For this test, E) NOTA means “None of These Answers.” As usual, \( i = \sqrt{-1} \) and all functions are restricted to their traditional domains and ranges unless otherwise specified. Good luck and have fun!

1) Simplify the following: \( \log_4(256) \cdot \log_8(32768) \cdot \log_{1/2}(32) \).
   
   A) \(-300\)  
   B) \(-100\)  
   C) \(100\)  
   D) \(300\)  
   E) NOTA

2) Compute the product of the solutions for the equation: \( [\log_9(x)]^2 - \log_{1/27}(x) + 5 = 0 \).
   
   A) \(\log_3(12)\)  
   B) \(12\)  
   C) \(3^{12}\)  
   D) \(3\)  
   E) NOTA

3) What is the maximum value of the function \( y = \frac{13}{1 + e^{-1.4x}} \)?
   
   A) 1.4  
   B) \(\ln(13)\)  
   C) 13  
   D) Infinity  
   E) NOTA

4) How many integer values of \( x \) satisfy the following inequality: \( 2^{16} < 3^x < 2^{32} \)? Hint: \( \log_3 2 \approx 0.631 \).
   
   A) 0  
   B) \(2\)  
   C) \(5\)  
   D) \(10\)  
   E) NOTA

5) Consider the function \( f(x) = 2 \log_3 (x - 3) \). How many of the following statements are true?
   
   I. The domain of \( f \) is \( x \geq 3 \)
   
   II. The range of \( f \) is all Real numbers
   
   III. The inverse of \( f \) is \( f^{-1}(x) = 3^{x/2} + 3 \)
   
   A) 0  
   B) \(1\)  
   C) \(2\)  
   D) \(3\)  
   E) NOTA

6) For what value of \( x \) is the following equation true: \( \sum_{n=0}^{\infty} 2^{nx} = \frac{3}{2} \)?
   
   A) \(-1\)  
   B) \(-\log_2(3)\)  
   C) \(-2\)  
   D) \(-\log_2(5)\)  
   E) NOTA

7) Compute \( i^{1+6i+5+3i+2+4+8+9+63-30+12} \).
   
   A) \(i\)  
   B) \(-1\)  
   C) \(-i\)  
   D) \(1\)  
   E) NOTA

8) Compute the distance from the point \( 0 + 0i \) to \( (3 + 4i)^4 \).
   
   A) \(-527\)  
   B) \(-336\)  
   C) \(125\)  
   D) \(625\)  
   E) NOTA
9) How many times do the graphs of $y = 2^{x^2 - 1}$ and $y = 4^{1-x^2}$ intersect?
A) 0 B) 1 C) 2 D) 3 E) NOTA

10) Given that $\log 5 \approx 0.699$, which of the following is closest to $\log(3125)$?
A) 2.401 B) 3.495 C) 5 D) 9 E) NOTA

11) Which of the following intervals contains the value of $2000 \cdot \log 2005$? Note: $\log 2 \approx 0.301$
A) $[10^4, 2 \cdot 10^4]$ B) $[2 \cdot 10^4, 3 \cdot 10^4]$ C) $[3 \cdot 10^4, 4 \cdot 10^4]$ D) $[4 \cdot 10^4, 5 \cdot 10^4]$ E) NOTA

12) Which of the following is the largest number: $6^{99}, 7^{75}, 8^{50}$?
A) $6^{99}$ B) $7^{75}$ C) $8^{50}$ D) All equal E) NOTA

13) Find the sum of the solutions for $4e^x + \frac{5}{e^x} = 9$.
A) 0 B) $\frac{5}{4}$ C) $\ln \frac{4}{5} + 1$ D) $\ln \frac{5}{4} + 1$ E) NOTA

14) How many times do the graphs of $y = 2^x$ and $y = x^2$ intersect?
A) 1 B) 2 C) 3 D) 4 E) NOTA

15) Which of the following conditions is not a restriction on the equation $y = \log_b(a)$?
A) $b > 0, b \neq 1$ B) $a \geq 0$ C) $y \in \mathbb{R}$ D) All are true E) NOTA

16) Compute the unique value of $t$ such that $\sum_{n=0}^{t} \log_5 (n + 1) = 1 + \log_5(5040)$.
A) $t = 8$ B) $t = 7$ C) $t = 6$ D) $t = 5$ E) NOTA

17) Compute the sum $\log \frac{1}{3} + \log \frac{2}{4} + \log \frac{3}{5} + \cdots + \log \frac{2020}{2022}$
A) $-\log (2043, 231)$ B) $-2\log (2022)$ C) $-\log (2022 \cdot 2023)$ D) $-\log (4044 \cdot 2021)$ E) NOTA
18) If \( a = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots \) and \( b = \frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \frac{4}{16} + \cdots \), compute \( 3^a 5^b \).

A) 3  B) 15  C) 75  D) 125  E) NOTA

19) For how many values of \( 0 < x < 2022 \) is \( (1 + i\sqrt{3})^x \) an integer?

A) 666  B) 673  C) 674  D) 689  E) NOTA

20) Given that \( 2022^{1/2} \) lies on the interval \((n, n + 1)\), find the sum of the digits of \( 11n \).

A) 3  B) 4  C) 8  D) 12  E) NOTA

21) Find the sum of the solutions of \((x^2 - 9x + 15)(x^2 + 4x + 4) = 1\).

A) 9  B) 7  C) 4  D) 0  E) NOTA

22) What is the units digit of \( 1^{2022} + 2^{2022} + 3^{2022} + 4^{2022} \)?

A) 0  B) 3  C) 8  D) 9  E) NOTA

23) How many ways are there to scramble the letters of the word LOGARITHM?

A) 5,040  B) 40,320  C) 362,880  D) 3,628,800  E) NOTA

24) Which of the following interest models gains the most interest over time?

A) Simple Interest \( (A = PRT) \)  B) Compound Annually \( (A = P(1 + r)^t) \)

C) Compound Monthly \( (A = P \left( 1 + \frac{r}{12} \right)^{12t}) \)  D) Compound Continuously \( (A = Pe^{rt}) \)  E) NOTA

25) Speaking of interest, suppose Brighten puts $10,000 in an interest account that is compounded annually (see answer choice B from last problem) with a 10% annual interest rate. Assuming he doesn’t deposit or withdraw any money during this time, how much money will he have after 4 years?

A) $13,310  B) $14,641  C) $16,105.10  D) $19,487.17  E) NOTA

26) Austin sends her math teacher email asking about the homework in an exponential pattern. On the first day, she sends 1 message, on the second day she sends 3 messages, 9 messages on the third day and so on. How many messages does she send in all the first seven days?

A) 2,187  B) 6,561  C) 1,093  D) 3,280  E) NOTA
27) Solve the equation: $8^x + 3 \cdot 4^x 3^x + 3 \cdot 2^x 9^x + 27^x = 2197$

A) 1  B) $\sqrt{2}$  C) $\sqrt{3}$  D) 2  E) NOTA

28) A right triangle has hypotenuse 5 and sides equal to $x$ and $\ln x$. How many values of $x$ make such a triangle possible?

A) 0  B) 1  C) 2  D) 3  E) NOTA

29) Given that $\sqrt{11} + 4\sqrt{7}$ can be written in the form $a + \sqrt{b}$, for integers $a$ and $b$, find $b^2 - a^2$

A) 33  B) 45  C) 48  D) 60  E) NOTA

30) The Fundamental Theorem of Algebra states that a polynomial function with real coefficients and degree $n$ will have $n$ complex roots (not necessarily distinct). Given that $f(x)$ is a 4th degree polynomial with two distinct roots and one double root, what are the nature of the roots of $f(x)^3$?

A) 12 total roots, the two distinct roots are now triple roots, the double root is now a sextuple root.
B) 12 total roots, all distinct.
C) 64 total roots, all distinct.
D) 7 total roots, no way to know if they are distinct.
E) NOTA