

Alpha ciphering answers and solutions
2026 Mu Alpha Theta National Convention

0. $7\frac{1}{2}$

1. -12

2. 116

3. $\sqrt{3}$

4. $\frac{37\sqrt{146}}{73}$

5. 714

6. $\frac{6}{25}$

7. $\frac{2}{9}$

8. $\frac{1}{3}$

9. $12x + 5y + 3z = 31$

10. $3\sqrt{10}$

11. $16\pi - 32$

12. $\frac{7}{18}, -\frac{7}{32}$

13. $\frac{\sqrt{3}}{3}$

14. $\frac{1}{8}$

0. $y = 4x^2 - 6x + 9$

$$x = -\frac{b}{2a} = \frac{6}{8} = \frac{3}{4} \quad y = 4\left(\frac{9}{16}\right) - 6\left(\frac{3}{4}\right) + 9 = \frac{27}{4} \quad \frac{3}{4} + \frac{27}{4} = \frac{30}{4} = 7\frac{1}{2}$$

1. The only way to get x^7 is the sum of four terms of $4(-3x)^1(x^2)^3 \rightarrow -12x^7 \Rightarrow -12$

or

using the multinomial coefficient, $\frac{4!}{3!1!0!}(1)^3(-3)^1(-1)^0 = -12$

2. $a = 4, e = \frac{5}{2} = \frac{c}{a} = \frac{10}{4} \quad a^2 + b^2 = c^2 \rightarrow 16 + b^2 = 100 \rightarrow b^2 = 84 = n. m = a^2 = 16$
 $2(16) + 84 = 116$

3. $\log_b \sqrt{x} = \sqrt{\log_b x} \rightarrow \frac{1}{2} \log_b x = \sqrt{\log_b x}$

Let $c = \sqrt{\log_b x}$, then $\frac{1}{2}c^2 = c \rightarrow \frac{1}{2}c^2 - c = 0 \rightarrow c\left(\frac{1}{2}c - 1\right) = 0 \rightarrow c = 0, 2$

$\sqrt{\log_b x} = 0 \quad \sqrt{\log_b x} = 2 \quad 1 + b^4 = 10$

$\log_b x = 0 \quad \log_b x = 4 \quad b^4 = 9$

$x = 1 \quad x = b^4 \quad b = \sqrt[4]{9} = (3^2)^{\frac{1}{4}} = \sqrt{3}$

4. $D = \frac{|3(7) - 4(11) + 11(10) - 13|}{\sqrt{3^2 + 4^2 + 11^2}} = \frac{74}{\sqrt{146}} = \frac{74\sqrt{146}}{146} = \frac{37\sqrt{146}}{73}$

5. $1196 = 4 \times 13 \times 23 \quad 4 + 13 + 23 = 40 \quad a^2 + b^2 + c^2 = 16 + 169 + 529 = 714$

6. Using the power-reducing identities, $\cos^2 x = \frac{1 + \cos 2x}{2}$ and $\sin^2 x = \frac{1 - \cos 2x}{2}$.

$$\sin^2 x \cos^2 x = \left[\frac{1}{2} \left(1 - \frac{1}{5} \right) \right] \left[\frac{1}{2} \left(1 + \frac{1}{5} \right) \right] = \left(\frac{1}{4} \right) \left(\frac{4}{5} \right) \left(\frac{6}{5} \right) = \frac{6}{25}$$

7. We need the discriminant to be negative so that the graph never crosses the x -axis. We also know that the coefficient of x^2 is positive, so the parabola is opening upward.

$$b^2 - 4ac < 0 \rightarrow 4(c+4)^2 - 4(1)(-5c+64) < 0 \rightarrow 4c^2 + 32c + 64 + 20c - 256 < 0 \rightarrow$$

$$4c^2 + 52c - 192 < 0 \rightarrow c^2 + 13c - 48 < 0 \rightarrow (c+16)(c-3) < 0$$

$-16 < c < 3$, but with the given domain, we can only use $[-5, -2]$, which contains 8 integers.

The given domain contains 36 integers, so the probability is

$$\frac{8}{36} = \frac{2}{9}$$

8. $\sin x + \sin y = \frac{\sqrt{15}}{3}$ and $\cos x + \cos y = 1$. Square each side and add the equations:

$$(\sin x + \sin y)^2 = \sin^2 x + \sin^2 y + 2 \sin x \sin y = \frac{5}{3}$$

$$(\cos x + \cos y)^2 = \cos^2 x + \cos^2 y + 2 \cos x \cos y = 1$$

$$\rightarrow 2 + 2 \sin x \sin y + 2 \cos x \cos y = \frac{8}{3} \rightarrow \sin x \sin y + \cos x \cos y = \cos(x - y) = \frac{1}{3}$$

9. Let $A(1, 2, 3)$, $B(3, -1, 0)$, and $C(2, 2, -1)$. Now, $\overline{AB} = \langle 2, -3, -3 \rangle$ and $\overline{AC} = \langle 1, 0, -4 \rangle$.

$\overline{AB} \times \overline{AC} = \langle 12, 5, 3 \rangle$ = the normal vector. This gives us $12x + 5y + 3z = D$. We can find D by substituting any of the three given points. Using $(1, 2, 3)$, we get $12(1) + 5(2) + 3(3) = D = 31$, so our equation is $12x + 5y + 3z = 31$.

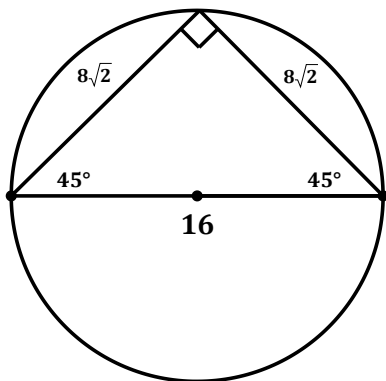
10. Solve each equation for x and y , square each side, and add the equations together. The result is

an ellipse, $\left(\frac{x-4}{7} \right)^2 + \left(\frac{y-8}{3} \right)^2 = 1$. For an ellipse, $a^2 - b^2 = c^2$, so $49 - 9 = c^2 = 40$ and $c = 2\sqrt{10}$.

Our three points are center $(4, 8)$, foci $(4 \pm 2\sqrt{10}, 8)$, and minor axis endpoints $(4, 8 \pm 3)$.

Using $A = \frac{1}{2}bh$, we have $A = \frac{1}{2}(2\sqrt{10})(3) = 3\sqrt{10}$.

11.



The 45° angles are inscribed angles, so their central angles are 90° . Using the segment area formula

$$A = \frac{1}{2}r^2(\theta - \sin\theta), \text{ we get } \frac{1}{2}(64)\left(\frac{\pi}{2} - 1\right) = 16\pi - 32.$$

Using basic geometry formulas, we can subtract the area of the triangle from the area of the semicircle and divide the difference by 2.

$$A = \frac{32\pi - \left(\frac{1}{2}\right)(8\sqrt{2})(8\sqrt{2})}{2} = \frac{32\pi - 64}{2} = 16\pi - 32$$

12. Using the infinite geometric series sum formula, we have

$$\frac{\cos x}{1 - \cos x} = 24 \rightarrow 25\cos x = 24 \rightarrow \cos x = \frac{24}{25}. \text{ We recognize this as a Pythagorean triple, so now we}$$

know that $\sin x = \pm \frac{7}{25}$. Again using the infinite geometric series sum formula, we have

$$\frac{\sin x}{1 - \sin x} = \frac{\frac{7}{25}}{1 - \frac{7}{25}} = \frac{7}{18} \text{ and } \frac{\sin x}{1 - \sin x} = \frac{-\frac{7}{25}}{1 + \frac{7}{25}} = -\frac{7}{32}. \text{ Our two answers are } \frac{7}{18} \text{ and } -\frac{7}{32}.$$

$$13. z = (1 + i\sqrt{3})^{100} = \left(2\operatorname{cis}\frac{\pi}{3}\right)^{100} = 2^{100}\operatorname{cis}\frac{100\pi}{3} = 2^{100}\operatorname{cis}\frac{4\pi}{3} = 2^{100}\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = -2^{99} - 2^{99}\sqrt{3}i.$$

$$\frac{\operatorname{Re}(z)}{\operatorname{Im}(z)} = \frac{-2^{99}}{-2^{99}\sqrt{3}} = \frac{\sqrt{3}}{3}$$

14. The feasible region for our solution is a triangle bounded by the positive x -axis, the positive y -axis, and the line $y = -x + \frac{1}{2}$. Our domain is a 1×1 square of area 1. The probability will be

$$\frac{\frac{1}{2}\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)}{1} = \frac{1}{8}.$$