Assume all variables are real unless stated otherwise. "NOTA" means that "None of The Above" answers are correct, and "DNE" means that the limit does not exist. Good luck and have fun!

1. Find $\lim_{x\to 0} \frac{x^{3/2}}{\sin x}$.

(A) $-\frac{3}{2}$ **(B)** 0 **(C)** $\frac{3}{4}$ **(D)** $\frac{3}{2}$ **(E)** NOTA

2. If $y = f(x)$ is a continuous function that satisfies $x^2 - 3y + 6xy + 9y^2 - x = 6$, find $\frac{dx}{dy}$ at the point $(-3, 2)$.

(A) -3 **(B)** 3 **(C)** $\frac{-1}{3}$ **(D)** $\frac{7}{39}$ **(E)** NOTA

3. Find $\lim_{x\to 3} \frac{x^3 - 8x^2 + 19x - 12}{2x^3 - 11x^2 + 19x - 10}$.

(A) $-\frac{2}{7}$ **(B)** 0 **(C)** $\frac{2}{7}$ **(D)** $\frac{2}{5}$ **(E)** NOTA

4. Find the sum of the values of x that satisfy the Mean Value Theorem for Derivatives for $f(x) = x^3 + 5x^2 + 8x + 2$ on the interval $(-3, -1)$.

(A) $-\frac{10}{3}$ **(B)** $-\frac{7}{3}$ **(C)** -2 **(D)** -1 **(E)** NOTA

5. Find the maximum value of $y = \sin\left(x - \frac{\pi}{6}\right) + \cos x$ over the interval $[0, 2\pi]$.

(A) 1 **(B)** $\sqrt{2}$ **(C)** $\frac{\sqrt{3}}{2}$ **(D)** $\sqrt{3}$ **(E)** NOTA

6. Find $\lim_{n\to\infty} \frac{n + n^2 \ln n + 3n^3}{3n + 6n^3 + 7}$.

(A) $\frac{1}{2}$ **(B)** 1 **(C)** 0 **(D)** DNE **(E)** NOTA

7. Let $f(x) = x\sin^3 x$. If $f'\left(\frac{\pi}{6}\right)$ can be expressed as $\frac{a + \pi\sqrt{b}}{c}$, find $a + b + c$.

(A) 13 **(B)** 20 **(C)** 21 **(D)** 24 **(E)** NOTA

8. Find the derivative of the polar function $r = \sqrt{8+8\cos\theta}$ when $\theta = \frac{\pi}{3}$.

(A) $-\frac{\sqrt{3}}{7}$ **(B)** $-\frac{\sqrt{3}}{5}$ **(C)** $-\frac{7}{\sqrt{3}}$ **(D)** $\frac{\sqrt{3}}{7}$ **(E)** NOTA

9. Approximate $3-\sqrt{7}$ using two iterations of Newton's Method with $f(x) = x^2 - 6x + 2$ and $x_0 = 1$.

(A) $\frac{25}{72}$ **(B)** $\frac{17}{48}$ **(C)** $\frac{63}{162}$ **(D)** $\frac{31}{88}$ **(E)** NOTA

10. Let R be the region bound by $r = a\sin(3\theta)$, and suppose that a is increasing at a rate of $3t^2$ m/s. If $a = 3$ when $t = 1$, find the rate of change of the area of R (in m^2/s) when $t = 2$.

(A) 30π **(B)** 36π **(C)** 48π **(D)** 60π **(E)** NOTA

11. If $f(x) = x^3 + 3x^2 + \frac{4}{x}$, approximate $f(1.01)$ using the tangent line to f centered at $x = 1$.

(A) 7.95 **(B)** 8.02 **(C)** 8.05 **(D)** 8.13 **(E)** NOTA

12. Find $\lim\limits_{x\to 0} \frac{xe^{x^2} - \tan x}{\sin x - x}$.

(A) -8 **(B)** -4 **(C)** -1 **(D)** 1 **(E)** NOTA

13. If $f(x) = x^{\ln x + 1}$, find $f'(e)$.

(A) 3 **(B)** 6 **(C)** $\frac{3}{e}$ **(D)** $3e$ **(E)** NOTA

14. Find $f^{(2025)}(0)$, where $f(x) = x^3 e^x$.

(A) $2023 \cdot 2024 \cdot 2026$ **(C)** $\frac{2023 \cdot 2024 \cdot 2025}{6}$ **(E)** NOTA

(B) $2023 \cdot 2024 \cdot 2025$ **(D)** $2022 \cdot 2023 \cdot 2024$

15. Now find $g^{(2025)}(0)$, where $g(x) = x^3 e^{x^3}$.

(A) $\frac{2025!}{1011!}$ **(B)** $\frac{2025!}{1012!}$ **(C)** $\frac{2025!}{675!}$ **(D)** $\frac{2025!}{674!}$ **(E)** NOTA

16. Define a function $y = f(x)$ such that $y'\cos x + y\sin x = x\cos^4 x$. If $f(0) = 0$ and $f\left(\frac{\pi}{4}\right)$ can be expressed as $\frac{\sqrt{2}}{a}(\pi^2 + b\pi + c)$, where a, b, and c are relatively prime integers, find $a+b+c$.

(A) 116 **(B)** 124 **(C)** 132 **(D)** 140 **(E)** NOTA

17. Find $\lim_{x\to\infty}\left(\sqrt[3]{x^6+2x^4-1} - \sqrt[4]{x^8+4x^6+2x^2+2}\right)$.

(A) $-\frac{1}{3}$ **(B)** $\frac{4}{3}$ **(C)** 0 **(D)** DNE **(E)** NOTA

Use the following information to answer questions 18-19: Tiger is standing on the exact center point of a 6-meter-long ladder leaned against a 17-meter-tall wall when, unsurprisingly, the bottom of the ladder starts slipping away from the wall. Let t_1 be the moment in time at which the base of the ladder is 2 meters away from the wall, and suppose the top of the ladder is sliding down at the rate of $2\sqrt{2}$ meters per second at this instant.

18. Find the rate of change, in m/s, of Tiger's horizontal distance from the wall at time t_1.

(A) 2 **(B)** $2\sqrt{2}$ **(C)** 4 **(D)** 8 **(E)** NOTA

19. In Tiger's defense, he had a perfectly justifiable reason for standing on the aforementioned wall: the Miami GP broadcast was being shown on a large screen nearby, but a 6.56-foot-tall fence obstructed his view. Suppose Tiger's eyes are exactly 2 meters above his feet and that the 3-meter-tall screen is mounted on a second vertical wall 6 meters away from the first (which his ladder was propped against) in a way that, any given, manages to stay exactly 2 meters above Tiger's head at its lowest point (just to torment him further). If the rate of change of Tiger's viewing angle of the screen at time t_1 can be expressed as the fraction $\frac{a}{b}$, compute $a+b$. (Assume that the thickness of the screen is negligible.)

(A) 143 **(B)** 161 **(C)** 163 **(D)** 181 **(E)** NOTA

20. If $f(x) = x^2 + 3x + 4$, find $\lim_{h\to 0}\frac{f(3h-1)-f(-h-1)}{2h}$

(A) 6 **(B)** -2 **(C)** 1 **(D)** 2 **(E)** NOTA

21. If $f(x) = \int_{x^2}^{2} t^2\log_3(t)\,dt$, find $f'(\sqrt{3})$.

(A) 3 **(B)** 9 **(C)** $-18\sqrt{3}$ **(D)** $18\sqrt{3}$ **(E)** NOTA

22. If $f(x) = \arctan\left(\frac{2}{3-x}\right)$, find $\lim_{x\to 3} f'(x)$.

(A) $\frac{1}{4}$ **(B)** $\frac{1}{2}$ **(C)** 2 **(D)** DNE **(E)** NOTA

23. If $f(x) = \frac{1}{2}x^4 + \frac{4}{3}x^3 + x^2 + x - 2$, find the positive difference between the values of $f'(x)$ at its two relative extrema.

(A) $\frac{1}{9}$ **(B)** $\frac{3}{8}$ **(C)** $\frac{8}{27}$ **(D)** $\frac{9}{16}$ **(E)** NOTA

24. Find $\lim_{n\to\infty}\left(\sum_{i=2}^{n/2} \frac{\pi}{n + n\tan^4(\frac{\pi i}{n})}\right)$.

(A) $\frac{\pi}{4}$ **(B)** $\frac{\pi}{2}$ **(C)** π **(D)** $\frac{3\pi}{2}$ **(E)** NOTA

25. Find the maximum volume of a cylinder inscribed in the ellipsoid formed by revolving $y = \pm\frac{4}{3}\sqrt{9-x^2}$ around the y-axis.

(A) $24\pi\sqrt{2}$ **(B)** $16\pi\sqrt{3}$ **(C)** $\frac{64\pi\sqrt{3}}{3}$ **(D)** $\frac{32\pi\sqrt{6}}{3}$ **(E)** NOTA

26. Evaluate $\int_0^{\frac{\pi}{2}} \cos^3\theta \ln(\sin\theta)\mathrm{d}\theta$.

(A) -1 **(B)** $-\frac{8}{9}$ **(C)** $-\frac{1}{3}$ **(D)** DNE **(E)** NOTA

27. Find the maximum value of $x + 3y$ if $x^2 + y^2 = 4$.

(A) $2\sqrt{10}$ **(B)** $2\sqrt{5}$ **(C)** $6\sqrt{10}$ **(D)** $\frac{3\sqrt{10}}{5}$ **(E)** NOTA

Use the following information for questions 28-29: The Euler-Mascheroni constant γ and the Gamma function $\Gamma(z)$ are defined as follows:

$$\gamma = -\int_0^\infty e^{-x}\ln(x)dx$$

$$\Gamma(z) = \int_0^\infty t^{z-1}e^{-t}dt$$

28. Find $\lim_{x\to 4}\Gamma(x)$

(A) 2 **(B)** 6 **(C)** 12 **(D)** 24 **(E)** NOTA

29. In terms of γ, evaluate

$$\lim_{x\to 1}\frac{\Gamma(x+1)-1}{\Gamma(x)-1}$$

(A) $1-\frac{1}{\gamma}$ **(B)** $1+\frac{1}{\gamma}$ **(C)** $-1+\frac{1}{\gamma}$ **(D)** $-1+\gamma$ **(E)** NOTA

30. Find $\lim_{x\to 0}(1+8\sin x)^{2\csc x}$

(A) 1 **(B)** e^4 **(C)** e^{16} **(D)** DNE **(E)** NOTA