

Important Instructions for this Test: The answer choice “E: NOTA” indicates that “None of These Answers” are correct. Problems are **not** in order of increasing difficulty, so don’t be afraid to skip around. Good luck and have fun!

1. Legosi is standing at the point $(3, -6)$, while Louis is standing at the point $(-2, 1)$. What is the square of the distance between them?

- A: 74 B: 144 C: 36 D: 26 E: NOTA

2. The lines $y = 2x + 1$ and $x = y - 2$ intersect at the point (a, b) compute $a + b$.

- A: 3 B: 4 C: 2 D: 1 E: NOTA

3. The line perpendicular to $y = 2x + 1$ passing through $(3, 5)$ has equation $Ax + By = C$, where A , B , and C are integers, A and B share no prime factors, and $A > 0$. Find C .

- A: 1 B: 13 C: 11 D: -7 E: NOTA

4. Identify the graph of $(x + y)(x - y) = 1$.

- A: Circle C: Parabola E: NOTA
B: Non-Circular Ellipse D: Hyperbola

5. Points $A(1, 2)$, B , C , and $D(10, -10)$ are equally spaced (from one point to the next) on a line in that order. Find C .

- A: $(4, -6)$ B: $(4, -3)$ C: $(7, -3)$ D: $(7, -6)$ E: NOTA

6. A line of the form $y = 2x + b$ splits the graph of $x^2 + y^2 = 8x + 14y$ into two regions of equal area. Compute b .

- A: -2 B: -1 C: 1 D: 2 E: NOTA

7. The point $(a, 0)$ is equidistant from the point $(6, 7)$ and $(10, 5)$. Find a .

- A: 8 B: 4 C: 5 D: 6 E: NOTA

8. Find the number of petals on the graph of the rose curve given by

$$r = \cos(\theta) \sin(\theta) (\cos^4(\theta) - \sin^4(\theta)).$$

- A: 8 B: 16 C: 4 D: 12 E: NOTA

9. Points $A(1, 2)$, $B(-3, 4)$, $C(0, 5)$ and D are the vertices of a parallelogram in clockwise order around the parallelogram. Find the coordinates of D .

- A: $(-2, 1)$ B: $(-4, 7)$ C: $(2, 2)$ D: $(4, 3)$ E: NOTA

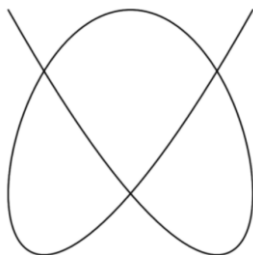
10. A hyperbola H has asymptotes $y = 3x - 1$ and $2y = x + 4$ and passes through the point $(2, 1)$. Find the other intersection point of H with the line $x = 2$.

- A: $(2, -1)$ B: $(2, 3)$ C: $(2, 7)$ D: $(2, 5)$ E: NOTA

11. In polar coordinates, find the midpoint of $(2, \frac{5\pi}{12})$ and $(-2, \frac{\pi}{12})$.

- A: $(2, \frac{\pi}{4})$ B: $(2, \frac{3\pi}{4})$ C: $(1, \frac{\pi}{4})$ D: $(1, \frac{3\pi}{4})$ E: NOTA

12. The graph of the parametric curve $x(t) = \cos(3t)$, $y(t) = \cos(4t)$ (where $0 \leq t \leq \pi$) is pictured below without the coordinate axes. This graph intersects itself at exactly one point in the first quadrant. Find the coordinates of this point.



- A: $(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$ B: $(\frac{\sqrt{3}}{2}, \frac{1}{2})$ C: $(\frac{\sqrt{3}}{2}, \frac{\sqrt{2}}{2})$ D: $(\frac{\sqrt{2}}{2}, \frac{1}{2})$ E: NOTA

13. A line segment with endpoints $(1, 2)$ and $(4, 3)$ is rotated counterclockwise by 60° about $(1, 2)$. Find the coordinates of the other endpoint (that was originally $(4, 3)$) after the rotation. As a hint, try using complex numbers.

- A: $(\frac{5}{2} + \frac{\sqrt{3}}{2}, \frac{5}{2} - \frac{3\sqrt{3}}{2})$ C: $(2 - \frac{3\sqrt{3}}{2}, \frac{3}{2} + 2\sqrt{3})$ E: NOTA
 B: $(\frac{5}{2} - \frac{\sqrt{3}}{2}, \frac{5}{2} + \frac{3\sqrt{3}}{2})$ D: $(2 + \frac{3\sqrt{3}}{2}, \frac{3}{2} - 2\sqrt{3})$

14. Triangle ABC has lengths $AB = 3$, $AC = 4$, and $BC = 5$. Points P , Q , and R are on $\overline{AB}$, $\overline{AC}$, and $\overline{BC}$ respectively such that length $AP = 1$ and triangle PQR is equilateral. Find length AQ . As a hint, consider the previous problem.

- A: $\frac{89\sqrt{3}-96}{39}$ B: $\frac{16\sqrt{3}-12}{39}$ C: $\frac{6\sqrt{3}-8}{11}$ D: $\frac{42\sqrt{3}-67}{11}$ E: NOTA

15. The ellipse E given by $\frac{x^2}{4} + y^2 = 1$ is rotated counterclockwise about the origin by an angle of $\frac{\pi}{4}$ to get a new ellipse E' . What is the largest y -coordinate of any point on E' ?

- A: $\sqrt{5}$ B: $\frac{3}{2}$ C: $\frac{\sqrt{10}}{2}$ D: 2 E: NOTA

16. Vectors $\vec{u}$, $\vec{v}$, and $\vec{w}$ satisfy $\vec{u} + \vec{v} = \langle 3, 6 \rangle$, $\vec{u} + \vec{w} = \langle -2, 1 \rangle$, and $\vec{v} + \vec{w} = \langle 5, -3 \rangle$. Compute $\vec{u} + \vec{v} + \vec{w}$.

- A: $\langle 6, 4 \rangle$ B: $\langle 4, 2 \rangle$ C: $\langle 2, 1 \rangle$ D: $\langle 3, 2 \rangle$ E: NOTA

17. Identify the conic section given by $r = \frac{4}{3+2\cos(\theta)}$.

- A: Circle C: Parabola E: NOTA
 B: Non-Circular Ellipse D: Hyperbola

18. Let S be the locus of all points P such that the sum of the distances from P to $(0,0)$ and $(4,6)$ is 14. Given that S encloses a finite region, compute the area enclosed by S .

- A: 42π B: 168π C: 35π D: 140π E: NOTA

19. Vectors $\vec{v}$ and $\vec{w}$ satisfy $||\vec{v}|| = 3$, $||\vec{w}|| = 4$, and $v \cdot w = -4$. Compute $||\vec{v} \times \vec{w}||$.

- A: $8\sqrt{2}$ B: 8 C: $6\sqrt{2}$ D: 6 E: NOTA

20. Find the area of the set of all points satisfying $|x| + 2|y| \leq 2$ and $2|x| + |y| \leq 2$.

- A: $\frac{8}{3}$ B: $\frac{16}{3}$ C: $\frac{8}{5}$ D: $\frac{16}{5}$ E: NOTA

21. Let $(d, 0)$ be the focus of the hyperbola $\frac{x^2}{9} - \frac{y^2}{16} = 1$ that lies on the positive x -axis. Compute the distance between the two intersection points of this hyperbola and the line $x = d$.

- A: $\frac{16}{3}$ B: $\frac{16}{9}$ C: $\frac{32}{3}$ D: $\frac{32}{9}$ E: NOTA

22. Let P be the parabola with vertex $(-3, -4)$ and focus $(0,0)$. P intersects the positive y -axis at the point $(0, k)$ where $k > 0$. Find k .

- A: 25 B: 24 C: 50 D: 48 E: NOTA

Please Use the Following Information to Answer Questions 23 to 25:

Suppose the conic $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ is rotated by some angle θ to get a new conic $A'x'^2 + B'x'y' + C'y'^2 + D'x' + E'y' + F' = 0$. If we assume $F' = F$ (otherwise we can multiply both sides by whatever constant we want), then $A + C = A' + C'$ and $B^2 - 4AC = B'^2 - 4A'C'$. Because of this, we say that $A + C$ and $B^2 - 4AC$ (and sometimes F) are *invariant* under rotation.

23. Suppose the conic $4x^2 + 16xy + 25y^2 = 1$ is rotated to get a conic of the form $A'x'^2 + C'y'^2 = 1$, where $A', C' > 0$. Compute $\sqrt{A'} + \sqrt{C'}$.

- A: $\sqrt{53}$ B: $\sqrt{41}$ C: $\sqrt{35}$ D: $\sqrt{37}$ E: NOTA

24. Compute the area enclosed by the ellipse $4x^2 + 16xy + 25y^2 = 2$. As a hint, suppose you rotated to get an ellipse of the form $A'x'^2 + C'y'^2 = F$. Find the area of this and use an appropriate invariant.

- A: $\frac{\pi}{3}$ B: $\frac{\pi}{6}$ C: $\frac{\pi}{12}$ D: $\frac{\pi}{8}$ E: NOTA

25. It turns out $A + C$ and $B^2 - 4AC$ are not the only invariants under rotation! There is another invariant involving the coefficients D and E . Which of the following expressions is invariant under rotation?

- A: $D + E$ B: $D - E$ C: $D^2 + E^2$ D: DE E: NOTA

26. Find the number of intersection points in the polar coordinate plane between the (polar) graphs of $r = 2 \cos(4\theta)$ and $r = 1$.

- A: 8 B: 16 C: 4 D: 0 E: NOTA

27. Let z be a complex number with nonzero real and imaginary parts. If the area enclosed by the triangle formed by vertices 0 , z , and z^3 in the complex plane has four times the area enclosed by the triangle formed by vertices 0 , z , and z^2 , find the real part of z .

- A: 1 B: 2 C: 3 D: 4 E: NOTA

28. Compute the area enclosed by the convex pentagon with vertices $(1, -3, 1)$, $(2, 2, -2)$, $(-3, 1, 1)$, $(0, -4, 2)$, and $(-4, -2, 3)$. As a hint, first compute the area if all of the z -coordinates were 0 , then adjust to have the points lie on the plane $x + y + 2z = 0$ instead.

- A: $23\sqrt{3}$ B: $13\sqrt{3}$ C: $\frac{13\sqrt{6}}{2}$ D: $\frac{23\sqrt{6}}{2}$ E: NOTA

29. The intersection of the plane $x + y + z = 3$ and the sphere $x^2 + y^2 + z^2 = 4^2$ is a circle. Find the area enclosed by this circle.

- A: 7π B: 9π C: 16π D: 13π E: NOTA

30. The intersection of the plane $x + y + z = 0$ and the ellipsoid $\frac{x^2}{1^2} + \frac{y^2}{2^2} + \frac{z^2}{4^2} = 1$ is an ellipse. Find the area enclosed by this ellipse.

- A: $\frac{8\pi}{\sqrt{21}}$ B: $\frac{8\pi}{\sqrt{7}}$ C: $\frac{8\pi}{\sqrt{3}}$ D: 8π E: NOTA