

For all questions, “E: NOTA” stands for “None of These Answers are correct.” All numbers can be assumed to be in base 10 unless specified otherwise. $\mathbb{Z}$ represents the set of integers, and $\mathbb{N}$ represents the set of natural numbers, defined as positive integers. Good luck, and have fun!

1. Evaluate: $2025^2 \bmod 22$

A: 1 B: 5 C: 11 D: 21 E: NOTA

2. For what integer value of n does $2025n$ have 28 positive integral factors?

A: 15 B: 45 C: 60 D: 75 E: NOTA

3. Given $2025!$ is evenly divisible by $5^a \times 9^b$ for positive integers a and b , what is the largest possible value of $a + b$?

A: 505 B: 757 C: 842 D: 1010 E: NOTA

4. Evaluate in base 10: $12_3 \times 23_4 + 34_5$

A: 35 B: 54 C: 74 D: 96 E: NOTA

5. A square-free number is defined as an integer not evenly divisible by any square number greater than 1. How many positive integral factors of 600 are square-free?

A: 8 B: 12 C: 16 D: 24 E: NOTA

6. Molly is trying to box cupcakes to sell at her bakery. When she puts them in boxes that hold 2, 3, or 5 cupcakes each, she has 1 left over, and when she puts them in boxes that hold 7 cupcakes each, she has 2 left over. Molly can put up to 40 cupcakes in a Big Box. Given that Molly has between 800 and 1000 cupcakes and wants to fill each Big Box with the same number of cupcakes, how many Big Boxes will Molly need to box all her cupcakes?

A: 23 B: 29 C: 31 D: 37 E: NOTA

Use the following information to answer questions 7-10:

Let $S(n)$ represent the sum of the positive integral factors of n , for $n \in \mathbb{N}$

For example: $S(20) = 1 + 2 + 4 + 5 + 10 + 20 = 42$

7. Evaluate: $S(2025)$

A: 2132 B: 2821 C: 3751 D: 3937 E: NOTA

8. Evaluate: $S\left(S\left(S\left(S(7)\right)\right)\right)$

A: 24 B: 60 C: 64 D: 128 E: NOTA

9. Let a and b be the pair of positive integers with the smallest possible sum such that $S(a) = S(b)$ and $a \neq b$. What is the value of $|a - b|$?

A: 1 B: 3 C: 4 D: 5 E: NOTA

10. What is the sum of the 3 smallest positive values of x such that $\frac{S(x)}{x}$ is an integer?

A: 19 B: 27 C: 33 D: 35 E: NOTA

11. What is the units digit of $3^{14^{15}}$?

A: 1 B: 3 C: 7 D: 9 E: NOTA

12. What is the hundreds digit of $7^{7^{7^7}}$?

A: 0 B: 2 C: 6 D: 8 E: NOTA

13. How many integers n on the interval $[1,100]$ are there such that there exists a prime number p where $\frac{1}{n}$ is a terminating decimal in base p ?

A: 13 B: 25 C: 36 D: 50 E: NOTA

14. The fraction $\frac{2}{13}$ can be written as the repeating decimal $.\overline{A}$, with A being a 6-digit number. How many distinct prime factors does A have?

A: 2 B: 3 C: 4 D: 5 E: NOTA

15. What is the sum of the distinct prime factors of $2^{16} + 51^2 - 101$?

A: 103 B: 127 C: 211 D: 308 E: NOTA

16. A dartboard is made up of three rings, worth 5, 9, and 11 points for hitting them, with no other ways to gain or lose points. If you hit the dartboard on one of the rings, that value is added to your score. What is the largest integer score that is unobtainable given an infinite number of attempts?

A: 17 B: 24 C: 31 D: 79 E: NOTA

17. Solve for x : $225_x = 142_{x+1} + 61_{x-2}$

A: 9 B: 10 C: 11 D: 12 E: NOTA

18. Find the number of positive integers less than or equal to 2025 that can be written as the sum of distinct positive and/or negative powers of 5. For example, $101 = 5^3 - 5^2 + 5^0$ works, as does $625 = 5^4$.

A: 31 B: 161 C: 1215 D: 2025 E: NOTA

19. How many solutions to the equation $\frac{2}{a} + \frac{2}{b} = 1$ are there for $a, b \in \mathbb{Z}$?

- A: 1 B: 3 C: 5 D: 6 E: NOTA

20. Two integers m and n are randomly selected on the interval $[1,100]$. What is the probability that $(m^2 + n^2) \equiv 0 \pmod{5}$?

- A: $\frac{1}{5}$ B: $\frac{9}{25}$ C: $\frac{2}{5}$ D: $\frac{16}{25}$ E: NOTA

21. How many solutions to $2x + 5y + 10z = 200$ are there for $x, y, z \in \mathbb{N}$?

- A: 171 B: 190 C: 210 D: 231 E: NOTA

22. What is the sum of all possible units digits of $(12_p)^{12_p}$ when it is simplified in base p , given p can be any prime number greater than 2?

- A: 8 B: 14 C: 16 D: 20 E: NOTA

23. How many ordered pairs of positive integers (a, b) with $a < b$ are there such that their least common multiple is 18 more than their greatest common factor?

- A: 3 B: 4 C: 6 D: 7 E: NOTA

24. What is the sum of the digits of the greatest common factor of 69733 and 31921?

- A: 7 B: 11 C: 13 D: 14 E: NOTA

Use the following information to answer questions 25-26:

An integer is defined as a “sequential number” if it can be written as the concatenation of an arithmetic sequence of non-negative integers that is at least 3 terms long. For example, 4321 and 510 are sequential numbers, being the sequences 4,3,2,1 and 0,5,10 respectively.

Note that while a sequence may begin with a zero, leading zeros of terms are omitted. For instance, 1,2,3 refers to the number 123 and not 1023, as 2 cannot be written as 02.

25. How many integers between 1 and 1000 are sequential numbers?

A: 45 B: 54 C: 63 D: 72 E: NOTA

26. What is the sum of all positive integers n where $n \leq 100$ and n^2 is a sequential number?

A: 60 B: 72 C: 96 D: 108 E: NOTA

27. Let $x = \frac{n^3 + 8n^2 + 15n + 13}{n+2}$ for $n \in \mathbb{Z}$. What is the sum of all positive prime values of x ?

A: 59 B: 77 C: 93 D: 106 E: NOTA

28. For $A, B, C \in \mathbb{N}$, what is the smallest possible value of $|A - B|$ given $A^2 + 10^B = C^2$?

A: 2 B: 3 C: 6 D: 12 E: NOTA

29. What is the sum of the digits of 88^{88} ?

A: 689 B: 733 C: 775 D: 804 E: NOTA

30. What is the product of the positive integral divisors of 2025?

A: 45^9 B: 45^{15} C: 2025^9 D: 2025^{15} E: NOTA