2009 Mu Combinatorics

Directions: The answer "E. NOTA" stands for "None of the above answers is correct."

1) Define $\binom{n}{r} = {}_nC_r$. Determine the value of x maximizes $\binom{-x^2+8x+9}{-x^2+6x}$.

A) 2 B) 3 C) 4 D) 5 E) NOTA

2) Thirteen cards labeled A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K are fed (in that order) into a reordering machine. If the reordering machine always permutes the cards in the same way, what will the last three cards be after being fed through the reordering machine a third time given that the order of the cards after the second reordering is 10, 9, Q, 8, K, 3, 4, A, 5, J, 6, 2, 7?

A) 7, A, 3 B) 4, Q, 2 C) K, 8, 6 D) 9, 5, 2 E) NOTA

3) Define the **weight** of a cell in 4x4 tic-tac-toe to be the number of tic-tac-toes that run through that cell. How many distinct weights are there on the 4x4 tic-tac-toe board? It is no coincidence that this number is equal to the number of strategically indistinguishable first moves. (In 4x4 tic-tac-toe, a row, column, or diagonal of 4 cells constitutes a tic-tac-toe.)

A) 2 B) 3 C) 4 D) 8 E) NOTA

4) A real number n is chosen at random from the interval (0, 4). What is the average value of n^3?

A) 8 B) 16 C) 32 D) 48 E) NOTA

5) How many 7 digit telephone numbers that are divisible by 2, 3, and 5 can be made without repeating digits?

A) $9 \cdot 6!$ B) $12 \cdot 6!$ C) $27 \cdot 6!$ D) $30 \cdot 6!$ E) NOTA

6) Into how many distinct regions do 17 lines divide a plane, given that no two lines are parallel and that no three lines intersect at a single point?

A) 18 B) 154 C) 172 D) 191 E) NOTA

7) Find the 23rd derivative of $\left(2x^7 - x^3 + x^2\right)^5$ at $x = 0$.

A) $40 \cdot 23!$ B) $-40 \cdot 23!$ C) $80 \cdot 23!$ D) $-80 \cdot 23!$ E) NOTA

8) How many different ways can Ashley, Brad, Courtney, Dylan, Erin, and Frank stand in a line if Dylan must stand just behind Ashley or just in front of Courtney?

A) 180 B) 216 C) 240 D) 260 E) NOTA

9) Consider the seven element set {1, 2, 3, 4, A, L, M}. How many subsets of this set contain the element 2, but not both elements L and M?

A) 24 B) 36 C) 48 D) 56 E) NOTA

10) Find the coefficient of the x^{15} term in the expansion of $\frac{d}{dx}\left(\frac{x^2}{2}+2\right)^{12}$.

A) 165 B) 495 C) 1485 D) 7425 E) NOTA

11) Consider sets A, B, C, and D. Given $|A|=17$, $|B|=23$, $|C|=15$, $|D|=25$, $|C\cap B|=4$, and $|B\cap D|=7$. Let Q be the largest possible value of $|A\cap C\cap D|$ and let Z be the smallest possible value of $|A\cap B\cap D|$. What is Q – Z?

A) 8 B) 12 C) 15 D) 19 E) NOTA

12) A circle is partitioned into six congruent wedges. I label the wedges with a number from the set {0, 2, 5, 6, 8, 9} and makes sure no two wedges have the same number. How many distinct circles can be made in this way?

A) 60 B) 120 C) 360 D) 720 E) NOTA

13) For how many integer values of x where $0 < x < 1000$ is x^{2x} a perfect cube?

A) 333 B) 339 C) 340 D) 342 E) NOTA

14) Given that $f'(x)=\prod_{n=1}^{101}(n-x)^n$, how many relative maxima does $f(x)$ have?

A) 25 B) 26 C) 50 D) 51 E) NOTA

15) What is the minimum number of people needed to take this test to guarantee that at least three people will receive the same score?

A) 3 B) 290 C) 291 D) 292 E) NOTA

16) An ant walks on the coordinate lattice starting at (0,0) and either travels one unit in the positive x-direction or one unit in the positive y-direction with each step. How many paths can the ant take to get to (5, 8) if the ant is not allowed to pass through the lattice point (3, 2)?

A) 46 B) 280 C) 1277 D) 1287 E) NOTA

17) Determine the number of distinguishable permutations of letters in the word ACCISMUS.

A) 80640 B) 40320 C) 20160 D) 10080 E) NOTA

18) An ordered pair (m, n) of non-negative integers is called **simple** if the addition $m + n$ requires no carrying in base five. How many simple ordered pairs of three digit integers sum to 432_5?

A) 27 B) 36 C) 48 D) 60 E) NOTA

19) Ashten is reading a book where the pages are labeled in base 8. If the book has 179 (base 10) pages, then how many base 8 digits were used to number all 179 pages (starting with page 1_8)?

A) 467 B) 473 C) 474 D) 475 E) NOTA

20) Determine the number of ordered quadruples (x_1, x_2, x_3, x_4) of non-negative integers which satisfy $x_1 + x_2 + x_3 + x_4 \le 7$.

A) 20 B) 120 C) 330 D) 495 E) NOTA

21) Evaluate $\sum_{n=1}^{\infty}\left(\left[\sum_{k=0}^{n}\binom{n}{k}\right]^{-1}\right)$.

A) 1 B) 2 C) .25 D) .5 E) NOTA

22) How many triples (*x*, *y*, *z*) of positive integers are there such that *xyz* = 9800?

A) 120 B) 180 C) 240 D) 360 E) NOTA

23) Amanda, Brad, Courtney, Dylan, Elise and Frank enter a race. There are no ties and they all finish the race. Given the results of the race below, how many different outcomes are possible?

Elise finished three places ahead of Courtney. Brad finished somewhere ahead of Courtney. Amanda finished either second or fifth. Frank finished in the top 3.

A) 4 B) 6 C) 10 D) 12 E) NOTA

24) Consider the set $S = \{1, 2, \ldots 12, 13\}$. Let *x* be the number of subsets of *S* with an odd number of elements. What is the sum of the digits of *x*? (The empty set has 0 elements.)

A) 14 B) 15 C) 18 D) 19 E) NOTA

25) 7^n divides $\frac{1029!}{(490!)^2}$ evenly. Determine the largest possible value of *n*.

A) 6 B) 7 C) 8 D) 9 E) NOTA

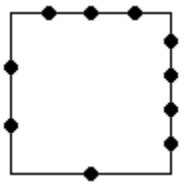
26) For how many values of *n* is $n^2 + 12$ divisible by 13? (Hint: $n^2 = n^2 + 1 - 1$)

A) 2 B) 3 C) 5 D) 6 E) NOTA

27) In which row of Pascal's triangle will you find three consecutive entries that occur in the ratio 2:4:7? (Take the top row of Pascal's triangle to be the 0^{th} row)

A) 30 B) 31 C) 32 D) 33 E) NOTA

28) The sides of a square have non-overlapping dots (as shown to the right. In how many ways can three dots be chosen so that, if connected with line segments, they form the vertices of a triangle?



A) 50 B) 72 C) 115 D) 165 E) NOTA

29) Consider the following: The function $f(x)$ is continuous and greater than 0 on [1, 9]. How many ordered triples (a, b, c) satisfy $\int_1^9 f(x)dx = \int_1^a f(x)dx + \int_b^c f(x)dx$, and $1 \le a \le b \le c$, where a, b, and c are positive integers?

A) 7 B) 8 C) 9 D) 10 E) NOTA

30) Ten cards are labeled 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. In addition, those cards labeled with a prime number are colored blue. Jessie draws two cards at random and without replacement. How many ways can she draw a 6 or a blue card on her second draw?

A) 25 B) 30 C) 45 D) 55 E) NOTA

Tie-breakers

T1) How many three digit integers don't have a 0 in them?

T2) How many factors of 144 are multiples of 2 or 3?

T3) $\sum_{i=0}^{3}\sum_{j=0}^{i}\sum_{k=0}^{j}1$