

Where applicable, “E) NOTA” indicates that none of the above answers is correct.

1. A fair coin is flipped repeatedly until the first heads appears. What is the probability that the number of flips is even?

- A) $\frac{1}{4}$ B) $\frac{1}{3}$ C) $\frac{1}{2}$ D) $\frac{2}{3}$ E) NOTA

2. A train leaves Dallas at 2 pm, traveling a constant 60 miles per hour directly toward Texarkana, which is 180 miles away. Another train leaves Texarkana at 3 pm, traveling a constant 90 miles per hour directly toward Dallas. At what time on that same day do the trains meet? Disregard the length of the train.

- A) 4 pm B) 4:15 pm C) 4:20 pm D) 4:30 pm E) NOTA

3. A radioactive substance decays at 5% per year. If 200 grams of the substance are present now, how many grams of the substance, to the nearest gram, will remain after three years?

- A) 170 B) 171 C) 172 D) 173 E) NOTA

4. One car leaves Dallas traveling due north at 40 miles per hour. Two hours later, another car leaves from the same point in Dallas, traveling due east at 60 miles per hour. How far apart, in miles, are the cars two hours after the second car traveling east departs that common point in Dallas?

- A) 160 B) 170 C) 180 D) 200 E) NOTA

5. A company invests \$5000 at 4% annual interest, compounded quarterly. What is the total value of this investment after one year, to the nearest dollar?

- A) \$5200 B) \$5202 C) \$5203 D) \$5205 E) NOTA

6. A bag contains 5 solid red, 4 solid blue, and 3 solid green marbles. If two marbles are drawn from the bag without replacement, what is the probability that both marbles are the same color?

- A) $\frac{19}{66}$ B) $\frac{23}{66}$ C) $\frac{25}{66}$ D) $\frac{29}{66}$ E) NOTA

7. A different bag contains 6 solid red, 5 solid blue, and 4 solid green marbles. If three marbles are drawn without replacement, what is the probability that two of the three colors are represented in the drawn marbles?

- A) $\frac{8}{13}$ B) $\frac{41}{65}$ C) $\frac{42}{65}$ D) $\frac{43}{65}$ E) NOTA

8. A satellite dish is shaped like a paraboloid of revolution, with the receiver placed at the focus of the parabola, 2 meters above the bottom of the dish. If the dish is 6 meters wide at its opening at the top, how deep, in meters, is the dish from the opening of the dish to the dish's bottom?

- A) 1 B) 1.125 C) 1.25 D) 1.5 E) NOTA

9. If a right cylindrical water tank with radius of 3 meters is being filled at a rate of 2 cubic meters per minute, how fast, in meters per minute, is the water level rising in the tank?

- A) $\frac{2}{9\pi}$ B) $\frac{2}{3\pi}$ C) $\frac{4}{9\pi}$ D) $\frac{4}{3\pi}$ E) NOTA

10. Billy Madison has four test scores while in 10th grade: 82, 88, 91, and 94. In order to pass to the 11th grade and come one step closer to taking over his father's company, Billy must make an average of at least 90 on his tests that count toward his final grade for the course. His teacher agrees to drop his lowest score, but only after Billy takes his fifth and final 10th grade test. What must Billy score on the fifth test in order to pass to the 11th grade?

- A) 90 B) 88 C) 87 D) 86 E) NOTA

11. A business sells 200 widgets per week at a price of \$10 each. For every \$1 decrease in price that they make, an additional 20 widgets are sold. At that price, in dollars, will the business's revenue be maximized?

- A) \$6 B) \$7 C) \$8 D) \$9 E) NOTA

12. Dallas's population grows by 8% annually. To the nearest year, in how many years will Dallas's population double?

- A) 8 B) 9 C) 10 D) 11 E) NOTA

For questions 13-14, a Ferris wheel with radius of length 15 meters revolves uniformly, performing a complete revolution in 3 minutes. People riding the Ferris wheel enter carts on the edge of the Ferris wheel on a platform 3 meters off the flat ground.

13. Find the linear speed, in meters per second, of a cart on this Ferris wheel.

- A) $\frac{\pi}{6}$ B) π C) 5π D) 10π E) NOTA

14. If the cart starts (at $t = 0$) at the bottom of the wheel, which expression, in meters, gives the height of the cart above ground after t minutes of revolution?

- A) $15\sin\left(\frac{2\pi t}{3}\right)$ B) $18 + 15\cos\left(\frac{2\pi t}{3}\right)$ C) $15 + 15\sin\left(\frac{2\pi t}{3}\right)$ D) $18 - 15\cos\left(\frac{2\pi t}{3}\right)$
E) NOTA

15. A theater has 30 rows numbered with integers from 1 to 30, inclusive (row 1 is the closest to the stage, row 30 is the farthest from the stage). Each row has 20 seats. Tickets for a performance at this theater cost \$75 per seat in the fifth row, which is the optimal row for viewing the performance. For all other seats, the price per seat in a specific row is reduced from the row 5 pricing by \$1 for every row that row of seats is away from row 5 (e.g., seats in row 13 are \$67 per seat, while seats in row 1 are \$71 per seat). If a performance is a sell-out (i.e., all tickets are sold), how much revenue is generated from these ticket sales?

- A) \$37,050 B) \$37,500 C) \$38,300 D) \$39,300 E) NOTA

16. A company that manufactures light bulbs has measured the life span of their bulbs and finds that distribution to be approximately normally distributed with mean 1000 hours and standard deviation 100 hours. If the company offers a warranty to replace bulbs that fail before a life span of 850 hours, which of the following percentages is the closest to the percentage of bulbs the company expects to have to replace?

- A) 16% B) 13% C) 10% D) 7% E) 3%

17. Find the minimum distance from the point $(2,0)$ to the graph of $y = \sqrt{x+4}$.

- A) $\frac{\sqrt{17}}{2}$ B) $\frac{\sqrt{19}}{2}$ C) $\frac{\sqrt{23}}{2}$ D) $\frac{5}{2}$ E) NOTA

For questions 18-20, three standard, fair, six-sided dice are to be rolled.

18. What is the probability that the sum of the rolled faces is 13?

- A) $\frac{7}{72}$ B) $\frac{1}{12}$ C) $\frac{1}{9}$ D) $\frac{5}{36}$ E) NOTA

19. What is the probability that the sum of the rolled faces is 13, given that one of the rolled faces is a 6?

- A) $\frac{1}{6}$ B) $\frac{15}{91}$ C) $\frac{5}{22}$ D) $\frac{6}{35}$ E) NOTA

20. What is the expected number of rolls of these three dice before rolling a sum of 13? Round your answer to the nearest integer.

- A) 7 B) 9 C) 10 D) 12 E) NOTA

21. A farmer has 120 linear meters of fencing to form a rectangular pen against the side of a large barn. If no fencing is needed against the barn, find the maximum area, in square meters, that could be inside this pen.

- A) 1600 B) 1650 C) 1750 D) 1800 E) NOTA

22. Three cities are connected pairwise by straight roads of lengths 13, 14, and 15 miles. A new water treatment plant will be located so that it is the same distance from all three cities. How many miles of pipeline are required to connect the plant to all three cities?

- A) 21 B) 24 C) 27 D) 30 E) NOTA

23. Odysseus is trying to “thread the needle” through Scylla and Charybdis. As he passes between them, he decides he needs to stay the same distance away from both of them for a short stretch of time. To figure out his path, Odysseus plots Scylla as a point in the coordinate plane and Charybdis as a disk of radius 4 kilometers whose closest point to Scylla is 2 kilometers away. The path that Odysseus takes in his ship follows a path that is part of which of the following?

- A) hyperbola B) parabola C) circle D) line E) NOTA

For questions 24-26, twenty identical machines have been made in a factory, five of which are defective.

24. If an inspector chooses four machines at random, what is the probability that exactly two of those four are defective?

- A) $\frac{208}{969}$ B) $\frac{70}{323}$ C) $\frac{11}{51}$ D) $\frac{2}{9}$ E) NOTA

25. An inspector chooses five machines at random. She then checks to see if any are functional (not defective), and if they are, she puts all 5 machines back and then again selects 5 machines at random. What is the expected number of times she will need to do this until she selects all 5 defective machines?

- A) 10,336 B) 13,566 C) 14,212 D) 15,504 E) NOTA

26. What is the minimum number of machines the inspector must select before she can guarantee she has selected two defective machines?

- A) 13 B) 15 C) 17 D) 19 E) NOTA

27. A circle with radius of length r rolls without slipping in one direction only around the outside of a larger fixed circle with radius of length R , always staying externally tangent to that larger circle. After completing one full revolution about the larger circle, how many rotations about its own center has the smaller circle made?

- A) $\frac{R}{r}$ B) $\frac{R-r}{r}$ C) $\frac{R+r}{r}$ D) $\frac{2R}{r}$ E) NOTA

28. A degree-5 polynomial $p(x)$ has leading coefficient 1 and real zeros 1 , $\cos\theta$, $\sin\theta$, $\sec\theta$, and $\csc\theta$, for some angle θ for which all four trigonometric functions are defined. Find $p(0)$.

- A) $-\tan\theta$ B) $\tan\theta$ C) -1 D) 1 E) NOTA

29. A tightrope walker is standing on top of a platform that is 30 feet tall. A stilt walker walks on level ground close to the platform, and the tightrope walker notices that he has to use an angle of depression of 30° to see the top of the stilt walker's head, while he has to use an angle of depression of 75° to see the point of contact with the stilts to the ground. If the tightrope walker's eyes are 5 feet above the platform, how tall is the stilt walker, from the ground to the top of her head?

- A) $\frac{35(4\sqrt{3}-6)}{13}$ B) $\frac{35(5\sqrt{3}-6)}{13}$ C) $20(3-\sqrt{3})$ D) $\frac{70(3-\sqrt{3})}{3}$ E) NOTA

30. In how many locations in the coordinate plane do the graphs of $r = 5\cos\theta$ and $r = 2 + 2\cos\theta$ intersect?

- A) 2 B) 3 C) 1 D) 0 E) NOTA