

All uppercase letter variables are positive integers unless otherwise stated. All fractions containing uppercase letter variables are in lowest terms. NOTA means “None of the Above.” ~~~~~ Good luck, and have fun! ~~~~~

1. Which of the following products is cheapest?
A. A camera that originally cost \$65, and is now 60% off.
B. A calculator that originally cost \$55, and is now 50% off.
C. A jacket that originally cost \$45, and is now 40% off.
D. A Rubik’s cube that originally cost \$35, and is now 30% off.
E. NOTA

For questions 2 through 4, let S be the set of possible rational roots of the polynomial $f(x) = 24x^{2026} - 47x^9 + 150$ according to the Rational Root Theorem.

2. Find the sum of the elements of S .
A. 372 B. 744 C. 930 D. 1860 E. NOTA
3. Find the number of elements in S .
A. 20 B. 40 C. 45 D. 90 E. NOTA
4. Let the product of the elements of S be $(-1)^n \cdot 2^x \cdot 3^y \cdot 5^z$ for integers n, x, y , and z with $n = 1$ or $n = 2$. Find $n(x + 2y + 3z)$.
A. 0 B. 192 C. 384 D. 768 E. NOTA
5. Ursula and Croix go on a date to the coffee shoppe. Both of them order mochas, which consist of coffee mixed with steamed chocolate milk. Ursula likes her mocha dark, with 80% coffee and 20% milk, while Croix likes her mocha to be an even mix of coffee and milk. If in total the couple drinks 8 cups of coffee and 4 cups of milk in total, then the number of cups of mocha Croix drinks, in lowest terms, is equal to $\frac{L}{N}$. Find $L + N$.
A. 17 B. 19 C. 23 D. 28 E. NOTA
6. If $\tan \theta + \tan \varphi = 6$ for $0 < \theta, \varphi < \frac{\pi}{2}$, find the maximum possible negative value of $\tan(\theta + \varphi)$.
A. -3 B. $-\frac{6}{5}$ C. $-\frac{3}{4}$ D. $-\frac{1}{2}$ E. NOTA

7. Line ℓ_1 has slope 1, and line ℓ_2 has slope 7. Line ℓ_3 bisects the acute angle formed by lines ℓ_1 and ℓ_2 . Find the slope of line ℓ_3 .
A. 2 B. $\sqrt{7}$ C. $2\sqrt{2}$ D. 3 E. NOTA
8. Let the sum of the two prime factors of 4,101,649 equal N . Find the remainder when N is divided by 6.
Hint: You may use the fact that $2025^2 = 4100625$.
A. 2 B. 3 C. 4 D. 5 E. NOTA
9. Find the length of the minor axis of the ellipse $8x^2 + 2\sqrt{3}xy + 6y^2 = 9$.
A. 1 B. $\frac{3}{\sqrt{5}}$ C. 2 D. $\frac{6}{\sqrt{5}}$ E. NOTA
10. Equilateral triangle AHS is inscribed in rectangle $ABCD$. If H is on $\overline{BC}$ such that $|\overline{BH}| = 1$ and S is on $\overline{CD}$ such that $|\overline{SD}| = \sqrt{3}$, then find the area enclosed by triangle AHS .
A. $4\sqrt{3}$ B. $5\sqrt{3}$ C. $6\sqrt{3}$ D. $7\sqrt{3}$ E. NOTA
11. $\{a_n\}_{n \geq 3}$, $\{b_n\}_{n \geq 3}$, and $\{c_n\}_{n \geq 3}$ are sequences of real numbers such that for all $n \geq 3$, $a_n < b_n < c_n$, $a_n + b_n + c_n = 3n - 1$, $a_nb_n + b_nc_n + c_na_n = 3n + 1$, and $a_nb_nc_n = 3$. Find $\lim_{n \rightarrow \infty} \frac{c_n}{n}$.
A. 1 B. $\frac{1}{2} + \sqrt{2}$ C. $\frac{1 + \sqrt{10}}{2}$ D. 3 E. NOTA
12. The ratio of the measures of the exterior angles of a triangle is 3:4:5. Find the measure of the smallest interior angle of the triangle.
A. 12° B. 15° C. 18° D. 24° E. NOTA
13. If $\sqrt{14 + 8\sqrt{3}} = \sqrt{A} + \sqrt{B}$, find $|A - B|$.
A. 2 B. 3 C. 4 D. 6 E. NOTA

14. An infinite geometric series has sum 20. If the common ratio is doubled, then the sum of the series becomes 26. The first term of the original series has value $\frac{A}{B}$, written in simplest form. Find $A + B$.
A. 19 B. 39 C. 59 D. 69 E. NOTA
15. The solutions to the equation $20x^2 + 26x + c = 0$ are $\sin \theta$ and $\cos \theta$ for some θ . Find c .
A. $-\frac{69}{5}$ B. $-\frac{69}{10}$ C. $\frac{69}{10}$ D. $\frac{69}{5}$ E. NOTA
16. $f(x)$ is a polynomial with integer coefficients such that $f(5) = f(9) = 25$. Find the minimum possible value of $|f(0)|$.
A. 0 B. 5 C. 20 D. 25 E. NOTA
17. Let $m = \frac{1}{1^5} + \frac{1}{3^5} + \frac{1}{5^5} + \frac{1}{7^5} + \dots$ and $n = \frac{1}{2^5} + \frac{1}{4^5} + \frac{1}{6^5} + \frac{1}{8^5} + \dots$. Find the remainder when $\frac{m}{n}$ is divided by 6.
A. 2 B. 3 C. 4 D. 5 E. NOTA
18. A hemisphere with radius 1 is inscribed in a square pyramid with height 7. The base of the hemisphere rests on the base of the pyramid. Find the slant height of the pyramid.
A. $4\sqrt{3}$ B. $5\sqrt{2}$ C. $\frac{49\sqrt{3}}{12}$ D. $\frac{49\sqrt{2}}{10}$ E. NOTA
19. 2026 is the length of one leg in a Pythagorean triple. Let L be the length of the other leg in this triple. Find the remainder when L is divided by 6.
A. 2 B. 3 C. 4 D. 5 E. NOTA
20. One of the sides of an isosceles triangle is $\frac{5}{2}$ times as long as another side. The perimeter of the triangle is 36. Find the sum of all possible unique side lengths of this triangle.
A. 21 B. 28 C. 42 D. 49 E. NOTA

21. Find the product of the real solutions to $x^{3-\log_2 x} = \pi$.
A. 3 B. 4 C. 8 D. 9 E. NOTA
22. $f(x) = ax^2 + bx + c$ is a polynomial such that $f(7) = 20$, $f(8) = 26$, and $f(9) = 2026$. Find $100a + 10b + c$.
A. 4032 B. 6020 C. 6032 D. 6176 E. NOTA
23. The graphs of $y = |ax - 1|$ and $y = bx$ are drawn in the coordinate plane, where a and b are positive constants with $a \geq b$. The graphs intersect at exactly one point. Which of the following must be true?
A. The x -coordinate and y -coordinate of the intersection are both fixed.
B. The x -coordinate of the intersection is fixed, but the y -coordinate of the intersection depends on the value of a .
C. The x -coordinate of the intersection depends on the value of a , but the y -coordinate of the intersection is fixed.
D. The x -coordinate and y -coordinate of the intersection both depend on the value of a .
E. NOTA
24. Evaluate: $\frac{1}{1+\frac{1}{1+\frac{1}{1+\dots}}}$ - $\sqrt{1 + \sqrt{1 + \sqrt{1 + \dots}}}$.
A. $-\sqrt{5}$ B. -1 C. 0 D. 1 E. NOTA
25. In quadrilateral $ABCD$, $|\overline{AB}| = 3$, $|\overline{BC}| = 5$, $|\overline{CD}| = 7$, and $|\overline{DA}| = 13$. Find the value of $|\overline{BD}|$, given that it is an integer.
A. 7 B. 9 C. 10 D. 11 E. NOTA
26. Let P be a permutation of the letters in the word "UNCOPYRIGHTABLES". Find the probability that in P , the 'C' appears earlier than the 'O', which appears earlier than the 'U', which appears earlier than both the 'N' and the 'T'.
A. $\frac{1}{120}$ B. $\frac{1}{60}$ C. $\frac{1}{30}$ D. $\frac{1}{15}$ E. NOTA

For Questions 27 and 28, let $A = \begin{bmatrix} 1 & -2 & 2 \\ 2 & -1 & 2 \\ 2 & -2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$.

27. Find the sum of the entries of AB .

A. 10 B. 12 C. 14 D. 15 E. NOTA

28. Find the largest entry of A^5B .

Hint: Find $A(AB)$.

A. 41 B. 61 C. 77 D. 85 E. NOTA

29. A straight bet on a roulette table pays 35 to 1, meaning if your number is rolled on the roulette wheel, you will profit \$35 for every \$1 you bet; otherwise, you lose your bet. In the long run, this is a profitable bet for the casino, since there are 38 sections of a roulette wheel that can be rolled, each with equal probability. Find the expected value of the amount of money a casino profits if a player plays a \$1 straight bet on one spin of a roulette wheel.

A. $\$ \frac{2}{38}$ B. $\$ \frac{2}{37}$ C. $\$ \frac{3}{38}$ D. $\$ \frac{3}{37}$ E. NOTA

30. The probability that you lose a straight bet on a single spin of a roulette wheel is $\frac{37}{38}$. The probability that you lose n consecutive straight bets on a roulette wheel is therefore $\left(\frac{37}{38}\right)^n$. Consider a betting strategy where you play straight bets on a roulette wheel until you win. If your bet wins before you lose 35 consecutive times, then you profit. $\left(\frac{37}{38}\right)^{34} \approx 40.38\%$, so the probability you profit is 1 minus this, or 59.62%. Which of the following is true?

A. This is an unprofitable roulette strategy in the long run because $40.38\% < 50\%$.
B. This is an unprofitable strategy because it assumes you stop betting as soon as you win.
C. This is an unprofitable strategy because in the long run, you will lose more money in 40.38% of cases than you profit in 59.62% of cases.
D. This is a profitable roulette strategy in the long run.
E. NOTA