

Alpha Complex Numbers
2026 Mu Alpha Theta National Convention
Solutions

1. A	6. D	11. A	16. A	21. A	26. D
2. C	7. A	12. B	17. A	22. C	27. A
3. D	8. B	13. B	18. A	23. B	28. E (-2)
4. D	9. C	14. A	19. D	24. C	29. B
5. C	10. E	15. C	20. B	25. D	30. B

1. Argand A

$$2. \frac{\sqrt{(-5)^2}}{\sqrt{-5^2}} = \frac{|-5|}{\sqrt{-25}} = \frac{5}{5i} = \frac{1}{i} = -i \quad C$$

$$3. (1-2i)^2 + (1-2i) + 2 = -3 - 4i + 1 - 2i + 2 = -6i \quad D$$

$$4. \text{ Since } z^2 = [(3-2i)x + (1-i)y]^2 = -4, z \text{ must be a pure imaginary number. } z = 3x - 2xi + y - yi = (3x+y) + (-2x-y)i \text{ To be pure imaginary, } 3x+y=0 \rightarrow y=-3x \rightarrow z = (-2x-(-3x))i = xi \\ (xi)^2 = -4 \rightarrow -x^2 = -4 \rightarrow x = \pm 2, y = \mp 6 \quad D$$

$$5. (x^2 + 5x + 4)(x^2 + 5x + 6) = 120 \Rightarrow x^4 + 10x^3 + 35x^2 + 50x - 96 = (x-1)(x+6)(x^2 + 5x + 16) = 0 \\ x = 1, -6, -\frac{5 \pm \sqrt{39}}{2}i \quad C$$

$$6. i^{1!} + i^{2!} + i^{3!} + i^{4!} + i^{5!} + \dots + i^{100!} \rightarrow [i^1 + i^{2 \cdot 1} + i^{3 \cdot 2 \cdot 1}] + [i^{4 \cdot 3 \cdot 2 \cdot 1} + i^{4 \cdot 5 \cdot 3 \cdot 2 \cdot 1} + \dots + i^{4 \cdot 100 \cdot 99 \cdot \dots \cdot 3 \cdot 2 \cdot 1}] \rightarrow \\ [i - 1 + (-1)] + 1(97) = 95 + i \quad D$$

$$7. \sum_{n=1}^{10} i^n + \prod_{n=1}^{10} i^n = (i + i^2 + i^3 + \dots + i^{10}) + i^{1+2+3+\dots+10} = (i-1) + i^3 = -1 \quad A$$

$$8. \text{ Let } z = 2 - i. -i\bar{z} = -i(2+i) = 1 - 2i \quad B$$

$$9. \left(\frac{3+i}{1+2i}\right)^{10} = \left(\frac{(3+i)(1-2i)}{(1+2i)(1-2i)}\right)^{10} = \left(\frac{5-5i}{5}\right)^{10} = (1-i)^{10} = (-2i)^5 = -32i \quad C$$

$$10. z = \frac{-8i \pm \sqrt{(8i)^2 - 4(1)(-c)}}{2(1)} = \frac{-8i \pm \sqrt{-64 + 4c}}{2} \text{ Looking at the discriminant, we can find all that}$$

we need to know pertaining to the answer choices. If $c = 16$, we have equal roots. If $c > 16$, we have complex imaginary roots (which eliminates choice D), but they are not conjugates since the sign on the imaginary part is the same for both. If $c < 16$, we have pure imaginary roots, unless $c = 0$, in which case one of the roots is real. E

11. $(x + yi)(x - yi) = 5 + (x + y - 3)i$ The left-hand side of the equation is real, so we get

$$\begin{cases} x^2 + y^2 + 5 \\ x + y = 3 \end{cases} \cdot (x + y)^2 = x^2 + y^2 + 2xy \rightarrow 9 = 5 + 2xy \rightarrow xy = 2 \quad A$$

12. We are given $A\bar{A} = 1$, $B\bar{B} = 1$, and $A + B = 2i$. This leads to $\bar{A} = \frac{1}{A}$ and $\bar{B} = \frac{1}{B}$.

$$\begin{cases} \bar{A} + \bar{B} = \frac{1}{A} + \frac{1}{B} = \frac{A+B}{AB} \rightarrow \frac{2i}{AB} = -2i \rightarrow AB = -1 \\ \bar{A} + \bar{B} = \overline{A+B} = \overline{2i} = -2i \end{cases}$$

$$A^2 + B^2 = (A+B)^2 - 2AB = (2i)^2 - 2(-1) = -4 + 2 = -2 \quad B$$

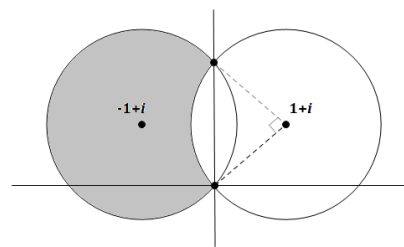
13. The inequalities can be written as $|z - (-1 + i)| \leq \sqrt{2}$ and

$|z - (1 + i)| \geq \sqrt{2}$, giving us two congruent circles of radius

$\sqrt{2}$, one with center $(-1, 1)$ and the other with center $(1, 1)$.

The common region is the shaded region in the first circle.

Our area will be $\pi(\sqrt{2})^2 - 2\left(\frac{1}{4}\pi(\sqrt{2})^2 - \frac{1}{2}(\sqrt{2})^2\right) = 2 + \pi.$ B



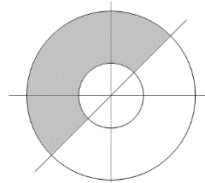
14. $|z + 1| = |z - i| \rightarrow |z - (-1)| = |z - (i)|$ which is the set of points equidistant from $-1 + 0i$ and $0 + i$.

This set of points is the perpendicular bisector of the line connecting these two points, which is the same as the perpendicular bisector of the segment joining $1 + 0i$ and $0 - i$. A

15. This will be the common region to $y \geq x$, $x^2 + y^2 \geq 1$, and

$y \geq x$, $x^2 + y^2 \leq 3$. This is half the area of the annulus:

$$\frac{1}{2}(9\pi - \pi) = 4\pi. \quad C$$



16. $z + zi = (a + bi) + (a + bi)i = (a - b) + (a + b)i$, so $\overline{z + zi} = (a - b) - (a + b)i = 3(a - bi) + 5i$. Simplifying, we have $\overline{z + zi} = (a - b) - (a + b)i = 3a + (-3b + 5)i$. Setting up a system of equations, we now have

$$\begin{cases} a - b = 3a + 0b \\ -a - b = -3b + 5 \end{cases} \rightarrow \begin{cases} -2a = b \\ a = 2b - 5 \end{cases} \rightarrow a = -4a - 5 \rightarrow a = -1, b = 2 \quad z = -1 + 2i, \bar{z} = -1 - 2i \quad A$$

17. Using DeMoivre's Theorem, we have

$$\frac{(\cos 2x - i \sin 2x)^4 (\cos 4x + i \sin 4x)^{-5}}{(\cos 3x + i \sin 3x)^{-2} (\cos 3x - i \sin 3x)^{-9}} \rightarrow \frac{\left[(\cos x + i \sin x)^{-2}\right]^4 \left[(\cos x + i \sin x)^4\right]^{-5}}{\left[(\cos x + i \sin x)^3\right]^{-2} \left[(\cos x + i \sin x)^{-3}\right]^{-9}}$$

$$(\cos x + i \sin x)^{-8-20+6-27} = (\cos x + i \sin x)^{-49} = \cos 49x - i \sin 49x \quad A$$

$$18. A^1 = \begin{bmatrix} 1+i & 1 \\ -i & 0 \end{bmatrix}, A^2 = \begin{bmatrix} i & 1+i \\ 1-i & -i \end{bmatrix}, A^3 = \begin{bmatrix} 0 & i \\ 1 & 1-i \end{bmatrix}, A^4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I. A^{4n+1} = A \text{ for } n=1, 5, 9, \dots, 97. \\ 97 = 1 + (n-1)4 \rightarrow 24 = n-1 \rightarrow n=25 \quad A$$

$$19. 6i(-3+3) - (-3i)(4i+20) + 1(12-60i) = 0 - 12 + 60i + 12 - 60i = 0 \quad D$$

$$20. \text{ For } n=2: P^2 = \begin{bmatrix} \omega^2 & \omega^3 \\ \omega^3 & \omega^4 \end{bmatrix} \begin{bmatrix} \omega^2 & \omega^3 \\ \omega^3 & \omega^4 \end{bmatrix} = \begin{bmatrix} -\omega^2 & -1 \\ -1 & -\omega \end{bmatrix} \neq \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\text{For } n=3: P^2 = \begin{bmatrix} \omega^2 & \omega^3 & \omega^4 \\ \omega^3 & \omega^4 & \omega^5 \\ \omega^4 & \omega^5 & \omega^6 \end{bmatrix} \begin{bmatrix} \omega^2 & \omega^3 & \omega^4 \\ \omega^3 & \omega^4 & \omega^5 \\ \omega^4 & \omega^5 & \omega^6 \end{bmatrix}$$

$$= \begin{bmatrix} \omega^4 + \omega^2 + 1 & \omega(\omega^4 + \omega^2 + 1) & \omega(\omega^4 + \omega^2 + 1) \\ \omega(\omega^4 + \omega^2 + 1) & \omega^4 + \omega^2 + 1 & \omega(\omega^4 + \omega^2 + 1) \\ \omega(\omega^4 + \omega^2 + 1) & \omega(\omega^4 + \omega^2 + 1) & \omega^4 + \omega^2 + 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

When n is a multiple of 3, $P^2 = 0$. Among the given choices, only 44 is NOT a multiple of 3. B

21. Let the roots be r_1, r_2 , and $a \pm bi$. $r_1 + r_2 + (a+bi) + (a-bi) = 14$ We know that $2 < r_1 < 3$ and $5 < r_2 < 6$, so $(2+5)7 < r_1 + r_2 < 9(3+6)$. Also considering the imaginary roots, we have

$$7 + 2a < 14 < 9 + 2a \rightarrow \frac{5}{2} < a < \frac{7}{2}. \quad a \text{ is close to } 3. \quad A$$

22. $z_1 z_2$ has magnitude $|z_1||z_2|$ and argument $\arg(z_1) + \arg(z_2)$. Among the choices, C is best.

$$23. \frac{x^3}{x^2} = x = \frac{2+11i}{3+4i} \cdot \frac{3-4i}{3-4i} = \frac{6+44-8i+33i}{9+16} = \frac{50+25i}{25} = 2+i \rightarrow 2+1=3 \quad B$$

24. If we graph the 7 points, ω^1 and ω^6 are conjugate pairs, ω^2 and ω^5 are conjugate pairs, and ω^3 and ω^4 are conjugate pairs. Since we are concerned with ω^1, ω^2 , and ω^4 , we will group them together.

$$\begin{cases} \omega^1 + \omega^2 + \omega^4 = a+bi \\ \omega^3 + \omega^5 + \omega^6 = a-bi \end{cases} \rightarrow 2a = \omega^1 + \omega^2 + \omega^3 + \omega^4 + \omega^5 + \omega^6 = -1, \text{ since the sum of the roots is } -1.$$

$$\text{This gives us } a = -\frac{1}{2}. \text{ Now, } (a+bi)(a-bi) = a^2 + b^2 = \omega^4 + \omega^6 + \omega^7 + \omega^5 + \omega^7 + \omega^8 + \omega^7 + \omega^9 + \omega^{10} \\ \rightarrow \omega^4 + \omega^6 + 1 + \omega^5 + 1 + \omega^1 + 1 + \omega^2 + \omega^3 \rightarrow 3 + \omega^1 + \omega^2 + \omega^3 + \omega^4 + \omega^5 + \omega^6 = 3 + (-1) = 2, \text{ using sum}$$

of roots.

$$a^2 + b^2 = 2 = \left(-\frac{1}{2}\right)^2 + b^2 \rightarrow b^2 = \frac{7}{4} \rightarrow b = \frac{\sqrt{7}}{2} \quad C$$

25. $z = e^{\frac{2\pi i}{5}}$ is a fifth root of unity. The sum of the n^{th} roots of unity is $1 + z + z^2 + z^3 + \dots + z^{n-1} = 0$.

Using this, we can rewrite the given expression:

$$\begin{aligned} & 1 + z + z^2 + z^3 + 5z^4 + 4z^5 + 4z^6 + 4z^7 + 4z^8 + 5z^9 \\ &= 1 + z + z^2 + z^3 + 5z^4 + 4 + 4z + 4z^2 + 4z^3 + 5z^4 \\ &= 5 + 5z + 5z^2 + 5z^3 + 10z^4 \\ &= 5(1 + z + z^2 + z^3 + 2z^4) \end{aligned}$$

For $n=5$, $1 + z + z^2 + z^3 + z^4 = 0$. Using this, we have $5(0 + z^4) = 5z^4 = 5\left(e^{\frac{2\pi i}{5}}\right)^4 = 5e^{\frac{8\pi i}{5}} = -5e^{\frac{3\pi i}{5}} \quad D$

$$26. e^{2x} = i = e^{i\left(\frac{\pi}{2} + 2\pi n\right)} \rightarrow 2x = \frac{\pi}{2}i + 2\pi in \rightarrow x = \frac{\pi}{4}i + \pi in \quad D$$

$$\begin{aligned} 27. \sin^{-1}(A + Bi) &= x + yi \rightarrow A + Bi = \sin(x + yi) = \sin x \cos(yi) + \cos x \sin(yi) \\ &= \sin x \cosh y + (\cos x \sinh y)i \end{aligned}$$

Equating the real parts and the imaginary parts, we have $\frac{A}{B} = \frac{\sin x \cosh y}{\cos x \sinh y} = \frac{\tan x}{\tanh y} \quad A$

$$28. x + \frac{1}{x} = 2\cos 36^\circ \rightarrow x^2 - 2x\cos 36^\circ + 1 = 0$$

$$x = \frac{2\cos 36^\circ \pm \sqrt{4\cos^2 36^\circ - 4}}{2} = \frac{2\cos 36^\circ \pm 2\sqrt{\cos^2 36^\circ - 1}}{2} = \cos 36^\circ \pm \sqrt{\cos^2 36^\circ - 1}$$

$$x = \cos 36^\circ \pm \sin 36^\circ i = \text{cis} 36^\circ, \text{cis}(-36^\circ)$$

$$x = e^{\frac{\pi i}{5}} \rightarrow x^5 = e^{\pi i}, x^{-5} = e^{-\pi i} \quad \text{or} \quad x = e^{-\frac{\pi i}{5}} \rightarrow x^5 = e^{-\pi i}, x^{-5} = e^{\pi i} \quad \text{Using either,}$$

$$x^5 + \frac{1}{x^5} = (-1 + 0i) + (-1 + 0i) = -2. \quad E \quad \text{Note: If } x + \frac{1}{x} = 2\cos \theta, x^n + \frac{1}{x^n} = 2\cos(n\theta).$$

$$29. \ln\left[-(i\cos 5 + \sin 5)^2\right] \rightarrow \ln\left[-(-\cos^2 5 + \sin^2 5 + 2i\sin 5\cos 5)\right] \rightarrow$$

$$\ln\left[\cos^2 5 - \sin^2 5 - 2i\sin 5\cos 5\right] \rightarrow \ln[\cos 10 - i\sin 10] \rightarrow \ln[\cos(-10) + i\sin(-10)] =$$

$$= \ln[\cos(4\pi - 10) + i\sin(4\pi - 10)] = \ln e^{(4\pi - 10)i} = (4\pi - 10)i \quad E$$

$$30. z = \frac{z-2}{z-1} \rightarrow z^2 - 2z + 2 = 0 \quad z = \frac{2 \pm \sqrt{4 - 4(1)(2)}}{2(1)} = \frac{2 \pm \sqrt{-4}}{2} = \frac{2 \pm 2i}{2} = 1 \pm i \quad B$$