

All logarithms must be real-valued unless otherwise specified. Where applicable, “E) NOTA” indicates that none of the above answers is correct.

1. Evaluate: $\log_2 128 - \log_3 27$

- A) 3 B) 4 C) 5 D) 6 E) NOTA

2. Solve for x : $5^x = 0.04$

- A) 1 B) 0 C) -1 D) -2 E) NOTA

3. Evaluate: $\log_2 \left(2048^{\frac{1}{5}} \right)$

- A) 2 B) $\frac{11}{5}$ C) 10 D) 11 E) NOTA

4. What is the greatest integer x that satisfies $5^x \mid 2500$?

- A) 2 B) 3 C) 4 D) 5 E) NOTA

5. If $a = \log_3 2$, express $\log_9 2916$ in terms of a .

- A) $2 + a$ B) $(2 + a)^2$ C) $3 + a$ D) $(3 + a)^2$ E) NOTA

6. Solve for x : $3^{2x-1} = \frac{1}{9}$

- A) $-\frac{1}{2}$ B) 0 C) $\frac{1}{2}$ D) 1 E) NOTA

7. Simplify: $2^{3\log_2 3}$

- A) 3 B) 11 C) 24 D) 27 E) NOTA

8. Solve for x : $4^x = 8^{x-1}$

- A) 3 B) 4 C) 5 D) 6 E) NOTA

9. Simplify: $\log_3 \left(\frac{81^{x+1}}{9^{2x-3}} \right)$

- A) $-x + 4$ B) $3x - 2$ C) -3 D) 10 E) NOTA

10. How many integers n satisfy the inequality $2^n < 100$?

- A) 5 B) 6 C) 7 D) 8 E) NOTA

11. Let $f^n(x) = \underbrace{(f \circ f \circ \dots \circ f)}_{n \text{ copies of } f}(x)$. If $f(x) = \ln x$, find the greatest integer n such that $f^n(3)$ is defined.

- A) 2 B) 3 C) 4 D) no greatest value E) NOTA

12. If $\log_b 81 = 4$, find the value of $\log_3 b$.

- A) 1 B) $\frac{\sqrt{2}}{2}$ C) $\frac{1}{4}$ D) $\frac{1}{6}$ E) NOTA

13. Evaluate: $\sum_{k=2}^{10} 2^k$

- A) 2046 B) 2047 C) 2048 D) 2049 E) NOTA

14. Find the sum of the solutions to the equation $\log_8(x-3) + \log_8(x-5) = 1$.

- A) 5 B) 6 C) 7 D) 8 E) NOTA

15. Find the product of the solutions to the equation $\log_x 256 = \log_2 x$.

- A) $2^{2\sqrt{2}}$ B) 4 C) 8 D) 1 E) NOTA

16. Find the sum of the greatest possible integer m and the least possible integer n such that $3^m < 2026 < 3^n$.

- A) 7 B) 9 C) 11 D) 13 E) NOTA

17. Solve for x : $5^x + 5^{x-1} = 150$

- A) 2 B) 3 C) 4 D) 5 E) NOTA

18. What is the greatest integer n that satisfies $3^n \mid 100!$?

- A) 46 B) 47 C) 48 D) 49 E) NOTA

19. Solve for x : $3^x - 3^{x-2} = 648$

- A) 6 B) 7 C) 8 D) 9 E) NOTA

20. If $\log_2 x + \log_x 2 = 7$, find the value of $(\log_2 x)^2 + (\log_x 2)^2$.

- A) 41 B) 43 C) 47 D) 48 E) NOTA

21. The equation $\log_2 x + \log_4 x + \log_8 x = 11$ has an integer solution. Find the least integer $n > 11$ such that $\log_2 x + \log_4 x + \log_8 x = n$ also has an integer solution.

- A) 16 B) 42 C) 88 D) 121 E) NOTA

22. In how many consecutive zeros does the base 10 number $2026!$ have when it is written in base 12?

- A) 995 B) 996 C) 1009 D) 1010 E) NOTA

23. Find the sum of the solutions to the equation $9^x - 4 \cdot 3^x + 3 = 0$.

- A) 0 B) 1 C) 3 D) 4 E) NOTA

24. How many integers n satisfy the inequality $2^{10} \leq n^2 + n + 1 < 2^{11}$?

- A) 26 B) 28 C) 30 D) 32 E) NOTA

25. Evaluate: $\prod_{k=2}^{2047} (\log_{k^2} (k+1)^2)$

- A) 11 B) 12 C) 121 D) 144 E) NOTA

26. Integers $m, n > 1$ satisfy the equation $2^m = 5^n + 3$. Find the least possible value of the sum $m + n$.

- A) 6 B) 10 C) 12 D) 16 E) NOTA

27. When written as a base 10 numeral, find the tens digit of 7^{45^2+1} ?

- A) 0 B) 2 C) 4 D) 6 E) NOTA

28. Simplify: $\frac{\log_{10} 75 - \log_{100} 9}{\log_{1000} 125}$

- A) 1 B) 2 C) 3 D) 4 E) NOTA

29. When written as a base 10 numeral, how many digits does 3^{20} have? Assume that $\log 3 \approx 0.48$.

- A) 7 B) 8 C) 9 D) 10 E) NOTA

30. Consider the equation $50^x - 3 \cdot 25^x - 2 \cdot 10^x + 6 \cdot 5^x - 8 \cdot 2^x + 24 = 0$. If the product of the real solutions can be written in the form $\log_a b$, where a and b are positive integers, find the least possible value of $a + b$.

- A) 6 B) 10 C) 12 D) 14 E) NOTA