

Theta Matrices and Determinants

2022 MAΘ National Convention

The abbreviation "NOTA" found in choice E of each question means "None of the Above [answers]" and should be chosen if choices A, B, C and D are incorrect.

1. $\begin{bmatrix} 4 & 3 \\ -5 & 8 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 2 \end{bmatrix} =$

A. $\begin{bmatrix} 12 & 6 \\ -15 & 16 \end{bmatrix}$

B. $\begin{bmatrix} 12 & -15 \\ 6 & 16 \end{bmatrix}$

C. $\begin{bmatrix} 18 \\ 1 \end{bmatrix}$

D. $\begin{bmatrix} 1 \\ 18 \end{bmatrix}$

E. NOTA

2. Evaluate the determinant of the matrix:

$$\begin{vmatrix} \log_8 2 & \log_2 4 \\ \log_2 0.25 & \log_2 8 \end{vmatrix}$$

A. 5 B. 3

C. 1 D. -3

E. NOTA

3. Solve over the reals: $\begin{vmatrix} 1 & 2 & -1 \\ 0 & 1 & 3 \\ 4 & 5 & y \end{vmatrix} = (27 - y)$

A. 33 B. 28

C. 14 D. 7

E. NOTA

4. Given $\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 35 \\ 0 \end{bmatrix}$, find the value of x .

A. 5 B. 6

C. 8 D. 10

E. NOTA

5. For the system $\begin{cases} x - y + 3z = -5 \\ 2x + y - z = 16 \\ 3x + 2y = 23 \end{cases}$, which represents the value of x ?

A. $\begin{vmatrix} 1 & -1 & 3 \\ 2 & 1 & -1 \\ 3 & 2 & 0 \\ -5 & 1 & 1 \\ 16 & 1 & 1 \\ 23 & 1 & 1 \end{vmatrix}$

B. $\begin{vmatrix} -5 & -1 & 3 \\ 16 & 1 & -1 \\ 23 & 2 & 0 \\ 1 & -1 & 3 \\ 2 & 1 & -1 \\ 3 & 2 & 0 \end{vmatrix}$

C. $\begin{vmatrix} 1 & -1 & 3 \\ 2 & 1 & -1 \\ 3 & 2 & 0 \\ -5 & -1 & 3 \\ 16 & 1 & -1 \\ 23 & 2 & 0 \end{vmatrix}$

D. $\begin{vmatrix} 1 & -1 & 3 \\ 2 & 1 & -1 \\ 3 & 2 & 0 \\ -5 & 1 & 1 \\ 16 & 0 & 1 \\ 23 & 1 & 0 \end{vmatrix}$

E. NOTA

6. For the system $\begin{cases} x - y + 3z = -5 \\ 2x + y - z = 16 \\ 3x + 2y = 23 \end{cases}$, when solving with Cramer's Rule, $y = \frac{32}{D}$.

Find the value of D .

A. 10 B. 8

C. 6 D. 4

E. NOTA

7. Given $\begin{vmatrix} 2a & 10 \\ -1 & 3 \end{vmatrix} = 52$, find the value of $(3a)$.

A. -31 B. -21

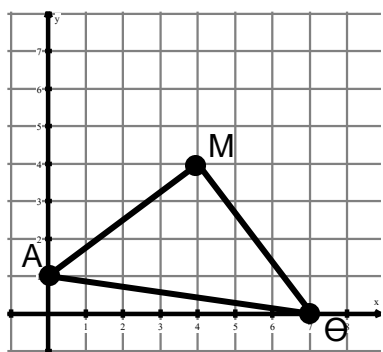
C. 21 D. 31

E. NOTA

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8. For $\triangle MA\theta$ shown, vertices are $M(4, 4)$, $A(0, 1)$ and $\theta(7, 0)$.



Which represents the area of $\triangle MA\theta$?

A. $\begin{vmatrix} 4 & 4 & 0 \\ 0 & 1 & 0 \\ 7 & 0 & 0 \end{vmatrix}$ B. $\frac{1}{2} \begin{vmatrix} 4 & 4 & 0 \\ 0 & 1 & 0 \\ 7 & 0 & 0 \end{vmatrix}$

C. $\begin{vmatrix} 4 & 4 & 1 \\ 0 & 1 & 1 \\ 7 & 0 & 1 \end{vmatrix}$ D. $\frac{1}{2} \begin{vmatrix} 4 & 4 & 1 \\ 0 & 1 & 1 \\ 7 & 0 & 1 \end{vmatrix}$

E. NOTA

9. For the system $\begin{cases} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{cases}$, the

solution (x, y) has y -value $\frac{\begin{vmatrix} 2 & 23 \\ 3 & 25 \end{vmatrix}}{\begin{vmatrix} 2 & -3 \\ 3 & 5 \end{vmatrix}}$.

Find the value of x .

- A. 10 B. 2
C. -1 D. 5
E. NOTA

10. $A = \begin{bmatrix} 4 & (1-\sqrt{x}) & 3 \\ 1 & \sqrt{x} & 5 \\ 2 & \sqrt{x} & -1 \end{bmatrix}$. If $|A| = -217$

then find the value of $\sqrt{x}(\sqrt{x} + 5)$.

- A. $\sqrt{6}$ B. $6 + 5\sqrt{6}$
C. 36 D. 66
E. NOTA

11. $M = \begin{bmatrix} 1 & 2 & -1 & 5 \\ 0 & 1 & 0 & 2 \\ 1 & 1 & 3 & 4 \\ 5 & 4 & 1 & 2 \end{bmatrix}$. Find $|M|$.

- A. 50 B. 40
C. -40 D. -50
E. NOTA

12. Given that $f\left(\begin{pmatrix} x & 6 \\ 2 & 1 \end{pmatrix}\right) = 4 - 2x$ then find $f(-30)$.

- A. 0 B. 40
C. 60 D. 120
E. NOTA

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13. Given that $\begin{vmatrix} 1 & -1 \\ n & n-1 \\ 6 & n+1 \end{vmatrix} = -\frac{3}{2}$, find the

sum of the values of n , when solved over the set of **integers**.

- A. -2 B. -1
C. 0 D. 2
E. NOTA

14. Which represents the inverse of

$$\begin{bmatrix} 4 & 3 \\ 2 & -1 \end{bmatrix}?$$

- A. $\begin{bmatrix} 0.1 & 0.2 \\ 0.3 & -0.4 \end{bmatrix}$ B. $\begin{bmatrix} 0.4 & 0.3 \\ 0.2 & -0.1 \end{bmatrix}$
C. $\begin{bmatrix} 0.1 & 0.3 \\ 0.2 & -0.4 \end{bmatrix}$ D. $\begin{bmatrix} -0.1 & 0.2 \\ 0.3 & -0.4 \end{bmatrix}$
E. NOTA

15. $f(x) = \begin{vmatrix} 2 & 1 \\ 5 & x \end{vmatrix} \cdot x - \begin{vmatrix} x & 9 \\ 3 & -1 \end{vmatrix} - 33$.

What is the minimum value of the function f ?

- A. -3 B. -8
C. -9 D. -12
E. NOTA

16. Given:

$$\begin{bmatrix} 3 & 4 \\ -1 & 7 \end{bmatrix}^2 + \begin{bmatrix} 3 & 4 \\ -1 & 7 \end{bmatrix} = \begin{bmatrix} a & b+1 \\ c-1 & d \end{bmatrix}.$$

Find $a+b+c+d$.

- A. 91 B. 93
C. 96 D. 98
E. NOTA

17. $\begin{bmatrix} (x+1) & (4-x) & 3 \\ 1 & -5 & (x+3) \end{bmatrix} + \begin{bmatrix} 12 & 3 & 1 \\ 7 & 5 & 3 \end{bmatrix}$
cannot give which resultant matrix?

- A. $\begin{bmatrix} 33 & -13 & 4 \\ 8 & 0 & 26 \end{bmatrix}$ B. $\begin{bmatrix} 12 & 8 & 4 \\ 8 & 0 & 5 \end{bmatrix}$
C. $\begin{bmatrix} 0 & 20 & 4 \\ 8 & 0 & -10 \end{bmatrix}$ D. $\begin{bmatrix} 14 & 6 & 4 \\ 8 & 0 & 7 \end{bmatrix}$
E. NOTA

18. $f(x) = \begin{vmatrix} x & 2 \\ 1 & 1 \end{vmatrix} \cdot x$. What are the roots of $f(x-1)$?

- A. 1, 3 B. 0, 2
C. -1, 1 D. -2, 0
E. NOTA

19. $f(x) = \left(\sum_{n=1}^3 \begin{vmatrix} n & (n-1) \\ -1 & 2 \end{vmatrix} \right) x$. Find $f(11)$.

- A. 22 B. 154
C. 165 D. 176
E. NOTA

20. $M = \begin{bmatrix} (x-1) & 1 & 2 \\ 3 & (x-1) & 1 \\ 1 & 2 & (x-1) \end{bmatrix}$.

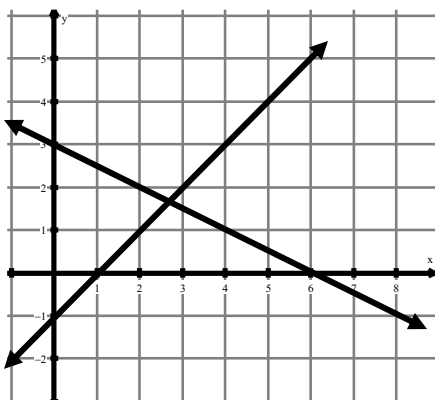
If $|M| = 13 + 2(x-1)$ then find the sum of the values of x .

- A. 1 B. 2
C. 3 D. 4
E. NOTA

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21.



L_1 has intercepts $x=1$ and $y=-1$.

L_2 has intercepts $x=6$ and $y=3$. Which

matrix equation below represents the system of L_1 and L_2 ?

A. $\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$ B. $\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 2 \\ 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \end{bmatrix}$ D. $\begin{bmatrix} 1 & 2 \\ 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$

E. NOTA

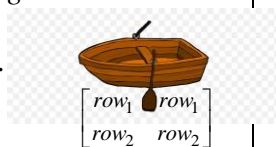
22. $\begin{bmatrix} 6 & (9-a) \\ (a+10) & 12 \end{bmatrix} - 2 \begin{bmatrix} 3 & a+1 \\ (5a) & 2 \end{bmatrix} = M$.

The sum of the elements of matrix M is 1. Find the value of a .

- A. 2 B. 3
C. 4 D. 5
E. NOTA

Why did Jerome keep destroying the oars in the team boats?

He did not like row operations.



23. For $x=100$ and $y=101$,

$$\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}^x + \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix}^y =$$

- A. -2 B. 0
C. 1 D. 2
E. NOTA

How many matrices does it take to screw in a light bulb? Only $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, but you might have to apply it more than once.



24. For $\begin{vmatrix} (x+2)^2 & 2 \\ 1 & (x+2)^2 \end{vmatrix} = 14$, give the least possible real value of x .

- A. -4 B. -2
C. $-\sqrt{14}$ D. $-2\sqrt{3}$
E. NOTA

25. Evaluate: $\left(\begin{vmatrix} (1-i) & 0 \\ 0 & (1-i) \end{vmatrix} \right)^5$

for $i = \sqrt{-1}$.

- A. 32 B. $32i$
C. $-32i$ D. -32
E. NOTA

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26. $M = \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{4} \\ 0 & \frac{1}{6} & -\frac{1}{2} \\ -\frac{1}{4} & \frac{1}{12} & -\frac{1}{8} \end{bmatrix}$, and the inverse of

M contains all integers. If the sums of the elements of the rows of M^{-1} are S_1 , S_2 and S_3 , which is **not** equal to S_1 , S_2 or S_3 ?

- A. 18 B. 0
 C. 4 D. 8
 E. NOTA

27. $M = \begin{bmatrix} (x+2)^3 & 0 \\ 1 & (x+2)^2 \end{bmatrix}$. $|M|$, written

as $a_1x^n + a_2x^{n-1} + \dots + a_{n+1}x^0$, has how many non-zero terms?

- A. 4 B. 5
 C. 6 D. 7
 E. NOTA

28. For matrices $A = \begin{bmatrix} 3 & 4 & 6 \\ 5 & -1 & 0 \\ 1 & 2 & 7 \end{bmatrix}$,

$B = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 1 & -2 \\ 2 & 4 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 2 & 4 \\ 7 & -8 & 2 \\ 1 & 5 & -1 \end{bmatrix}$,

the matrix $ABC^T = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$.

Give $a_{11} + a_{22} + a_{33}$.

- A. 128 B. 329
 C. 2011 D. 2022
 E. NOTA

29. $\begin{bmatrix} 4 \\ -2 \end{bmatrix} \cdot \begin{bmatrix} 3 & n \end{bmatrix} = n$ is true for what value of n ?

- A. 12 B. 8
 C. 6 D. 4
 E. NOTA

30. $M = \begin{bmatrix} \left(2x + \frac{1}{x}\right)^3 & 0 \\ 1 & \left(2x + \frac{1}{x}\right)^3 \end{bmatrix}$.

$|M|$ is written as

$a_1x^n + a_2x^{n-1} + \dots + a_{n+1}x^0$. What is the non-zero constant term?

- A. 160 B. 120
 C. 64 D. 32
 E. NOTA

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