

For this test, E) NOTA means “None of These Answers.” As usual, $i = \sqrt{-1}$ and all functions are restricted to their traditional domains and ranges unless otherwise specified. Good luck and have fun!

1) Simplify the following: $\log_4(256) \cdot \log_8(32768) \cdot \log_{1/2}(32)$.

- A) **-300** B) **-100** C) **100** D) **300** E) **NOTA**

2) Compute the product of the solutions for the equation: $[\log_9(x)]^2 - \log_{1/27}(x) + 5 = 0$.

- A) **$\log_3(12)$** B) **12** C) **3^{12}** D) **3** E) **NOTA**

3) What is the maximum value of the function $y = \frac{13}{1+e^{-1.4x}}$?

- A) **1.4** B) **$\ln(13)$** C) **13** D) **Infinity** E) **NOTA**

4) How many integer values of x satisfy the following inequality: $2^{16} < 3^x < 2^{32}$? Hint: $\log_3 2 \approx 0.631$.

- A) **0** B) **2** C) **5** D) **10** E) **NOTA**

5) Consider the function $f(x) = 2 \log_3(x - 3)$. How many of the following statements are true?

I. The domain of f is $x \geq 3$

II. The range of f is all Real numbers

III. The inverse of f is $f^{-1}(x) = 3^{x/2} + 3$

- A) **0** B) **1** C) **2** D) **3** E) **NOTA**

6) For what value of x is the following equation true: $\sum_{n=0}^{\infty} 2^{nx} = \frac{3}{2}$?

- A) **-1** B) **$-\log_2(3)$** C) **-2** D) **$-\log_2(5)$** E) **NOTA**

7) Compute $i^{1+6145+312+48963-3012}$.

- A) **i** B) **-1** C) **$-i$** D) **1** E) **NOTA**

8) Compute the distance from the point $0 + 0i$ to $(3 + 4i)^4$.

- A) **-527** B) **-336** C) **125** D) **625** E) **NOTA**

9) How many times do the graphs of $y = 2^{|x^2-1|}$ and $y = 4^{1-x^2}$ intersect?

- A) 0 B) 1 C) 2 D) 3 E) NOTA

10) Given that $\log 5 \approx 0.699$, which of the following is closest to $\log(3125)$?

- A) 2.401 B) 3.495 C) 5 D) 9 E) NOTA

11) Which of the following intervals contains the value of $2000 \cdot \log 2000^5$? Note : $\log 2 \approx 0.301$

- A) $[10^4, 2 \cdot 10^4]$ B) $[2 \cdot 10^4, 3 \cdot 10^4]$ C) $[3 \cdot 10^4, 4 \cdot 10^4]$
D) $[4 \cdot 10^4, 5 \cdot 10^4]$ E) NOTA

12) Which of the following is the largest number: $6^{99}, 7^{75}, 8^{50}$?

- A) 6^{99} B) 7^{75} C) 8^{50} D) All equal E) NOTA

13) Find the sum of the solutions for $4e^x + \frac{5}{e^x} = 9$.

- A) 0 B) $\frac{5}{4}$ C) $\ln \frac{4}{5} + 1$ D) $\ln \frac{5}{4} + 1$ E) NOTA

14) How many times do the graphs of $y = 2^x$ and $y = x^2$ intersect?

- A) 1 B) 2 C) 3 D) 4 E) NOTA

15) Which of the following conditions is **not** a restriction on the equation $y = \log_b(a)$?

- A) $b > 0, b \neq 1$ B) $a \geq 0$ C) $y \in \mathbb{R}$ D) All are true E) NOTA

16) Compute the unique value of t such that $\sum_{n=0}^t \log_t(n+1) = 1 + \log_t(5040)$.

- A) $t = 8$ B) $t = 7$ C) $t = 6$ D) $t = 5$ E) NOTA

17) Compute the sum $\log \frac{1}{3} + \log \frac{2}{4} + \log \frac{3}{5} + \dots + \log \frac{2020}{2022}$.

- A) $-\log(2,043,231)$ B) $-2\log(2022)$ C) $-\log(2022 \cdot 2023)$
D) $-\log(4044 \cdot 2021)$ E) NOTA

18) If $a = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots$ and $b = \frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \frac{4}{16} + \dots$, compute $3^a 5^b$.

- A) 3 B) 15 C) 75 D) 125 E) NOTA

19) For how many values of $0 < x < 2022$ is $(1 + i\sqrt{3})^x$ an integer?

- A) 666 B) 673 C) 674 D) 689 E) NOTA

20) Given that $2022^{1/2}$ lies on the interval $(n, n + 1)$, find the sum of the digits of $11n$.

- A) 3 B) 4 C) 8 D) 12 E) NOTA

21) Find the sum of the solutions of $(x^2 - 9x + 15)^{x^2 + 4x + 4} = 1$.

- A) 9 B) 7 C) 4 D) 0 E) NOTA

22) What is the units digit of $1^{2022} + 2^{2022} + 3^{2022} + 4^{2022}$?

- A) 0 B) 3 C) 8 D) 9 E) NOTA

23) How many ways are there to scramble the letters of the word LOGARITHM?

- A) 5,040 B) 40,320 C) 362,880 D) 3,628,800 E) NOTA

24) Which of the following interest models gains the most interest over time?

- A) Simple Interest ($A = PRT$) B) Compound Annually ($A = P(1 + r)^t$)
C) Compound Monthly ($A = P\left(1 + \frac{r}{12}\right)^{12t}$) D) Compound Continuously ($A = Pe^{rt}$) E) NOTA

25) Speaking of interest, suppose Brighten puts \$10,000 in an interest account that is compounded annually (see answer choice B from last problem) with a 10% annual interest rate. Assuming he doesn't deposit or withdraw any money during this time, how much money will he have after 4 years?

- A) \$13,310 B) \$14,641 C) \$16,105.10 D) \$ 19,487.17 E) NOTA

26) Austin sends her math teacher email asking about the homework in an exponential pattern. On the first day, she sends 1 message, on the second day she sends 3 messages, 9 messages on the third day and so on. How many messages does she send in all the first seven days?

- A) 2,187 B) 6,561 C) 1,093 D) 3,280 E) NOTA

27) Solve the equation: $8^x + 3 \cdot 4^x 3^x + 3 \cdot 2^x 9^x + 27^x = 2197$

- A) 1 B) $\sqrt{2}$ C) $\sqrt{3}$ D) 2 E) NOTA

28) A right triangle has hypotenuse 5 and sides equal to x and $\ln x$. How many values of x make such a triangle possible?

- A) 0 B) 1 C) 2 D) 3 E) NOTA

29) Given that $\sqrt{11 + 4\sqrt{7}}$ can be written in the form $a + \sqrt{b}$, for integers a and b , find $b^2 - a^2$

- A) 33 B) 45 C) 48 D) 60 E) NOTA

30) The Fundamental Theorem of Algebra states that a polynomial function with real coefficients and degree n will have n complex roots (not necessarily distinct). Given that $f(x)$ is a 4th degree polynomial with two distinct roots and one double root, what are the nature of the roots of $f(x)^3$?

- A) 12 total roots, the two distinct roots are now triple roots, the double root is now a sextuple root.
B) 12 total roots, all distinct.
C) 64 total roots, all distinct.
D) 7 total roots, no way to know if they are distinct.
E) NOTA