

- For $z = a + bi$, $\operatorname{Re}(z) = a$, $\operatorname{Im}(z) = b$, and $\bar{z} = a - bi$
- $\sinh(x) = \frac{e^x - e^{-x}}{2}$ and $\cosh(x) = \frac{e^x + e^{-x}}{2}$
- $\cos\theta + i\sin\theta$ is abbreviated as $\operatorname{cis}\theta$
- An imaginary number z has $\operatorname{Im}(z) \neq 0$, while a pure imaginary number has $\operatorname{Im}(z) \neq 0$ and $\operatorname{Re}(z) = 0$
- The argument of a complex number falls within the principal argument of $(-\pi, \pi]$

1. The complex coordinate plane is named for which mathematician?

- A. Argand B. Boole C. Cayley D. Dirichlet E. NOTA

2. Simplify: $\frac{\sqrt{(-5)^2}}{\sqrt{-5^2}}$.

- A. 1 B. -1 C. $-i$ D. i E. NOTA

3. If $z = 1 - 2i$, in which quadrant—or on which axis—does $z^2 + z + 2$ lie?

- A. Quadrant II B. Quadrant III C. real axis D. imaginary axis E. NOTA

4. For real numbers x and y , $[(3 - 2i)x + (1 - i)y]^2 = -4$. Which of the following is a possible value of y ?

- A. 2 B. -3 C. -4 D. -6 E. NOTA

5. Which of the following is a solution to $(x^2 + 5x + 4)(x^2 + 5x + 6) = 120$?

- A. $-\frac{5}{2} + \frac{\sqrt{97}}{2}i$ B. $-\frac{5}{2} + \frac{\sqrt{7}}{2}i$ C. $-\frac{5}{2} + \frac{\sqrt{39}}{2}i$ D. $-\frac{5}{2} + \frac{\sqrt{47}}{2}i$ E. NOTA

6. Simplify: $i^{1!} + i^{2!} + i^{3!} + i^{4!} + i^{5!} + \dots + i^{100!}$.

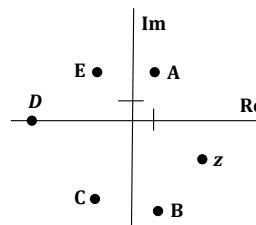
- A. $98 - i$ B. $25 + 25i$ C. $51 - 49i$ D. $95 + i$ E. NOTA

7. Find the sum: $\sum_{n=1}^{10} i^n + \prod_{n=1}^{10} i^n$.

- A. -1 B. $1 + i$ C. $-i$ D. $1 - 2i$ E. NOTA

8. In the figure, z is in Quadrant IV. Which of the following could be $-i\bar{z}$?

- A. point A B. point B C. point C
D. point D E. point E



9. Simplify: $\left(\frac{3+i}{1+2i}\right)^{10}$.

- A. 1 B. $32i$ C. $-32i$ D. $\frac{1}{32}i$ E. NOTA

10. For $f(x) = x^2 + 8ix - c$, which of the following statements must be true?
- If $c > 16$, both roots of f are complex conjugates
 - If $c = 16$, both roots of f are rational
 - If $c < 16$, both roots of f are pure imaginary
 - If $c > 0$, both roots of f are irrational
 - NOTA
11. For real x and y , $(x + yi)(x - yi) = 5 + (x + y - 3)i$. Find the product xy .
- 2
 - 4
 - 1
 - 3
 - NOTA
12. For complex numbers A and B , $A\bar{A} = 1$, $B\bar{B} = 1$, and $A + B = 2i$. Find the value of $A^2 + B^2$.
- 2
 - 2
 - 6
 - 6
 - NOTA
13. Find the area of the region of all points z in the complex plane such that $|z + 1 - i| \leq \sqrt{2}$ and $|z - 1 - i| \geq \sqrt{2}$
- $1 + \pi$
 - $2 + \pi$
 - $2\pi - 2$
 - $2\pi - 1$
 - NOTA
14. In the complex plane, which of the following is the graph of $|z + 1| = |z - i|$?
- the perpendicular bisector of the segment joining $1 + 0i$ and $0 - i$
 - two parallel lines
 - two perpendicular lines
 - two points
 - NOTA
15. Find the area in the complex plane of the set $S = \{z = a + bi \mid a < b \text{ and } 1 \leq |z| \leq 3\}$.
- 16π
 - 8π
 - 4π
 - 2π
 - NOTA
16. If $\overline{z + zi} = 3\bar{z} + 5i$, find $\bar{z}$.
- $-1 - 2i$
 - $-1 + 2i$
 - $2 - i$
 - $-2 + i$
 - NOTA
17. Which of the following is equivalent to $\frac{(\cos 2x - i \sin 2x)^4 (\cos 4x + i \sin 4x)^{-5}}{(\cos 3x + i \sin 3x)^{-2} (\cos 3x - i \sin 3x)^{-9}}$?
- $\cos 49x - i \sin 49x$
 - $\cos 49x + i \sin 49x$
 - $\cos 23x - i \sin 23x$
 - $\cos 21x + i \sin 21x$
 - NOTA
18. If $A = \begin{bmatrix} 1+i & 1 \\ -i & 0 \end{bmatrix}$, for how many values of n , $n \in \{1, 2, 3, \dots, 100\}$, does $A^n = A$?
- 25
 - 24
 - 20
 - 16
 - NOTA
19. If $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + yi$, find (x, y) .
- (3, 1)
 - (1, 3)
 - (0, 3)
 - (0, 0)
 - NOTA

20. Let ω be an imaginary cube root of unity, and let $P = [p_{ij}]$ be an $n \times n$ matrix ($n \geq 2$) with $p_{ij} = \omega^{i+j}$. For which of the following values of n is $P^2 \neq 0$?

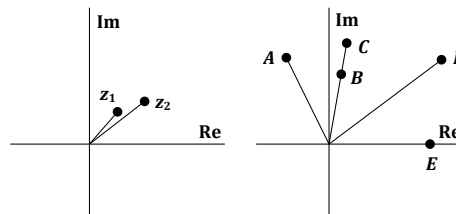
A. 33 B. 44 C. 63 D. 78 E. NOTA

21. One real root of $x^4 - 14x^3 + 72x^2 - 162x + 133 = 0$ is between 2 and 3, and the other real root is between 5 and 6. The imaginary roots are in the form $a + bi$, where a and b are nonzero real numbers. What integer is closest in value to a ?

A. 3 B. 2 C. -2 D. 4 E. NOTA

22. z_1 and z_2 represent two complex numbers whose lengths are greater than 1 and which are graphed on identically scaled axes. Which point in the second figure might reasonably represent the product $z_1 z_2$?

A. A B. B C. C
D. D E. E



23. If $x = a + bi$, $x^2 = 3 + 4i$, and $x^3 = 2 + 11i$, find the value of $a + b$.

A. 2 B. 3 C. 5 D. 6 E. NOTA

24. If $1, \omega, \omega^2, \omega^3, \dots, \omega^6$ are the 7th roots of unity in increasing order of argument, find $\text{Im}(\omega + \omega^2 + \omega^4)$.

A. $\frac{\sqrt{7}}{7}$ B. $\frac{\sqrt{7}}{8}$ C. $\frac{\sqrt{7}}{2}$ D. $\frac{\sqrt{7}}{6}$ E. NOTA

25. If $z = e^{\frac{2\pi i}{5}}$, find the value of $1 + z + z^2 + z^3 + 5z^4 + 4z^5 + 4z^6 + 4z^7 + 4z^8 + 5z^9$.

A. $4e^{\frac{2\pi i}{5}}$ B. $4e^{\frac{3\pi i}{5}}$ C. $-5e^{\frac{2\pi i}{5}}$ D. $-5e^{\frac{3\pi i}{5}}$ E. NOTA

26. Find the complex roots of $e^{2x} = i$.

A. $\frac{i}{2} \left(-\frac{\pi}{2} + 2n\pi \right)$ B. $2i \left(-\frac{\pi}{2} + n\pi \right)$ C. $\frac{i}{2} \left(\frac{\pi}{2} + n\pi \right)$ D. $\frac{i}{2} \left(\frac{\pi}{2} + 2n\pi \right)$ E. NOTA

27. If $\sin^{-1}(A + Bi) = x + yi$, find $\frac{A}{B}$.

A. $\frac{\tanh x}{\tanh y}$ B. $\frac{\tanh x}{\tan y}$ C. $\frac{\tanh x}{\tanh y}$ D. $\frac{\cos x}{\cosh y}$ E. NOTA

28. If $x + \frac{1}{x} = 2\cos 36^\circ$, find the value of $x^5 + \frac{1}{x^5}$.

A. $\frac{1}{4}$ B. $-\frac{8}{3}$ C. $\frac{5}{12}$ D. 0 E. NOTA

29. Simplify: $\ln\left[-(i\cos 5 + \sin 5)^2\right]$.

- A. $10i$ B. $-10i$ C. $\ln 10 + \pi i$ D. $2e^{5i}$ E. NOTA

30. A point z_0 is called a fixed point of a complex function $f(z)$ if $f(z_0) = z_0$. Find all fixed points of the Mobius transformation $f(z) = \frac{z-2}{z-1}$.

- A. $1 \pm 2i$ B. $1 \pm i$ C. $2 \pm i$ D. $-1 \pm \sqrt{2}i$ E. NOTA