

- Unless otherwise specified, all matrices and vectors have real entries
- “NOTA” stands for “None of These Answers are correct.” Good luck! :^)

1. Evaluate the determinant: $\begin{vmatrix} 2026 & 2024 \\ 2025 & 2023 \end{vmatrix}$

- A) 2 B) -4 C) -2 D) 4 E) NOTA

2. If $A = \begin{bmatrix} 1 & 4 \\ -5 & 2 \end{bmatrix}$ and $A + 2B = \begin{bmatrix} 5 & -8 \\ -7 & 10 \end{bmatrix}$, what is B^{-1} ?

- A) $\begin{bmatrix} 4 & -2 \\ -12 & 3 \end{bmatrix}$ C) $\begin{bmatrix} -2 & 0 \\ \frac{1}{2} & 3 \end{bmatrix}$ E) NOTA
- B) $\begin{bmatrix} -2 & 6 \\ 1 & -4 \end{bmatrix}$ D) $\begin{bmatrix} 2 & 3 \\ \frac{1}{2} & 1 \end{bmatrix}$

3. Evaluate: $\begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} 5 & 3 \\ -1 & 3 \end{bmatrix}$

- A) $\begin{bmatrix} 1 & 15 \\ -13 & 3 \end{bmatrix}$ C) $\begin{bmatrix} -1 & 5 \\ -3 & 3 \end{bmatrix}$ E) NOTA
- B) $\begin{bmatrix} -1 & 29 \\ -7 & 5 \end{bmatrix}$ D) $\begin{bmatrix} 1 & -15 \\ -3 & 3 \end{bmatrix}$

4. Which of the following vectors is perpendicular to $\langle 5, 1, -2 \rangle$?

- A) $\langle 3, -3, 5 \rangle$ C) $\langle -5, -1, 2 \rangle$ E) NOTA
- B) $\langle -1, 3, 1 \rangle$ D) $\langle 2, 4, 7 \rangle$

5. Let $A = \begin{bmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}^{2026}$. What is the value of $A_{1,2}$?

- A) $-\frac{\sqrt{3}}{2}$ B) $-\frac{1}{2}$ C) $\frac{1}{2}$ D) 1 E) NOTA

6. What is the cosine of the smaller angle formed by the vectors $\langle 1, 7 \rangle$ and $\langle -4, 4 \rangle$?

- A) $\frac{3}{5}$ B) $\frac{4}{5}$ C) $\frac{\sqrt{5}}{5}$ D) $\frac{2\sqrt{5}}{5}$ E) NOTA

7. What is the projection of $\langle 1, 3, -2 \rangle$ onto $\langle -3, 5, 1 \rangle$?

- A) $\frac{1}{7} \langle 12, -20, -4 \rangle$ C) $\frac{1}{7} \langle -6, 10, 2 \rangle$ E) NOTA
- B) $\frac{1}{7} \langle -12, 20, 4 \rangle$ D) $\frac{1}{7} \langle 6, -10, -2 \rangle$

For questions 8 to 10: Refer to the matrix $M = \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ 2 & 1 & 2 \end{bmatrix}$

8. Let $M^2 = N$. What is the sum of the elements of N ?

- A) -16 B) -6 C) 14 D) 24 E) NOTA

9. What is the product of the eigenvalues of M ?

- A) -12 B) -2 C) 8 D) 15 E) NOTA

10. Let $M^{-1} = P$. What is the value of $P_{1,3}$?

- A) $\frac{1}{6}$ B) $\frac{1}{4}$ C) $\frac{1}{2}$ D) $-\frac{1}{3}$ E) NOTA

11. What is the volume of a tetrahedron with vertices at $(1, -2, -1)$, $(2, 1, 6)$, $(4, -4, -6)$, and $(-1, 7, 3)$?

- A) 24 B) 32 C) 48 D) 64 E) NOTA

12. What is the sum of all values of A that make the following system of equations have no solutions?

$$\begin{aligned} Ax + y - 2z &= 4 \\ 3x + Ay + 3z &= 6 \\ -8x - 2y + Az &= -8 \end{aligned}$$

- A) 12 B) 2 C) 0 D) -4 E) NOTA

13. Let A be a square matrix with $|A| = 6$ and $|3A| = 162$. What is the value of $|-2A^{-1}|$?

- A) $\frac{4}{3}$ B) $-\frac{1}{48}$ C) $-\frac{4}{3}$ D) $\frac{2}{3}$ E) NOTA

14. If $A = \begin{bmatrix} 6 & -2 \\ 9 & -3 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -1 & 5 \\ 2 & 5 & 4 \end{bmatrix}$, and $C = \begin{bmatrix} -4 & 4 \\ 1 & 3 \\ 9 & 5 \end{bmatrix}$, which of the following sets of operations is defined?

- A) $A^{-1} + BC$ C) $B^T AC$ E) NOTA
B) $AB + CA$ D) $B^T C^T - A$

15. What is the shortest distance between the planes $4x - y + 8z = 16$ and $4x - y + 8z = 7$?

- A) $\frac{5\sqrt{3}}{9}$ B) 1 C) $\frac{7\sqrt{3}}{9}$ D) 3 E) NOTA

16. If $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 4$, what is $\begin{vmatrix} 3a + 6g & 3d & 3g \\ c + 2i & f & i \\ b + 2h & e & h \end{vmatrix}$?

- A) 24 B) -12 C) 12 D) -24 E) NOTA

17. Which of the following matrices are diagonalizable?

$$L = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad M = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \quad N = \begin{bmatrix} 6 & -1 \\ 1 & 4 \end{bmatrix} \quad O = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

- A) M & N only B) L , M , & N only C) L & O only D) L & M only E) NOTA

18. Let $A = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$ be the matrix that represents the reflection across the line $y = 2x$ on $\mathbb{R}^2$ with respect to the standard basis. What is $\frac{50abc}{d}$?

- A) -32 B) -36 C) -24 D) -20 E) NOTA

19. Let A and B be $n \times n$ symmetric matrices. Which of the following is a necessary and sufficient condition for the product AB to be symmetric?

- A) A and B are both invertible
 B) $AB = BA$
 C) $A^2 = B^2$
 D) $A + B$ is symmetric
 E) NOTA

20. Let A be the 2×2 diagonal matrix with nonnegative entries so that $A = 2I + (2I + (2I + (\dots)^{-1})^{-1})^{-1}$, where I is the 2×2 identity matrix. What is the determinant of A ?

- A) $2 + \sqrt{2}$ B) $1 + \sqrt{2}$ C) $1 + 2\sqrt{2}$ D) $2 + 2\sqrt{2}$ E) NOTA

For questions 21 and 22: Refer to the matrix $A = \begin{bmatrix} 2 & 3 & -5 & 10 \\ -1 & -1 & 1 & -3 \\ 1 & 2 & -4 & 7 \end{bmatrix}$

21. What is the reduced row echelon form of A ?

A) $\begin{bmatrix} 1 & 0 & -1 & -3 \\ 0 & 1 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

C) $\begin{bmatrix} 1 & 2 & -4 & 7 \\ 0 & 1 & -2 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

E) NOTA

B) $\begin{bmatrix} 1 & 0 & 2 & -1 \\ 0 & 1 & -3 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

D) $\begin{bmatrix} 1 & 0 & 2 & 6 \\ 0 & 1 & 4 & -9 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

22. Which of the following sets form a basis for the null space of A ?

- A) $\left\{ \begin{bmatrix} -2 \\ 3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -4 \\ 0 \\ 1 \end{bmatrix} \right\}$ C) $\left\{ \begin{bmatrix} 2 \\ 4 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 6 \\ -9 \\ 0 \\ 1 \end{bmatrix} \right\}$ E) NOTA
- B) $\left\{ \begin{bmatrix} 2 \\ -3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 4 \\ 0 \\ 1 \end{bmatrix} \right\}$ D) $\left\{ \begin{bmatrix} -2 \\ -4 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -6 \\ 9 \\ 0 \\ 1 \end{bmatrix} \right\}$

23. What is the reflection of vector $\vec{v} = \langle 2, -1, 3 \rangle$ across the plane $x + 2y - 2z = 0$?

- A) $\langle \frac{10}{3}, \frac{5}{3}, \frac{1}{3} \rangle$ C) $\langle \frac{1}{3}, \frac{5}{3}, \frac{8}{3} \rangle$ E) NOTA
- B) $\langle \frac{8}{3}, \frac{1}{3}, \frac{5}{3} \rangle$ D) $\langle \frac{5}{3}, \frac{1}{3}, \frac{10}{3} \rangle$

24. Let A be a nilpotent matrix. Which of the following statements is always true?

- A) All eigenvalues of A are real and positive
 B) The trace of A is 0
 C) A is diagonalizable
 D) The determinant of A is 1
 E) NOTA

25. Evaluate the determinant:

$$\begin{vmatrix} 1 & 2 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 & -2 \\ 4 & 0 & 0 & 0 & 1 \end{vmatrix}$$

- A) 49 B) 26 C) 32 D) -47 E) NOTA

26. Let l be the line that passes through the point $(2, -1, 3)$ and is perpendicular to both of the vectors $\langle 1, 2, -1 \rangle$ and $\langle 4, -1, 2 \rangle$. At what point does line l intersect the plane $x + y + z = 6$?

- A) $(1, -2, -3)$ C) $\left(\frac{5}{3}, \frac{-7}{3}, 2\right)$ E) NOTA
- B) $(3, 3, -1)$ D) $\left(\frac{3}{2}, 0, \frac{9}{2}\right)$

27. A tech company tracks users across two competing services: App A and App B. Every year:

- 80% of App A users stay with App A, while 20% switch to App B.
- 70% of App B users stay with App B, while 30% switch to App A.

Initially, there are 2 App A users for every App B user. But after enough years pass, the proportions of users for each app will eventually become stable. Once this happens, what will be the proportion of app A users?

- A) 70% B) 40% C) 60% D) 50% E) NOTA

28. Let A be a 20×20 matrix such that $A_{i,j} = 6i - 2j$. What is the trace of A ?

- A) 1680 B) 840 C) 800 D) 1600 E) NOTA

29. Let $\vec{a}$, $\vec{b}$, and $\vec{c}$ be three-dimensional vectors such that $\vec{a} \times \vec{b} \neq \vec{0}$ and $\vec{a} \cdot \vec{c} = \vec{0}$. Which of the following statements is always true?

- A) $(\vec{a} \times \vec{b}) \cdot \vec{c} = \vec{0}$
B) $\vec{a} \cdot (\vec{b} \times \vec{c}) = \vec{a} \cdot (\vec{c} \times \vec{b})$
C) $(\vec{a} \times \vec{b}) \times \vec{a} = \vec{0}$
D) $(\vec{a} \times \vec{b}) \cdot (\vec{a} + \vec{b}) = \vec{0}$
E) NOTA

30. Solve for x in terms of θ :

$$\sin(\theta)x + \cos(\theta)y = \cot(\theta)$$

$$\cos(\theta)x - \sin(\theta)y = \tan(\theta)$$

- A) $\sin(2\theta)$ C) $\cos(2\theta)$ E) NOTA
B) $\sin(\theta)\tan(\theta)$ D) $\cos(\theta)\cot(\theta)$