Welcome to ”The Most Magical Place on Earth”! Around this enchanted lots are dozens of wonderful shapes to find area and volume for. As you complete the test, remind yourself to read each question and its answer choices carefully. As always, “NOTA” stands for “None of the above.” Have fun and good luck!

1. To help Player A triumph over Player B, find the area between $y = 4782.26$ and $y = 4799.17$ from $x = 0$ to $x = 1$.

(A) -16.91 **(B)** 16.91 **(C)** 17.09 **(D)** 379.07 **(E)** NOTA

2. The logo for this enchanted land is an interesting shape. Find the area of a regular hexagon with side length $\sqrt[4]{12}$.

(A) 6 **(B)** 8 **(C)** 9 **(D)** 12 **(E)** NOTA

3. Another logo for this place is 3-dimensional! It can be represented as a cube inscribed in a sphere inscribed in a unit cube. The square of the volume of the inner cube is $\frac{a}{b}$, for relatively prime positive integers a, b. Find $a + b$.

(A) 11 **(B)** 26 **(C)** 28 **(D)** 32 **(E)** NOTA

4. Anna wants to find the area of a right triangle but doesn’t know any sides! All she knows is the triangle has angles θ_1, θ_2, and θ_3 such that $\sin(\theta_1 + \theta_2) = \frac{12}{13}$ and $\sin(\theta_2 + \theta_3) = \frac{5}{13}$. If the triangle has perimeter 10, find the area.

(A) $\frac{10}{3}$ **(B)** $\frac{20}{3}$ **(C)** 10 **(D)** $\frac{40}{3}$ **(E)** NOTA

Please Use the Following Information to Answer Questions 5 - 7: A magical ”bashy” ellipse is created in the complex plane according to the following equation, where z is a complex number:

$$|z - 8i| + |z - 6| = 26$$

5. Find the area bounded by this ”bashy” ellipse.

(A) 39π **(B)** 48π **(C)** 60π **(D)** 156π **(E)** NOTA

6. The two latera recta intersect the ellipse at 4 points. These 4 points and the ellipse’s vertices form the vertices of a ”bashier” polygon. Find the area of this polygon.

(A) $\frac{288}{13}$ **(B)** $\frac{2304}{13}$ **(C)** $\frac{2880}{13}$ **(D)** $\frac{5184}{13}$ **(E)** NOTA

7. A "bashy" solid is created by revolving the ellipse around its major axis. Find the volume of the resulting ellipsoid.

(A) 1040π **(B)** 2496π **(C)** 2592π **(D)** 2704π **(E)** NOTA

8. Gaston bakes some funnel cakes by revolving an infinite amount of circles with center $(2^n, 2^n)$ and radius $(\frac{1}{5})^n$ around the x-axis for all non-negative integers n. If the sum of the volumes of his funnel cakes can be written as $\frac{a\pi^c}{b}$ for relatively prime integers a, b, find $a + b + c$.

(A) 49 **(B)** 50 **(C)** 74 **(D)** 75 **(E)** NOTA

9. Scar realizes he can draw unique shapes with absolute value functions. Find the area bounded by:

$$3x + 2y + 9|x| + 4|y| = 36.$$

(A) 108 **(B)** 112 **(C)** 120 **(D)** 126 **(E)** NOTA

10. Donald's beak is the region bounded above by $f(x) = 1 - x^2$ and below by $g(x) = x^2$. Find the area of this region.

(A) $\frac{\sqrt{2}}{6}$ **(B)** $\frac{\sqrt{2}}{4}$ **(C)** $\frac{\sqrt{2}}{3}$ **(D)** $\frac{2\sqrt{2}}{3}$ **(E)** NOTA

11. Scrooge is trying to find some trivial areas. Find $\lim\limits_{n\to\infty} \int_0^1 nx^n \, dx$.

(A) -1 **(B)** 0 **(C)** 1 **(D)** ∞ **(E)** NOTA

12. Find the area bounded by $f(x) = 2x^6 - 6x^5 + 15x^4 - 20x^3 + 15x^2 - 6x + 1$ and the x-axis from $x = 1$ to $x = 2$.

(A) $\frac{127}{7}$ **(B)** $\frac{128}{7}$ **(C)** $\frac{254}{7}$ **(D)** $\frac{256}{7}$ **(E)** NOTA

13. Mufasa has a region R bounded by $f(x) = k^x$, the x-axis, $x = 0$, and $x = 2$ for some real number k. The line $x = 1$ splits R into two regions with the ratio of the the regions' areas as $\sqrt{17} : \sqrt{6}$. Suppose k^2 can be written as $\frac{a}{b}$ for relatively prime positive integers a, b. Find $a + b$.

(A) 11 **(B)** 17 **(C)** 23 **(D)** 29 **(E)** NOTA

14. Find the area of the region inside both $r = 1 + \cos\theta$ and $r = 1 - \cos\theta$.

(A) $\frac{3\pi-8}{8}$ **(B)** $\frac{3\pi-4}{8}$ **(C)** $\frac{3\pi-8}{4}$ **(D)** $\frac{3\pi-8}{2}$ **(E)** NOTA

15. Mater drives around a path, resembled by the equation $(x^2 + y^2)^4 = 8x^2y^2$. Find the area bounded by Mater's path.

(A) $\sqrt{2}$ **(B)** 2 **(C)** $2\sqrt{2}$ **(D)** 4 **(E)** NOTA

Please Use the Following Information to Answer Questions 16-17: Stitch randomly chooses two points uniformly at random on the unit circle given by $x^2 + y^2 = 1$. These two points and the origin form the vertices of a triangle, T.

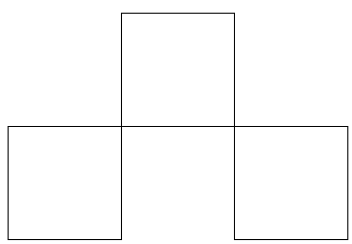
16. Find the average value of the triangle's area.

(A) $\frac{1}{\pi}$ **(B)** $\frac{4}{3\pi}$ **(C)** $\frac{3}{2\pi}$ **(D)** $\frac{2}{\pi}$ **(E)** NOTA

17. Find the average value of the triangle's perimeter.

(A) $\frac{2\pi+4}{\pi}$ **(B)** $\frac{2\pi+6}{\pi}$ **(C)** $\frac{3\pi+4}{\pi}$ **(D)** $\frac{3\pi+6}{\pi}$ **(E)** NOTA

18. Lilo decides to one-up Stitch and creates three regions, each represented by a unit square, and configures them like so:



Lilo randomly chooses one point inside each of the three squares. Find the expected area of the triangle formed.

(A) $\frac{1}{3}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{2}{3}$ **(D)** 1 **(E)** NOTA

19. Noticing an interesting trend about parabolas, Jiminy remarks that the area bounded by a parabola and its latus rectum can be expressed as a function $f(a)$, where a is the focal distance. Evaluate:

$$\int_0^3 f(a)\, da$$

(A) 12 **(B)** 18 **(C)** 24 **(D)** 30 **(E)** NOTA

20. Slinky has an infinite number of cubes, each with side length $\frac{1}{n}$ for all non-negative integers n. Slinky begins stacking them one on top of the other, in order of decreasing side length, with no overhangs. What is the total surface area of Slinky's final structure?

(A) $\frac{2\pi^2}{3}$ **(B)** $\frac{2\pi^2+3}{3}$ **(C)** $\frac{\pi^2+12}{3}$ **(D)** $\frac{2\pi^2+6}{3}$ **(E)** NOTA

21. Help Remy by finding the total volume bounded by $z = e^{\frac{-(x+y)}{\sqrt{5}}}$ above the positive quadrant when $x, y \geq 0$.

(A) $\frac{1}{5}$ **(B)** $\frac{2}{\sqrt{5}}$ **(C)** $2\sqrt{5}$ **(D)** 5 **(E)** NOTA

22. Bolt takes takes two right circular cylinders with radius 2 and intersects them at right angles, creating a unique **Steinmetz solid**. Noticing the cross-sections, you tell Bolt you can calculate the volume. Find its volume.

(A) $\frac{32}{3}$ **(B)** $\frac{64}{3}$ **(C)** $\frac{128}{3}$ **(D)** $\frac{256}{3}$ **(E)** NOTA

23. Dash takes a regular hexagon with side length 4 and revolves it around its longest diagonal to form a 3-dimensional object. Find the volume of this object.

(A) 56π **(B)** 60π **(C)** 64π **(D)** 68π **(E)** NOTA

24. Abu's solid is created by revolving $y = \frac{\sin x}{x^2}$ from $x = 0$ to $x = \infty$ around the y-axis. Find the volume of Abu's solid.

(A) $\frac{\pi}{2}$ **(B)** π **(C)** $\frac{\pi^2}{2}$ **(D)** π^2 **(E)** NOTA

Please Use the Following Information to Answer Questions 25 - 27: Goofy is inventing a series of new geometric objects. He begins with a sphere of radius 1. Then, he forms a rigid wire frame in the shape of a regular n-gon inscribed in a great circle of the sphere. When the sphere is pushed through this wire frame, it is sliced into $n+1$ distinct solids: n congruent outer regions, each called a **Goober-n**, and a central remaining region called the **Goofball-n**.

For example, when $n = 3$, the wire is an equilateral triangle inscribed in the great circle, and the sphere is split into 3 congruent Goober-3s and a Goofball-3.

25. The volume of a Goober-3 can be written as $\frac{a\pi}{b}$ for relatively prime positive integers a, b. Find $a+b$.

(A) 13 **(B)** 17 **(C)** 23 **(D)** 29 **(E)** NOTA

26. The volume of a Goofball-3 can be written as $\frac{a\pi}{b}$ for relatively prime positive integers a, b. Find $a+b$.

(A) 11 **(B)** 17 **(C)** 41 **(D)** 45 **(E)** NOTA

27. The surface area of a Goober-n can be expressed as a polynomial $S_1(x)$, where $x = \cos(\frac{\pi}{n})$. Find $S_1(\frac{1}{3})$.

(A) $\frac{11\pi}{9}$ **(B)** $\frac{14\pi}{9}$ **(C)** $\frac{17\pi}{9}$ **(D)** $\frac{20\pi}{9}$ **(E)** NOTA

28. Linguine has a unit circle M, and randomly chooses two points on the circle. Drawing a chord between the points splits M into two regions. Find the average area of the smaller region.

(A) $\frac{\pi}{2}$ **(B)** $\frac{\pi^2}{2+2\pi}$ **(C)** $\frac{\pi^2-4}{4\pi}$ **(D)** $\frac{\pi^2-2}{\pi}$ **(E)** NOTA

29. Let the radius of Mickey's cylindrical coin be 1. It has equal probability of landing heads or tails. Assume that the thickness of the coin is non-negligible so the coin can land on its side. When flipped, the coin has a $\frac{1}{5}$ probability of landing on its side. Find the thickness of the coin.

(A) $\frac{1}{16}$ **(B)** $\frac{1}{8}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{1}{2}$ **(E)** NOTA

30. Find the greatest integer less than the area of Joe's triangle with side lengths of 13, 14, and 15.

(A) 21 **(B)** 42 **(C)** 84 **(D)** 168 **(E)** NOTA