Theta Matrices and Determinants Answers and Solutions

ANSWERS:

1. C	7. C	13. A	19. C	25. C
2. A	8. D	14. C	20. C	26. D
3. D	9. A	15. B	21. D	27. C
4. A	10. D	16. B	22. A	28. B
5. B	11. D	17. C	23. B	29. D
6. B	12. B	18. A	24. A	30. A

SOLUTIONS:

1. **C**. $\begin{bmatrix} 4 & 3 \\ -5 & 8 \end{bmatrix} \bullet \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 4(3)+3(2) \\ -5(3)+8(2) \end{bmatrix} = \begin{bmatrix} 18 \\ 1 \end{bmatrix}$

2. **A.** $\begin{bmatrix} \log_8 2 & \log_2 4 \\ \log_2 0.25 & \log_2 8 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 2 \\ -2 & 3 \end{bmatrix}$.

   The determinant is 1+4=5

3. **D.** $\begin{vmatrix} 1 & 2 & -1 \\ 0 & 1 & 3 \\ 4 & 5 & y \end{vmatrix} = (27-y)$.

   $(y+24+0)-(-4+15+0)=27-y$.

   $2y=14$. y=7.

4. **A.** $\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix} \bullet \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 35 \\ 0 \end{bmatrix}$, $x+3y=35$ and $2x-y=0$. y=2x so $x+3(2x)=35$. x=5.

5. **B.** The denominator matrix, by Cramer's Rule, uses the coefficients of the non-constant terms. Choice B is the only possible answer (except E), so we check the numerator matrix. It should be the same as the denominator except for the "x" coefficients, which should be replaced by the constant terms. Answer is B.

6. **B.** By Cramer's Rule again,

   $$y = \frac{\begin{vmatrix} 1 & -5 & 3 \\ 2 & 16 & -1 \\ 3 & 23 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & -1 & 3 \\ 2 & 1 & -1 \\ 3 & 2 & 0 \end{vmatrix}}.$$ The numerator is 32

   and denominator is 8. D = 8.

7. **C.** $\begin{vmatrix} 2a & 10 \\ -1 & 3 \end{vmatrix} = 52$, $6a+10=52$ so $6a=42$ and $(3a)$ =21.

8. **D.** The formula is $\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$ given that the result is positive, as area is. The answer is D.

9. **A.** Using the clue given by the Cramer's Rule "set-up" we have equations $\begin{cases} 2x-3y=23 \\ 3x+5y=25 \end{cases}$. You can evaluate the given y-value and substitute, or you can just solve for x. The solution is (10, -1) and the answer is x=10.

10. **D.** $A = \begin{bmatrix} 4 & \left(1-\sqrt{x}\right) & 3 \\ 1 & \sqrt{x} & 5 \\ 2 & \sqrt{x} & -1 \end{bmatrix}$ has

    determinant

    $\left(-4\sqrt{x}+10\left(1-\sqrt{x}\right)+3\sqrt{x}\right)-\left(6\sqrt{x}+20\sqrt{x}-(1-\sqrt{x}\right)$

    = -217. $-38\sqrt{x}=-217-10-1$.

    $\sqrt{x}=\frac{228}{38}=6$. Since we want $\sqrt{x}\left(\sqrt{x}+5\right)$

    this gives $6(6+5)=66$

11. **D.** Using row 2, we have

$$(-1)(0)\begin{vmatrix}2&-1&5\\1&3&4\\4&1&2\end{vmatrix}+(1)(1)\begin{vmatrix}1&-1&5\\1&3&4\\5&1&2\end{vmatrix}+$$

$$(-1)(0)\begin{bmatrix}1&2&5\\1&1&4\\5&4&2\end{bmatrix}+(1)(2)\begin{bmatrix}1&2&-1\\1&1&3\\5&4&1\end{bmatrix}.$$

= $1\big((6-20+5\big)-(75+4-2))$ +
$2\big((1+30-4\big)-(-5+12+2))$
= $1(-9-77)+2(27-9)$ = -86+36= -50.

12. **B.** $f\left(\begin{vmatrix}x&6\\2&1\end{vmatrix}\right)=4-2x$ so $f(x-12)=4-2x$. The inverse function for $y=x-12$ is $x=y-12$ or $y=x+12$. So $f((x+12)-12)=4-2(x+12)$ = $-2x-20$ and for x= -30, we have 40.

13. **A.** $\begin{vmatrix}\frac{1}{n}&\frac{-1}{n-1}\\6&n+1\end{vmatrix}=-\frac{3}{2}$, $\frac{n+1}{n}+\frac{6}{n-1}=-\frac{3}{2}$.

Multiply by the LCD of $2n(n-1)$ to get $2(n-1)(n+1)+12(n)=-3n(n-1)$.
$2\left(n^2-1\right)+12n=-3n^2+3n$.
$5n^2+9n-2=0$. $(5n-1)(n+2)=0$ so over integers, n= -2.

14. **C.** The inverse of $\begin{bmatrix}a&b\\c&d\end{bmatrix}$ is $\begin{vmatrix}d&-b\\-c&a\end{vmatrix}$ multiplied by the determinant of the original, which gives $\frac{1}{-10}\begin{bmatrix}-1&-3\\-2&4\end{bmatrix}$=
$\begin{bmatrix}0.1&0.3\\0.2&-0.4\end{bmatrix}$

15. **B.** $f(x)=\begin{vmatrix}2&1\\5&x\end{vmatrix}\bullet x-\begin{vmatrix}x&9\\3&-1\end{vmatrix}-33$.
$f(x)=(2x-5)x-(-x-27)-33$.
$f(x)=2x^2-4x-6$ =
$2\left(x^2-2x-3\right)=2(x+1)(x-3)$ which has roots at x= -1 and x=3. That means the parabolic graph has a min at the middle of these x-coordinates, which is at x=1.
$f(1)=-8$

16. **B.** $\begin{bmatrix}3&4\\-1&7\end{bmatrix}^2+\begin{bmatrix}3&4\\-1&7\end{bmatrix}=\begin{bmatrix}a&b+1\\c-1&d\end{bmatrix}$

$\begin{bmatrix}3&4\\-1&7\end{bmatrix}\begin{bmatrix}3&4\\-1&7\end{bmatrix}+\begin{bmatrix}3&4\\-1&7\end{bmatrix}=\begin{bmatrix}a&b+1\\c-1&d\end{bmatrix}$

$\begin{bmatrix}5&40\\-10&45\end{bmatrix}+\begin{bmatrix}3&4\\-1&7\end{bmatrix}=\begin{bmatrix}a&b+1\\c-1&d\end{bmatrix}$

a = 8, b= 43, c= -10 and d=52. Sum is 93.

17. **C.** $\begin{bmatrix}(x+1)&(4-x)&3\\1&-5&(x+3)\end{bmatrix}+\begin{bmatrix}12&3&1\\7&5&3\end{bmatrix}$
= $\begin{bmatrix}(x+13)&(7-x)&4\\8&0&(x+6)\end{bmatrix}$. Note that for choice A, x=20. For choice B x= -1. For choice D, x=1. For choice C, x= -13 for the row1-column1 term but the row2-column3 term is wrong. Answer C.

18. **A.** $f(x)=\begin{vmatrix}x&2\\1&1\end{vmatrix}\bullet x$. $f(x)=(x-2)x$.
$f(x-1)$ = $(x-3)(x-1)$. Roots are 1, 3.

19. **C.** $f(x)=\left(\sum_{n=1}^{3}\begin{vmatrix}n&(n-1)\\-1&2\end{vmatrix}\right)x$ =
$f(x)=\left(\sum_{n=1}^{3}(3n-1)\right)x=$ $(2+5+8)x$ = 15x
so $f(11)$ = 165.

20. **C.** $M=\begin{bmatrix}(x-1) & 1 & 2\\ 3 & (x-1) & 1\\ 1 & 2 & (x-1)\end{bmatrix}$.

$|M|=13+2(x-1)$, so

$\left((x-1)^3+12+1\right)-\left(2(x-1)+2(x-1)+3(x-1)\right)$

= $13+2(x-1)$. $(x-1)^3+13-9(x-1)=13$

so $(x-1)^3=9(x-1)$ so x=1 or $(x-1)^2=9$ and x=10 or -8. 1+10-8=3.

21. **D.** You are really being asked for the equation of the lines. One line has slope 1 and contains (1, 0) so it is $x-y=1$. The other has slope $-\frac{1}{2}$ and contains (6, 0) so it is $x+2y=6$. The matrix equation that matches this is $\begin{bmatrix}1 & 2\\ 1 & -1\end{bmatrix}\bullet\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}6\\ 1\end{bmatrix}$, choice A.

22. **A.** $\begin{bmatrix}6 & (9-a)\\ (a+10) & 12\end{bmatrix}-2\begin{bmatrix}3 & a+1\\ (5a) & 2\end{bmatrix}=$ $\begin{bmatrix}0 & (7-3a)\\ (-9a+10) & 8\end{bmatrix}$. The sum of the elements is $-12a+25$ and this is equal to 1 when a=2.

23. **B.** $\begin{vmatrix}1 & 0\\ 0 & 1\end{vmatrix}^x+\begin{vmatrix}1 & 0\\ 0 & -1\end{vmatrix}^y$ = $1^{100}+(-1)^{101}=0$

24. **A.** $(x+2)^4-2=14$. $(x+2)^4=16$. $(x+2)=\pm 2$ which has least value for x at x= -4

25. **C.** $\left(\begin{vmatrix}(1-i) & 0\\ 0 & (1-i)\end{vmatrix}\right)^5$ = $\left((1-i)^2\right)^5$

= $(-2i)^5$ = $-32(i^5)=-32i$.

26. **D.** Using, $\left[\begin{array}{ccc:ccc}\frac{1}{2} & 0 & \frac{1}{4} & 1 & 0 & 0\\ 0 & \frac{1}{6} & -\frac{1}{2} & 0 & 1 & 0\\ -\frac{1}{4} & \frac{1}{12} & -\frac{1}{8} & 0 & 0 & 1\end{array}\right]$

Multiply row 2 by 6. Then row 1 by 2.

$\left[\begin{array}{ccc:ccc}1 & 0 & \frac{1}{2} & 2 & 0 & 0\\ 0 & 1 & -3 & 0 & 6 & 0\\ -\frac{1}{4} & \frac{1}{12} & -\frac{1}{8} & 0 & 0 & 1\end{array}\right]$.

Add ¼ of row 1, to row 3.

$\left[\begin{array}{ccc:ccc}1 & 0 & \frac{1}{2} & 2 & 0 & 0\\ 0 & 1 & -3 & 0 & 6 & 0\\ 0 & \frac{1}{12} & 0 & \frac{1}{2} & 0 & 1\end{array}\right]$

Third row times 12.

$\left[\begin{array}{ccc:ccc}1 & 0 & \frac{1}{2} & 2 & 0 & 0\\ 0 & 1 & -3 & 0 & 6 & 0\\ 0 & 1 & 0 & 6 & 0 & 12\end{array}\right]$.

Subtract last two rows.

$\left[\begin{array}{ccc:ccc}1 & 0 & \frac{1}{2} & 2 & 0 & 0\\ 0 & 0 & 3 & 6 & -6 & 12\\ 0 & 1 & 0 & 6 & 0 & 12\end{array}\right]$. 1/3 of row 2

and swap rows: $\left[\begin{array}{ccc:ccc}1 & 0 & \frac{1}{2} & 2 & 0 & 0\\ 0 & 1 & 0 & 6 & 0 & 12\\ 0 & 0 & 1 & 2 & -2 & 4\end{array}\right]$

-½ of row 3 added to row 1.

$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & -2 \\ 0 & 1 & 0 & 6 & 0 & 12 \\ 0 & 0 & 1 & 2 & -2 & 4 \end{array}\right]$. The sum of the rows of the inverse are 0, 18 and 4. Answer is D.

27. **C.** $|M|$ = $(x+2)^5$ which has 6 terms.

28. **B**. AB = $\begin{bmatrix} 27 & 34 & 22 \\ 2 & 9 & 22 \\ 21 & 32 & 21 \end{bmatrix}$. AB$C^T$ =

$\begin{bmatrix} 27 & 34 & 22 \\ 2 & 9 & 22 \\ 21 & 32 & 21 \end{bmatrix}\begin{bmatrix} 1 & 7 & 4 \\ 2 & -8 & 2 \\ 4 & 2 & -1 \end{bmatrix}$ =

$\begin{bmatrix} 183 & -39 & 175 \\ 108 & -14 & 25 \\ 169 & -67 & 160 \end{bmatrix}$, 183+(-14)+160=329

29. **D.** $\begin{bmatrix} 4 \\ -2 \end{bmatrix} \bullet \begin{bmatrix} 3 & n \end{bmatrix} = n$, $12-2n=n$ gives n=4.

30. **A.** $|M|$ = $\left(2x+\frac{1}{x}\right)^6$. Terms are of the form $C(6,n)(2x)^n\left(\frac{1}{x}\right)^{6-n}$. When n = 6-n we get a constant term. So n=3. This makes the term $C(6,3)(2x)^3\left(\frac{1}{x}\right)^3$ =

$\frac{6\bullet5\bullet4}{3\bullet2\bullet1}(2)^3$= 160