **B.** $4x-y+x=180$ and $4x-y=7y$. This gives $x=2y$ and $5x-y=180$. 10y-y=180 and y=20, x=40. $x+y$ = 60.

11. **A.** Area = $\frac{1}{2}(d_1)(d_2)=\frac{1}{2}(10)(24)=120$. Since area of a parallelogram is bh and side length is 13 (see diagram), $13h=120$ and height is $\frac{120}{13}$.



12. **C.** Using the Geometric Mean ratios/formulas,
$n^2=(3n-6)\left(\frac{3n}{n-2}\right)$. $n^2=3(n-2)\left(\frac{3n}{n-2}\right)$. $n^2=9n$.
n=0 or n=9. For lengths, n=9. That makes the missing leg of the largest right triangle $\sqrt{21^2-9^2}=\sqrt{360}=6\sqrt{10}$. $a+b=6+10=16$.

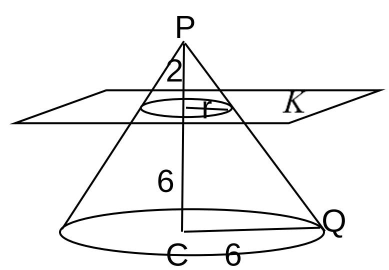
13. **A.** Look at ΔGHN. Fill in the side lengths for the 30-60-90 triangles and we see HG=$6\sqrt{3}$ which is equal to MP. Now look at ΔMPG.
MG = $\sqrt{\left(6\sqrt{3}\right)^2+3^3}=\sqrt{108+9}=\sqrt{117}=3\sqrt{13}$



14. **A.** Draw from P to S and from P to T. This shows ΔPST is equilateral since PS and PT are both radii, which is equal to 10. So from P to $\overline{ST}$, length is $5\sqrt{3}$. That makes from P to $\overline{RU}$ equal to $10-5\sqrt{3}$.

15. **C.** Let RS = x. Since X divides x into lengths in the ratio of 2:3, RX = $\frac{2}{5}x$ and XS = $\frac{3}{5}x$. Since Y divides x into the ratio of 3:4, RY= $\frac{3}{7}x$. XY = $\frac{3}{7}x-\frac{2}{5}x$ = $\frac{1}{35}x$ and this is equal to 4. So RS = x= 140.

16. **D.** Consider the triangle PCQ in the diagram to the right.
$\frac{2}{8}=\frac{r}{6}$ and r = 3/2. The small cone has slant height
$\sqrt{2^2+\left(\frac{3}{2}\right)^2}=\sqrt{4+\frac{9}{4}}=\sqrt{\frac{25}{4}}=\frac{5}{2}$. SA = $\frac{1}{2}(C)(slant)+\pi r^2$
= $\frac{1}{2}(3\pi)\left(\frac{5}{2}\right)+\frac{9}{4}\pi=\frac{24}{4}\pi=6\pi$.

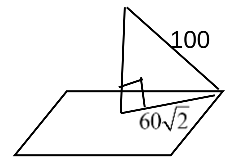


17. **D.** R$(-5,4)$ and S$(4,-2)$. The x-coordinates are 9 units apart. 9/3 means the trisection points will occur at 3 unit intervals: x= -5 + 3= -2 and x= -2 + 3 = 1. The y-values are 6 units apart, so 6/3=2 units distance for trisection points. That is y = 4 – 2 = 2 and y=2 – 2 =0. So points are T(-2, 2) and U(1, 0).

18. **D.** Looking at the points from #17 above, we have the triangle vertices at T(-2, 2) and U(1, 0) and P(2, 3). So TP = $\sqrt{4^2+1^2}=\sqrt{17}$. TU = $\sqrt{3^2+2^2}=\sqrt{13}$. PU = $\sqrt{1^2+3^2}=\sqrt{10}$ so 17+13+10 = 40.

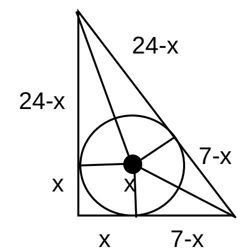
19. **C.** Area of the first square is 8(8) and the area of the second is $8\sqrt{2}\left(8\sqrt{2}\right)$. Reduce the common eights and the ratio of 1: 2.

20. **B.** If the base edge is 120 then the radius of the square is $60\sqrt{2}$. The lateral edge is 100 and so with the height we have a right triangle, shown to the right. h = $\sqrt{100^2-(60\sqrt{2})^2}=10\sqrt{100-72}=10\sqrt{28}=$ $20\sqrt{7}$. V = $\frac{1}{3}(120)(120)(20\sqrt{7})=$ $96000\sqrt{7}$



21. **D.** $2x+10=\frac{1}{2}(7x+10-(4y+2))$. $4x+20=7x-4y+8$. $3x-4y=12$ and we are told $x+y=32$. $3(32-y)-4y=12$. $7y=84$. y=12 and x=20. $x-y=8$.

22. **B.** See the diagram to the right. Let the radius be x. Tangent segments which intersect are congruent. Now let the hypotenuse $25=7-x+24-x$ which solves to x=3.

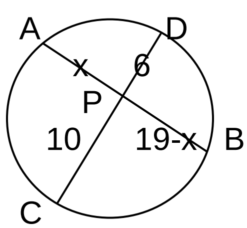


23. **A.** $\frac{1}{2}(2\pi r)+2r=12$. $\pi r+2r=12$. $r(\pi+2)=12$. $r=\frac{12}{\pi+2}$

The sector then has area $\frac{1}{6}\pi\left(\frac{12}{\pi+2}\right)^2=\frac{24\pi}{\pi^2+4\pi+4}=\frac{a\pi}{\pi^2+b\pi+c}$. $a+b+c$ = 24+4+4 = 32.

24. **D.** Extend $\overleftrightarrow{PR}$ to $\overleftrightarrow{ST}$ and let the point of intersection be U. Now look at ΔRUS. Using the remote interior angles to the exterior angle that is 90, we have angle RST has measure 30 degrees.

25. **A.** See diagram to the right. $10(6)=x(19-x)$. $x^2-19x+60=0$. $(x-4)(x-15)=0$. x is either 4 or 15. So since $AP<PB$, AP=4 and PB=15.

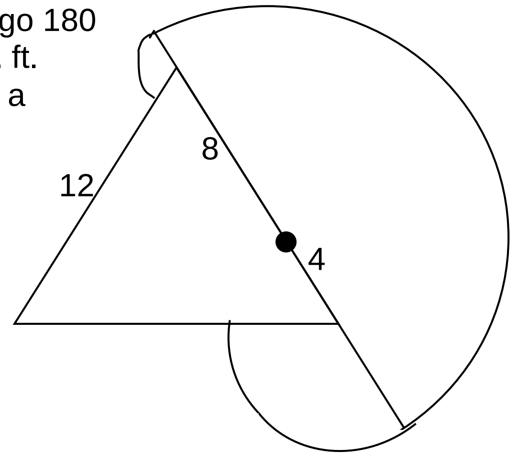


26. **C.** S is the vertex angle. $5n+20+2(2n+35)=180$. $9n=90$. n=10. Angle R has the measure 2n+35, as a base angle, so the answer is 55.

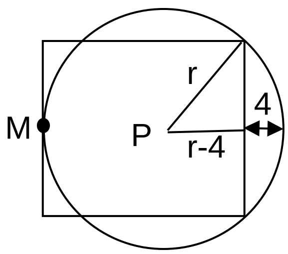
27. **C.** $\frac{4}{3}\pi r^3=36\pi$. r = 3. The diameter of the sphere is 6, which is also the space diagonal of the cube, which is $\left(side\sqrt{3}\right)$. $side\sqrt{3}=6$ means the side is $2\sqrt{3}$. One face has area 12, so the total surface area is 6(12)=72.

28. **D.** Opposite angles of a cyclic quadrilateral are supplementary, so $6x+30+180-(2x+50)=180$. x=5.

29. **C.** Pardon the misshapen arcs. The leash will go 180 degrees in a full radius of 12 ft. That is 72π sq. ft. Then at the top, it will go around the vertex, for a 120 degree sector of radius 4. That adds $\frac{1}{3}(16\pi)$ in area. At the bottom right, the leash allows 120 degrees with a radius of 8. That gives $\frac{1}{3}(64\pi)$. That gives a total of $\frac{296}{3}\pi$.



30.



Let P be the center of the circle. PQ = r. Consider the right triangle drawn with hypotenuse PQ. The horizontal leg is (r-4) and since MP=r, from M, past P to the square side is r+r-4 = 2r-4 so the square side is 2r-4 and half the side is r-2. So $(r-4)^2+(r-2)^2=r^2$. r =10 or 2. It cannot be 2, so r=10 and side of the square is 16. Area of the square is 256. Answer **B.**