

E) NOTA means “None of These Answers.” Any question that uses the population of the United States, assume the population is 330,000,000. For the land areas, we will assume the United States lies in a flat plane, ignoring the small amount of area the slight curvature of the earth would add.

YOU MAY USE A SCIENTIFIC CALCULATOR WITH THIS TEST

For the first 5 questions, we'll be considering the 2010 Census, as well as other demographic surveys.

1) In a small town in Indiana, there are 50 households. No household is the home of more than 4 people and no household is empty. If the town's population is 178, what is the maximum number of households containing 4 people?

- A) 40 B) 41 C) 42 D) 43 E) NOTA

2) At a high school in Florida, seniors are allowed to participate in one of three clubs: Archery, Basket weaving, and Cooking. There are 72 seniors at this school. There are 4 seniors in all three clubs, 7 in Archery and Basket weaving, 12 in Archery and Cooking, and 6 in Basket Weaving and Cooking. If there are 36 in Archery, 20 in Basket weaving, and 27 in Cooking, how many seniors are **not** participating in a club?

- A) 10 B) 15 C) 20 D) 32 E) NOTA

3) A region in Virginia is comprised of 32,768 people, living in around 15 square miles of land. Rounded to the nearest whole number, what is the population density of this area in people per square mile?

- A) 2205 B) 2185 C) 2022 D) 1738 E) NOTA

4) The average age of a small town in Iowa is exactly 34 years old. One year later, the average age is still exactly 34. Which of the following would **not** be a reason the average stayed the same instead of increasing by 1 as expected?

- A) Some people moved out of town.
 B) Some people had children.
 C) The town's borders changed, changing the sample population.
 D) The census data that the average was calculated from was inaccurately reported.
 E) NOTA

5) Consult the chart. If p is the smallest possible ratio of registered voters to electoral college votes, compute the sum of the digits of $\frac{p}{360}$.

State	TX	CA	AL	WA	WY	NE
# Registered	16,200,008	22,040,040	3,700,008	4,860,000	270,000	1,250,000
Electoral Votes	38	55	9	12	3	5

- A) 9 B) 7 C) 5 D) 4 E) NOTA

6) Daniel's internship at the VA hospital as a pharmacist pays \$19 per hour, with any work past 40 hours in one week paying at the normal overtime rate ($1.5 \cdot \$19 = \28.50). If Daniel works an integer number of hours in a 2 week pay period, and earns \$2118.50 before taxes, how many hours of overtime did he work in the pay period?

- A) 21 B) 19 C) 17 D) 15 E) NOTA

For questions 7-11, we'll be considering some geography. When two states are said to be x miles apart, I am referring to the minimum possible distance between a point in the first state and point in the second state.

7) The longest interstate highway in the US stretches from Boston to Seattle. I-90 is over 3,021 miles long, which is incidentally only 70 miles **shorter** than the distance from Texas to Hawaii. If a = the sum of the three prime factors of 3021, and b = the sum of the two prime factors of 3091, find $b - a$.

- A) 345 B) 234 C) 217 D) 148 E) NOTA

8) It is well known that the largest area:perimeter ratio of any closed shape is that of the circle. Given the chart below, which of the following states is "the most circular"? Phrased differently, which state has the highest area:perimeter ratio?

State	GA	SC	CO	WY	WV
Area	$57,513 \text{ mi}^2$	$30,061 \text{ mi}^2$	$103,642 \text{ mi}^2$	$97,093 \text{ mi}^2$	$24,038 \text{ mi}^2$
Perimeter	$3,288 \text{ mi}$	$3,437 \text{ mi}$	$1,302 \text{ mi}$	$1,257 \text{ mi}$	996 mi

- A) GA B) SC C) CO D) WY E) WV

9) The longest straight-line distance between two points in the state of Florida is about 598 miles. The longest such distance in Texas is about 808 miles, in California it's about 830 miles, and in Alaska its about 2,260 miles (that chain of islands goes way out). Suppose it is possible to walk each of these distances at a constant 3 miles per hour? First round to the nearest hour, then find is the total amount of time it would take to walk all of these routes?

- A) 1499 days B) 187 hours C) 62 days, 11 hours D) 62 days, 12 hours E) NOTA

10) The sum of the areas of Kentucky and Massachusetts is 47826 mi^2 ; the sum of the areas of Massachusetts and Pennsylvania is 52543 mi^2 ; the sum of the areas of Kentucky and Pennsylvania is 84229 mi^2 . Rounded to two decimals, what is the average of the areas of these three states?

- A) $30,766.33 \text{ mi}^2$ B) $92,299.50 \text{ mi}^2$ C) $184,598.67 \text{ mi}^2$ D) $432,413.33 \text{ mi}^2$ E) NOTA

11) Some of the largest cities by area are in Alaska, and they hold these titles because they've merged with county governments so that the city takes up the entire county. The largest city by area in the lower 48 is Tribune, Kansas at 778 mi^2 , and the largest in Alaska is Sitka, coming in at 2870 mi^2 . If one were to simplify these regions to a perfect square, call the side length of square-Sitka s and the side length of square-Tribune t . On what interval is the ratio $\frac{s}{t}$?

- A) $1.25 < \frac{s}{t} < 1.75$ B) $1.75 < \frac{s}{t} < 2.25$ C) $x > 2.25$ D) No such ratio E) NOTA

12) Americans are notoriously bad at estimating the number of households where the combined income of all inhabitants is over \$500,000 per year. Surveys show Americans estimate that over 26% of households make over \$500,000, when in reality the proportion is around 1%. Assuming 3 people per household, how many households make under \$500,000 per year?

- A) 1,100,000 B) 3,300,000 C) 10,890,000 D) 108,900,000 E) NOTA

The next 3 questions are about taxes (☹️). Below is a chart of tax percentages from 2020 for single individuals.

NOTE : The federal government allows you to deduct money from your gross income to generate your Adjusted Gross Income (AGI); things like charitable donations, paying interest on student loans, and medical expenses. If you do not have many of these deductions, you're allowed to take the standard deduction, which for these questions will be \$12,400.

13) America has a progressive tax system, meaning that all of the income you earn in a specific bracket is taxed at that rate, then any income above that is taxed at the next bracket up and so on. For instance, someone who makes \$20000 per year will have the first \$9,875 taxed at 10% and the remaining \$10,125 taxed at 12% for a total tax of $0.1 \cdot \$9,875 + 0.12 \cdot \$10,125 = \$2,202.50$ owed. How much does someone earning \$75,000 (AGI) owe come tax season to the nearest dollar?

Rate	Income Interval	Width of Interval
10%	\$0 - \$9,875	9,875
12%	\$9876 - \$40,125	30,250
22%	\$40,126 - \$85,525	45,400
24%	\$85,526 - \$163,300	77,775
32%	\$163,301 - \$207,350	44,050
35%	\$207,351 - \$518,400	311,050
37%	\$518,401 and up	-

- A) \$12,000 B) \$12,123 C) \$12,290 D) \$16,500 E) NOTA

14) Suppose Steve makes a charitable donation every 10 days for 1 year (36 total donations), in the amount of 2^{n-1} pennies on the n^{th} donation and he has no other deductions. Which is the higher deduction, the standard deduction or Steve's charity?

- A) Standard Deduction B) Charitable Deduction
C) Both are equal D) Impossible to tell E) NOTA

15) Many employers will take a percentage of your paycheck and set it aside, then pay the IRS your taxes for you. Occasionally, if they take too much out of your paycheck, you will have paid too much in taxes for the year and will actually get money back come tax time. Suppose an hourly employee works 40 hours per week, 50 weeks a year, at an hourly rate of \$19.50. If the employer withholds 20% of the paycheck to be paid in taxes, and the employee claims the standard deduction (\$12,400), what should their tax return be?

- A) \$4805.50 B) \$3720.50 C) \$3812.50 D) \$3924.50 E) NOTA

16) Suppose the four corners of the state of Colorado lie on the points (18884, 12034), (19219, 12034), (19219, 12345), and (18884, 12345). What is the area of the state of Colorado in square miles?

- A) 311 B) 335 C) 96,721 D) 104,185 E) NOTA

17) When a person from another country wishes to become an American citizen, they must take a test on American history, laws, and customs. The number of questions on the test is a random integer ranging from 48 to 75. Given that a test-taker needs at least an 80% to pass, what is the probability that the test-taker passes when they score 42 points?

- A) $\frac{1}{7}$ B) $\frac{5}{28}$ C) $\frac{3}{14}$ D) $\frac{1}{4}$ E) NOTA

18) How many unique ways are there to rearrange the letters of MISSISSIPPI?

- A) 28865 B) 30210 C) 34650 D) 11! E) NOTA

19) The distance between Savannah, Georgia and Charleston, South Carolina is 82 miles. Augusta, Georgia is 128 miles from Charleston and 110 miles from Savannah. What is the area of the triangle formed by these three cities?

- A) $195\sqrt{320}$ B) $320\sqrt{195}$ C) $395\sqrt{120}$ D) $120\sqrt{395}$ E) NOTA

For Questions 20-23, use the following information: one *bit* is defined to be either 0 (off) or 1 (on). An n -bit string is a string of n 0s or 1s.

20) How many possible unique 32-bit strings can be written?

- A) 32 B) 64 C) 32^2 D) 2^{32} E) NOTA

21) Cryptography is a field of mathematical research concerned with encrypting information so that the information can be transmitted privately without others being able to intercept and read it. One particular encryption algorithm is called SHA256, which takes a piece of information and outputs a 256-bit string. If 2^{33} humans could write down two 256-bit strings per second, and they do so in an orderly manner, how many seconds would it take for all these humans to write every possible string the SHA256 algorithm could output?

- A) 2^{256} B) 2^{223} C) 2^{222} D) $2^{256/33}$ E) NOTA

22) Consider the byte, defined to be an 8-bit string. The storage of information on computer hard drives breaks from the tradition of metric prefix naming, in that a kilobyte is not 1000 bytes (10^3) but 1024 (2^{10}). This continues through all prefixes, such that a megabyte is 1024 kilobytes, and so on. Let a be the amount of bytes in a gigabyte and let b be the number of grams in a gigagram. Find $\frac{a-b}{24}$.

- A) 3,072,576 B) 3,128,256 C) 3,144,576 D) 4,072,216 E) NOTA

23) Files on computers are strings of characters, each character stored as a bit or byte, depending on the computer handling the file. The pictures you most often save on your computer or phone are called jpg's, and they use an algorithm to compress the size of the file by looking for patterns in the strings of bits and rewriting them as shorter strings. Given that the uncompressed file size of a selfie that Jason takes is 343 megabytes (MB), and the encryption algorithm modifies the file size y to the value $y = 343 + \frac{-101}{1+e^{-x}}$, where x is the number of applications of the compression algorithm, what is the minimum file size of the selfie after an essentially unlimited number of compressions?

- A) 343 MB B) 289 MB C) 264 MB D) 242 MB E) NOTA

24) Consider the 50 US states, and include Puerto Rico and Washington DC. Of the 52 territories in consideration, 13 have fewer than 2,000,000 residents. Given this information, what is the minimum average of the remaining 39 territories, to the nearest integer?

- A) 23,384,615 B) 7,794,872 C) 7,600,000 D) 5,846,154 E) NOTA

25) Suppose the government has hired a science lab to create an at-home test that tests for the Covid-19. In trials, there were 1000 test patients. Of those 1000 test patients, only 10 had Covid. Of those 10, 9 received a positive test result, and 1 received a false negative. Of the 990 without covid, there were 89 false positives and 901 true negatives. Given that person A received a positive test result, what is the probability that person A had Covid?

- A) $\frac{49}{500}$ B) $\frac{9}{1000}$ C) $\frac{9}{89}$ D) $\frac{9}{98}$ E) NOTA

26) The most recent data from the Office of Personnel Management states that the federal government employs 1,869,986 people across all branches. The state with the most federal employees is California (of course). Virginia and Maryland take the number 2 and number 4 spots respectively, which makes sense considering Washington DC is surrounded by these two states. Which Lone Star state takes the number 3 spot in total federal government employees? (All of the following states could be considered 'Lone Star' states, as they all have exactly one star on their state flag)

- A) Massachusetts B) North Carolina C) Nevada D) Texas E) NOTA

27) Let the population of the US be p . How many factors does p^2 have?

- A) 2022 B) 2025 C) 2222 D) 2160 E) NOTA

For questions 28-30, we will be going out of this world!

28) The brightness of stars is determined a logarithmic scale called *magnitude*. The smaller the number, the brighter the star. The relationship between the ratio of the brightness of two stars is the difference in their magnitudes is given by $m_2 - m_1 = -2.5 \log \frac{b_2}{b_1}$ where m is the magnitude and b is the brightness. Approximately how many times brighter is the sun whose magnitude is -26.7 and Rigel whose magnitude is 0.12 ?

- A) 1.073×10^1 B) 2.682×10^1 C) 5.346×10^{10} D) 6.607×10^{26} E) NOTA

29) Exoplanet HD 108236D orbits star HD 108236. Assume the exoplanet is in the shape of a circle. If plotted on a grid, HD 108236D is centered at (h, k) where $h = k$, has a radius of 5, and passes through the origin. It is known the center of the planet is located along the line $y = -4$. What is a possible equation of HD 108236 in the form $(x - h)^2 + (y - k)^2 = r^2$?

- A) $(x - 3)^2 + (y + 4)^2 = 25$ B) $(x - 3)^2 + (y + 4)^2 = 5$ C) $(x + 4)^2 + (y - 3)^2 = 25$
D) $(x + 4)^2 + (y - 3)^2 = 5$ E) NOTA

30) A Mu Alpha Theta alumnus represents the organization well as she is on the developmental team for the Orion space capsule that, shortly, will take astronauts back to the Moon. She was a speaker at last year's MAΘ National Convention (not this year's) speaking about trans-lunar injection trajectories. What is her name? (Another hint : E) is not the answer.)

- A) Dr. Naomi Brownstein B) Dr. Kavita Chandra C) Dr. Ann Dietrich
D) Dr. Alberta Einstein E) NOTA