For all questions, choice E: NOTA means that none of the given answers is correct. $i = \sqrt{-1}$.

1. Find the 20th derivative of the function $f(x) = x^{21} - e^{2x} + \ln x$.

   (A) $(21!) - (2)^{20}e^{2x} + (-1)^{20}\dfrac{19!}{x^{20}}$

   (B) $(21!)x - (2)^{20}e^{2x} + (-1)^{19}\dfrac{18!}{x^{19}}$

   (C) $(20!)x - (2)^{20}e^{2x} + (-1)^{19}\dfrac{18!}{x^{19}}$

   (D) $(21!) - (2)^{19}e^{2x} + (-1)^{19}\dfrac{19!}{x^{20}}$
        (E) NOTA

2. Evaluate the limit $\displaystyle\lim_{x \to 7} \dfrac{\sqrt{x+2} - 3}{x-7}$, if it exists.

   (A) $\dfrac{1}{6}$ (B) 0   (C) 3   (D) DNE   (E) NOTA

3. The equation $3x^2 + 3y^2 - 24x + 18y = 225$ has two points with an x-value of 10. Find the sum of the values of $\dfrac{dy}{dx}$ at these two points.

   (A) -3 (B) $-\dfrac{3}{4}$ (C) $\dfrac{3}{4}$ (D) 0   (E) NOTA

4. Given $h(x) = a\cos x + b\tan x + c\sec x + d\cot x + e\sin x + f\csc x$, evaluate $h'\left(\dfrac{\pi}{4}\right)$ given that $a - e = -4$, $b - d = -3$, and $c - f = 2$.

   (A) -6 (B) $4\sqrt{2} - 6$ (C) $4\sqrt{2}$ (D) 6   (E) NOTA

5. If $f(x)$ is continuous and differentiable and $f(x) = \begin{cases} ax^4 + 5x & : x \le 2 \\ bx^2 - 3x & : x > 2 \end{cases}$, find the value of b.

   (A) 0.5   (B) 0   (C) 2   (D) 6   (E) NOTA

6. Evaluate the integral $\displaystyle\int_1^e \dfrac{\ln x}{x}\, dx$.

   (A) $\dfrac{1}{e}$ (B) $\dfrac{1}{2}$ (C) 1   (D) $\dfrac{e^2}{2}$ (E) NOTA

7. Evaluate $\displaystyle\int_0^5 |3x^2 - 18x + 15|\, dx$.

   (A) -18 (B) 7   (C) 18   (D) 39   (E) NOTA

8. The probability of Mrs. Funk's dog eating something that is bad for her, on any given day, is $\dfrac{3}{4}$. On a day where Mrs. Funk's dog does *not* eat something bad for her, the probability of Mrs. Funk having to clean up a mess when she gets home is $\dfrac{1}{4}$. On a day where Mrs. Funk's dog *does* eat something bad for her, the probability of Mrs. Funk having to clean up a mess is $\dfrac{7}{8}$. Find the probability that Mrs. Funk's dog eats something bad for her *given* that Mrs. Funk has to clean up a mess when she gets home.

   (A) $\dfrac{3}{4}$ (B) $\dfrac{7}{8}$ (C) $\dfrac{21}{23}$ (D) $\dfrac{23}{32}$ (E) NOTA

9. Evaluate $\displaystyle\int \dfrac{x^2 - 2}{x^3 - x}\, dx$.

   (A) $\ln \dfrac{x^2}{\sqrt{x^2 - 1}} + C$ (B) $2\ln x + C$ (C) $\ln(x^3 - x) + C$ (D) $\ln\sqrt{x^2 - 1} + C$ (E) NOTA

10. A cannonball is shot at a $30°$ angle from the ground with an initial velocity of 49 meters per second. If gravity is -9.8 m/s^2, how far is the cannonball from the cannon when it hits the ground?

   (A) $\dfrac{245}{2}$ meters   (B) $\dfrac{245\sqrt{3}}{2}$ meters   (C) $\dfrac{245}{6}$ meters   (D) $\dfrac{245\sqrt{6}}{6}$ meters   (E) NOTA

11. The length of the hypotenuse of a right triangle is changing at a rate of 2 inches per second. Side b is adjacent to angle θ, and remains at a constant length of 7 inches. Find the rate of change of θ when the hypotenuse is 25 inches.

(A) $\dfrac{1}{600}$ radians/second   (B) $\dfrac{2}{25}$ radians/second   (C) $\dfrac{1225}{576}$ radians/second (D) $\dfrac{7}{300}$ radians/second (E) NOTA

12. A particle travels according to the position function $p(t) = 6t^5 - 20t^3 - 90t$. Find the product of the t-coordinates of all real maximums and minimums of $p(t)$.

(A) 3   (B) -3 (C) $\sqrt{3}$ (D) $-\sqrt{3}$ (E) NOTA

13. Using Newton's method to approximate the roots of $x^3 + 4x^2 - 2 = 0$, let $x_0 = -1$. What is the value of x_2?

(A) $-\dfrac{4}{5}$ (B) $-\dfrac{227}{280}$ (C) $\dfrac{6}{125}$ (D) $\dfrac{-112}{25}$ (E) NOTA

14. What positive number's square exceeds its cube by the greatest amount?

(A) $\dfrac{1}{3}$ (B) $\dfrac{\sqrt{3}}{3}$ (C) $\dfrac{1}{2}$ (D) $\dfrac{\sqrt{5}}{4}$ (E) NOTA

15. Consider a rectangle of perimeter 12 inches. Form a cylinder by revolving this rectangle about one of its edges. What dimensions of the rectangle will result in a cylinder of maximum volume?

(A) 4×2 (B) 3×3 (C) 1×5 (D) 0.5×5.5 (E) NOTA

16. Evaluate the integral $\displaystyle\int \dfrac{x^3}{x-2}\,dx$.

(A) $\frac{1}{3}x^3 + x^2 + 4x + 8\ln|x-2| + C$ (B) $\frac{1}{3}x^3 - \frac{1}{8}x^4 + C$

(C) $\frac{1}{4}x^4 + \ln|x-2| + C$ (D) $\dfrac{2x^3 - 6x^2}{(x-2)^2} + C$ (E) NOTA

17. If $\displaystyle\int_1^{10} f(x)\,dx = 4$ and $\displaystyle\int_{10}^3 f(x)\,dx = 7$, then $\displaystyle\int_1^3 f(x)\,dx =$

(A) -3 (B) 0   (C) 3   (D) 11   (E) NOTA

18. Let f be the function given by $f(x) = \sqrt{x^4 - 16x^2}$. Find the equation of the line *normal* to the graph of f at $x = 5$.

(A) $y - 15 = \dfrac{34}{3}(x-5)$ (B) $y - 15 = -\dfrac{3}{34}(x-5)$

(C) $y - 5 = \dfrac{34}{3}(x-15)$ (D) $y - 5 = -\dfrac{3}{34}(x-15)$ (E) NOTA

19. Let R be the region in the first quadrant enclosed by the hyperbola $x^2 - y^2 = 9$, the x-axis, and the line $x = 5$. Find the volume of the solid generated by revolving R around the x-axis.

(A) $-\dfrac{206\pi}{3}$ (B) -18π (C) $\dfrac{44\pi}{3}$ (D) 18π (E) NOTA

20. Given the parametric equations $x = t^2 - 3t + 1$ and $y = t + 1$, find $\dfrac{d^2y}{dx^2}$ when $t = 2$.

(A) $\dfrac{-2}{25}$ (B) -2 (C) 1   (D) 2   (E) NOTA

21. Given the function $f(x) = \sin\dfrac{x}{2}$, find the sum of all values of x on the interval $[0, 2\pi)$ where the tangent line to the function is horizontal.

(A) $\dfrac{\pi}{2}$ (B) π (C) 2π (D) 4π (E) NOTA

22. Water is being pumped into an inverted conical tank at a constant rate. The tank has height 6 meters and the diameter at the top is 4 meters. If the water level is rising at a rate of 20 cm/min when the height of the water is 2 meters, find the rate at which the water is being pumped into the tank.

    (A) $\dfrac{4\pi}{15}$ cubic meters/minute   (B) $\dfrac{8\pi}{3}$ cubic meters/minute

    (C) $\dfrac{4\pi}{45}$ cubic meters/minute   (D) 8π cubic meters/minute   (E) NOTA

23. If $\displaystyle\int_0^a \sin x \, dx = 1 + \dfrac{\sqrt{2}}{2}$ and $0 \le a \le 2\pi$, find the sum of all possible values of a.

    (A) $\dfrac{3\pi}{4}$ (B) π (C) $\dfrac{7\pi}{4}$ (D) 2π (E) NOTA

24. Point P has Cartesian coordinates $(\sqrt{3}, -1)$. Which of the following could *not* be polar coordinates for this point?

    (A) $(2, \dfrac{11\pi}{6})$ (B) $(2, -\dfrac{\pi}{6})$ (C) $(-2, \dfrac{5\pi}{6})$ (D) $(-2, \dfrac{11\pi}{6})$ (E) NOTA

25. What is the equation of the plane perpendicular to the vector $\langle 8, 2, 3 \rangle$ that passes through the point $(7, 12, -5)$?

    (A) $8x + 2y + 3z = 65$ (B) $-8x + \dfrac{1}{2}y + 3z = -65$

    (C) $8x - \dfrac{1}{2}y + 3z = 35$ (D) $\dfrac{1}{8}x + \dfrac{1}{2}y + \dfrac{1}{3}z = \dfrac{125}{24}$ (E) NOTA

26. Which of the following is an eigenvector of the matrix $\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$?

    (A) $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

    (C) $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$ (D) $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ (E) NOTA

27. Evaluate $\displaystyle\sum_{n=1}^{\infty} \dfrac{2n}{3^{n+1}}$.

    (A) $\dfrac{1}{2}$ (B) $\dfrac{1}{3}$ (C) $\dfrac{2}{9}$ (D) $\dfrac{1}{6}$ (E) NOTA

28. $B(x)$ is an odd function such that $B(1) = 2$, $B(3) = 5$, and $B(-5) = -1$. Find the value of $\dfrac{B(B(B(-3)))}{3B(1) - 2B(3) - B(5)}$.

    (A) $-\dfrac{2}{3}$ (B) $-\dfrac{2}{5}$ (C) $\dfrac{2}{5}$ (D) $\dfrac{2}{3}$ (E) NOTA

29. Solve the differential equation given that $y(0) = 1$: $\dfrac{dy}{dx} = \dfrac{e^{2x}}{4y^3}$.

    (A) $y^4 = e^{2x}$ (B) $y^4 = \dfrac{1}{2}e^{2x} + \dfrac{1}{2}$ (C) $y^4 = \dfrac{1}{2}e^{2x} - \dfrac{1}{2}$ (D) $y^4 = 2e^{2x} - 1$ (E) NOTA

30. Given $f(x) = \sin x \cos x$, find $f'\left(\dfrac{\pi}{4}\right)$.

    (A) 1 (B) 0 (C) $\dfrac{1}{2}$ (D) $-\dfrac{1}{2}$ (E) NOTA