

Alpha Bowl solutions
2026 Mu Alpha Theta National Convention

- | | | |
|-----------------------------|--------------------------------------|--|
| 0. A. $\frac{1}{128}$ | 5. A. $-\frac{1}{4}$ | 10. A. 672 |
| 0. B. $\frac{3\sqrt{3}}{4}$ | 5. B. $-\frac{5}{4}$ | 10. B. -15 |
| 0. C. -15 | 5. C. 0 | 10. C. 24 |
| 0. D. 0 | 5. D. $-\frac{3}{2}$ | 10. D. -21 |
| 1. A. 14 | | 11. A. -3 |
| 1. B. $\frac{\pi}{6}$ | 6. A. $\frac{18\sqrt{5}}{5}$ | 11. B. $-6\sqrt{5}$ |
| 1. C. 0 | 6. B. 4 | 11. C. $\frac{69}{112}$ |
| 1. D. 4 | 6. C. $\left(\frac{18}{5}, 0\right)$ | 11. D. 3 |
| 2. A. $\frac{3}{7}$ | 6. D. 18 | 12. A. $\frac{11}{6}\pi$ |
| 2. B. $\frac{4}{5}$ | 7. A. 6 | 12. B. 3π |
| 2. C. $\frac{1}{10}$ | 7. B. 1014 | 12. C. 8π |
| 2. D. $\frac{4}{9}$ | 7. C. 6 | 12. D. $-\frac{3}{2}\pi, -\frac{\pi}{2}$ |
| | 7. D. 1 | 13. A. -3 |
| 3. A. $\sin x$ | 8. A. 282 | 13. B. $\frac{3\sqrt{21}}{7}$ |
| 3. B. 10,132 | 8. B. -1 | 13. C. $\frac{4}{5}\pi$ |
| 3. C. $8n-2$ | 8. C. 7 | 13. D. -49 |
| 3. D. $800\sqrt{3}$ | 8. D. 4 | 14. A. $x+3$ |
| 4. A. 12 | 9. A. $-\frac{132}{157}$ | 14. B. 9 |
| 4. B. 81 | 9. B. $\frac{\sqrt{3}}{2}$ | 14. C. 8,000 |
| 4. C. $\frac{7500}{\pi}$ | 9. C. 4π | 14. D. $\sqrt{37}$ |
| 4. D. $\sqrt{3}$ | 9. D. π^2 | |

0. A. $-2x = 4 \rightarrow x = -2$

$$\frac{2^{-2}}{32} = \frac{1}{4(32)} = \frac{1}{128}$$

0. B. The cube roots of unity are $(1, 0)$, $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$, and $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$.

$$\text{Area} = \frac{1}{2} \begin{vmatrix} 1 & 0 & 1 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 1 \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 1 \end{vmatrix} = \frac{3\sqrt{3}}{4}$$

0. C. $7+9-3 \times 20 \div 5-19=7+9-12-19=-15$

0. D. A tetrahedron has no space diagonals, so the product is 0.

1. $f(x) = (\log_{\sqrt{3}} 2187) \sin \left[4(\log 0.001)x - \pi e^{\ln 400} \right] + \log 10,000$

$$f(x) = 14 \sin(-12x - 400\pi) + 4$$

$$f(x) = -14 \sin 12 \left(x + \frac{100}{3} \pi \right) + 4$$

A. 14

B. $\frac{2\pi}{12} = \frac{\pi}{6}$

C. $\frac{100\pi}{3}$ is a multiple of the period $\frac{\pi}{6}$, so this equation can be written with no phase shift—0.

D. 4

2. A. Number of all 7-digit numbers possible: $\frac{7!}{2!3!2!} = 210$

Number of all 7-digit even numbers possible: $\frac{6!}{2!2!2!} = 90$ (The last digit must be 4 so we have six

positions to arrange.) The probability is therefore $\frac{90}{210} = \frac{3}{7}$.

B. Using complementary counting (getting 4, 5, or 6): $1 - \left(\frac{3}{6}\right)\left(\frac{2}{5}\right) = \frac{4}{5}$.

C. Alternate vertices produce equilateral triangles, so only two distinct equilateral triangles are

possible. There are ${}_6C_3 = 20$ ways to choose the three vertices. The probability is therefore $\frac{2}{20} = \frac{1}{10}$.

D. $0.2x \leq 1 \rightarrow x \leq 5$. $P = \frac{5-1}{10-1} = \frac{4}{9}$.

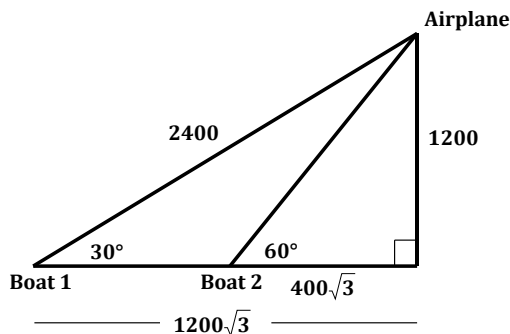
3. A. $\frac{p^2-1}{p^2+1} = \frac{\sec^2 x + 2 \sec x \tan x + \tan^2 x - 1}{\sec^2 x + 2 \sec x \tan x + \tan^2 x + 1} \rightarrow \frac{(\sec^2 x - 1) + \tan^2 x + 2 \sec x \tan x}{\sec^2 x + 2 \sec x \tan x + \sec^2 x} \rightarrow$
 $\frac{\tan^2 x + \tan^2 x + 2 \sec x \tan x}{2 \sec^2 x + 2 \sec x \tan x} \rightarrow \frac{2 \tan^2 x + 2 \sec x \tan x}{2 \sec^2 x + 2 \sec x \tan x} \rightarrow \frac{2 \tan x (\tan x + \sec x)}{2 \sec x (\sec x + \tan x)} \rightarrow \frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x}} = \sin x$

B. $a_{10} = 52$ $a_{17} = 52 + 7d$ and $a_{17} = 20 + (52 + 3d) = 72 + 3d$ $52 + 7d = 72 + 3d \rightarrow d = 5$, $a_1 = 7$
 $a_{2026} = 7 + 2025(5) = 7 + 10125 = 10132$

C. $S_n = \frac{n(a_1 + a_n)}{2} = 4n^2 + 2n$ $a_n = S_n - S_{n-1}$

$$S_{n-1} = 4(n-1)^2 + 2(n-1) = 4n^2 - 6n + 2 \quad a_n = (4n^2 + 2n) - (4n^2 - 6n + 2) = 8n - 2$$

D.



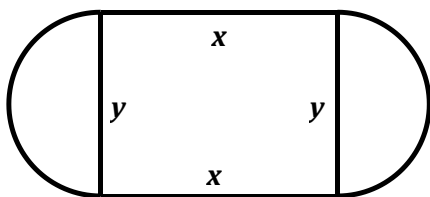
The distance between the boats is $800\sqrt{3}$.

4. A. $\log_{\frac{1}{2}} |\sin x| = 2 - \log_{\frac{1}{2}} |\cos x| \rightarrow \log_{\frac{1}{2}} |\sin x \cos x| = 2 \rightarrow |\sin x \cos x| = \frac{1}{4} \rightarrow 2 \sin x \cos x = \sin 2x = \pm \frac{1}{2}$

$$x = \frac{\pi}{12}, \frac{5}{12}\pi, \frac{7}{12}\pi, \frac{11}{12}\pi, \frac{13}{12}\pi, \frac{17}{12}\pi, \frac{19}{12}\pi, \frac{23}{12}\pi, \frac{25}{12}\pi, \frac{29}{12}\pi, \frac{31}{12}\pi, \frac{35}{12}\pi \quad 12 \text{ values}$$

B. $(0.05)^{\log_{\sqrt{20}}(0.1+0.01+0.001+\dots)} \quad 0.1+0.01+0.001+\dots = \frac{\frac{1}{10}}{1-\frac{1}{10}} = \frac{1}{9} \quad \left(\frac{1}{20}\right)^{2\log_{20}\left(\frac{1}{9}\right)} \rightarrow \left(\frac{1}{20}\right)^{-2\log_{20}9} \rightarrow 20^{\log_{20}9^2} = 81$

C.



$$2x + \pi y = 200 \quad \text{Area} = xy = x \left(\frac{200 - 2x}{\pi} \right)$$

The roots of the area function are 0 and 100, so the x-value that maximizes the expression is $x = 50$.

$$x = 50, y = \frac{100}{\pi} \quad \text{Total area} = \frac{5000}{\pi} + \pi \left(\frac{50}{\pi} \right)^2 = \frac{7500}{\pi}$$

D. Minimize the quadratic and you have minimized the square root. $x = -\frac{b}{2a} = 2 \rightarrow y = \sqrt{4 - 8 + 7} = \sqrt{3}$

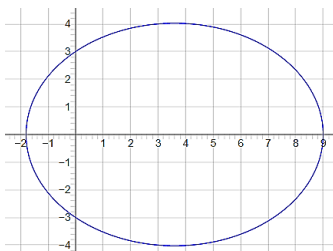
5. A. $\lim_{x \rightarrow 0} \frac{(2+x)^{-1} - 2^{-1}}{x} \rightarrow \lim_{x \rightarrow 0} \frac{\frac{1}{2+x} - \frac{1}{2}}{x} \rightarrow \lim_{x \rightarrow 0} \frac{2 - 2 - h}{2h(2+h)} \rightarrow \lim_{x \rightarrow 0} \frac{-1}{2(2+h)} = -\frac{1}{4}$

B. $\lim_{x \rightarrow -\infty} \frac{5e^{-5x} - 3e^{7x}}{6e^{7x} - 4e^{-5x}} \rightarrow \lim_{x \rightarrow -\infty} \frac{e^{-5x}(5 - 3e^{12x})}{e^{-5x}(6e^{12x} - 4)} = \frac{5 - 3(0)}{6(0) - 4} = -\frac{5}{4}$

C. $\lim_{x \rightarrow 7^-} e^{\frac{4}{x-7}}$ Looking at the exponent itself, that rational function has a vertical asymptote at $x = 7$, and the y-value approaches $-\infty$ from the left. So, $e^{-\infty} = 0$.

D. $\lim_{x \rightarrow 2} \frac{\sqrt{6-x}-2}{3-\sqrt{11-x}} \cdot \frac{\sqrt{6-x}+2}{\sqrt{6-x}+2} \cdot \frac{3+\sqrt{11-x}}{3+\sqrt{11-x}} \rightarrow \lim_{x \rightarrow 2} \frac{(2-x)(3+\sqrt{11-x})}{(-2+x)(\sqrt{6-x}+2)} = -\frac{6}{4} = -\frac{3}{2}$

6.



$$r = \frac{9}{3-2\cos\theta} = \frac{3}{1-\frac{2}{3}\cos\theta} \quad \text{The intercepts give the major axis}$$

endpoints and the endpoints of the latus rectum.

$$\text{Major axis endpoints: } \left(-\frac{9}{5}, 0\right), (9, 0) \quad \text{Center: } \left(\frac{18}{5}, 0\right)$$

$$\text{Endpoints of latus rectum: } (0, \pm 3) \quad \text{Foci: } (0, 0), \left(\frac{36}{5}\right)$$

$$\text{Eccentricity: } \frac{2}{3} \quad b^2 = a^2 - c^2 \rightarrow \left(\frac{27}{5}\right)^2 - \left(\frac{18}{5}\right)^2 = \frac{405}{25} = \frac{81}{5} \quad b = \frac{9\sqrt{5}}{5}$$

$$\text{A. Minor axis length} = 2b = \frac{18\sqrt{5}}{5}$$

$$\text{B. Eccentricity} = \frac{2}{3}, \text{ Focal width} = 6 \quad \text{Product} = 4$$

$$\text{C. Center: } \left(\frac{18}{5}, 0\right)$$

$$\text{D. Major axis length} = 9 + \frac{9}{5} = \frac{54}{5}, \text{ Distance between foci} = \frac{36}{5} \quad \text{Sum} = 18$$

$$7. \text{ A. } \frac{x^2 + 4x + 3}{x^2 - 2x - 8} \leq 0 \rightarrow \frac{(x+3)(x+1)}{(x-4)(x+2)} \leq 0 \quad \text{The solution set is } [-3, -2) \cup [-1, 4), \text{ so the integer solutions are } -3, -1, 0, 1, 2, 3 \rightarrow 6 \text{ values}$$

$$\text{B. } \log\left(\frac{2x-2026}{x+1}\right) \leq 0 \rightarrow \frac{2x-2026}{x+1} \leq 1 \rightarrow \frac{x-2027}{x+1} \leq 0 \quad \text{The domain for the log expression is } (-\infty, -1) \cup (1013, \infty), \text{ and the solution set for the rational expression is } (-1, 2027]. \text{ The intersection is } (1013, 2027], \text{ so the integers are 1014 through 2027, which is 1014 integers.}$$

$$\text{C. } n^{300} > 13^{200} \rightarrow \sqrt[100]{n^{300}} > \sqrt[100]{13^{200}} \rightarrow n^3 > 13^2 (=169) \quad \text{The smallest value of } n \text{ is 6.}$$

$$\text{D. } \cot^2 x - \csc x + 1 > 0 \rightarrow \csc^2 x - \csc x > 0 \rightarrow \csc x(\csc x - 1) > 0 \quad \text{Cosecant and cotangent are undefined at}$$

$$0, \pi, \text{ and } 2\pi. \csc x > 0 \text{ over } (0, \pi) \text{ and } \csc x < 0 \text{ over } (\pi, 2\pi). \csc x - 1 > 0 \text{ over } \left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right) \text{ and}$$

$$\csc x - 1 < 0 \text{ over } (\pi, 2\pi). \text{ The solution set is } \left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right) \cup (\pi, 2\pi). \frac{\frac{\pi}{2} + \frac{\pi}{2} + \pi}{\pi} + \frac{0}{\pi} = \frac{2\pi}{2\pi} = 1.$$

$$8. \text{ A. To be odd, the sum must be 2, 3, 5, or 7. There are nine entries in the matrix.}$$

$${}_9C_2 + {}_9C_3 + {}_9C_5 + {}_9C_7 = 282$$

$$\text{B. } \begin{bmatrix} 1 & -2 & x \\ x & 2 & -1 \end{bmatrix} \begin{bmatrix} 2 & x \\ -1 & 2 \\ 4 & -5 \end{bmatrix} = \begin{bmatrix} 4+4x & -4x-4 \\ 2x-6 & x^2+9 \end{bmatrix} \quad \det \begin{bmatrix} 4+4x & -4x-4 \\ 2x-6 & x^2+9 \end{bmatrix} = 4x^3 + 12x^2 + 20x + 12$$

$$4x^3 + 12x^2 + 20x + 12 = 0 \rightarrow x^3 + 3x^2 + 5x + 3 = 0$$

$$\text{Using sum of squares formula: } (-3)^2 - 2(5) = -1$$

$$\text{C. } \begin{vmatrix} x & 8 \\ 3 & 1 \end{vmatrix} = \begin{vmatrix} 4 & x \\ 3 & 1 \end{vmatrix} \rightarrow x - 24 = 4 - 3x \rightarrow x = 7$$

D. We need $\begin{bmatrix} 3 & x \\ -2 & -3 \end{bmatrix} \begin{bmatrix} 3 & x \\ -2 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 9-2x & 3x-3x \\ -6+6 & -2x+9 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \rightarrow 9-2x=1 \rightarrow x=4$

9. A. $\sin\left[2\operatorname{Arctan}\left(-\frac{11}{6}\right)\right] \sin 2x = 2\sin x \cos x \quad \sin x = -\frac{11}{\sqrt{157}}, \cos x = \frac{6}{\sqrt{157}}$

$$2\left(-\frac{11}{\sqrt{157}}\right)\left(\frac{6}{\sqrt{157}}\right) = -\frac{132}{157}$$

B. $\sin^{-1} x = \frac{\pi}{6} + \cos^{-1} x \rightarrow x = \sin\left(\frac{\pi}{6} + \cos^{-1} x\right) = \sin\left(\frac{\pi}{6}\right)\cos(\cos^{-1} x) + \cos\left(\frac{\pi}{6}\right)\sin(\cos^{-1} x) =$

$$x = \frac{1}{2}x + \left(\frac{\sqrt{3}}{2}\right)\left(\sqrt{1-x^2}\right) \rightarrow \frac{1}{2}x = \frac{\sqrt{3-3x^2}}{2} \rightarrow x^2 = 3-3x^2 \rightarrow x = \pm \frac{\sqrt{3}}{2} \quad \text{Now, check these!}$$

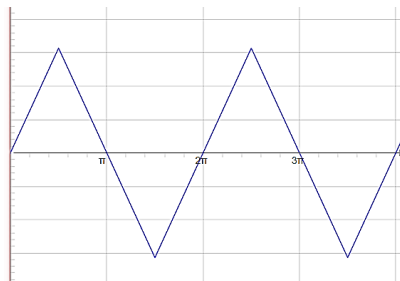
$$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6} + \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) \rightarrow \frac{\pi}{3} = \frac{\pi}{6} + \frac{\pi}{6} = \frac{\pi}{3} \quad \checkmark$$

$$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6} + \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) \rightarrow -\frac{\pi}{3} = \frac{\pi}{6} + \frac{\pi}{6} = \frac{\pi}{3} \quad \times$$

$$x = \frac{\sqrt{3}}{2}$$

C. The range for Arctan is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. As $x \rightarrow \infty$, the value of the function approaches $8\left(\frac{\pi}{2}\right) = 4\pi$.

D. The graph produces four congruent triangles with base width π and height $\frac{\pi}{2}$. $4\left[\frac{1}{2}(\pi)\left(\frac{\pi}{2}\right)\right] = \pi^2$.



10. A. $\left(8x^3 - 12x + \frac{6}{x} - \frac{1}{x^3}\right)^3 = \left[\left(2x - \frac{1}{x}\right)^3\right]^3 = \left(2x - \frac{1}{x}\right)^9 \quad \binom{9}{6}(2)^3(-1)^6 = (84)(8) = 672$

B. $\left(\frac{\sqrt{3}}{2} - \sqrt[3]{2}x\right)^5$ We are looking for the 4th term: $\binom{5}{3}\left(\frac{\sqrt{3}}{2}\right)^2\left(-\sqrt[3]{2}x\right)^3 = (10)\left(\frac{3}{4}\right)(-2)(x^3) = -15x^3 \rightarrow -15$

C. $(2)(3)(4) = 24$

D. $\binom{7}{r}(x)^{7-r}\left(-x^{-\frac{1}{2}}\right)^r \rightarrow \binom{7}{r}(-1)^r(x)^{7-\frac{3}{2}r} \rightarrow 7 - \frac{3}{2}r = -\frac{1}{2} \rightarrow \frac{3}{2}r = \frac{15}{2} \rightarrow r = 5. \quad \binom{7}{5}(-1)^5 = -21$

11. A. $g_1 = 2, g_2 = \frac{3}{-1} = -3, g_3 = \frac{-2}{4} = -\frac{1}{2}, g_4 = \frac{\frac{1}{2}}{\frac{3}{2}} = \frac{1}{3}, g_5 = \frac{\frac{4}{3}}{2} = 2$. The pattern repeats in groups of 4.

$$2026 \equiv 2 \pmod{4}, \text{ so } g_{2026} = g_2 = -3$$

$$B. \frac{(5\text{cis}153^\circ)(2\text{cis}49^\circ)(3\text{cis}238^\circ)}{\sqrt{5}\text{cis}260^\circ} = \frac{30\text{cis}440^\circ}{\sqrt{5}\text{cis}260^\circ} = 6\sqrt{5}\text{cis}180^\circ = -6\sqrt{5}$$

$$C. \sum_{n=2}^7 \frac{1}{n^2-1} = \sum_{n=2}^7 \frac{1}{(n+1)(n-1)} \quad \frac{1}{(n+1)(n-1)} = \frac{A}{n+1} + \frac{B}{n-1} \rightarrow 1 = A(n-1) + B(n+1)$$

$$\text{If } n=-1, A=-\frac{1}{2}. \text{ If } n=1, B=\frac{1}{2}.$$

$$\frac{1}{2} \sum_{n=2}^7 \left(\frac{1}{n-1} - \frac{1}{n+1} \right) = \frac{1}{2} \left[\left(1 - \frac{1}{3} \right) + \left(\frac{1}{2} - \frac{1}{4} \right) + \left(\frac{1}{3} - \frac{1}{5} \right) + \left(\frac{1}{4} - \frac{1}{7} \right) + \left(\frac{1}{5} - \frac{1}{7} \right) + \left(\frac{1}{6} - \frac{1}{8} \right) \right] =$$

$$\frac{1}{2} \left(1 + \frac{1}{2} - \frac{1}{7} - \frac{1}{8} \right) = \frac{69}{112}$$

D. Fermat's Little Theorem states that if a and p are coprime, then $a^{p-1} \equiv 1 \pmod{p}$. 5 and 2027 are coprime. This means that $5^{11-1} = 5^{10} \equiv 1 \pmod{11}$. Now we will start building up to 5^{2027} .

$$\begin{aligned} 5^{10} &\equiv 1 \pmod{11} & (5^2)^3 &= 5^6 \equiv 27 \pmod{11} = 5 \pmod{11} \\ (5^{10})^{202} &= 5^{2020} \equiv 1 \pmod{11} & 5^7 &= 5(5) \pmod{11} = 25 \pmod{11} = 3 \pmod{11} \\ 5^2 &\equiv 3 \pmod{11} & (5^{10})^{202} &= 5^{2020} \equiv 1 \pmod{11} \\ (5^2)^2 &= 5^4 \equiv 3^2 \pmod{11} = 9 \pmod{11} & 5^2 &\equiv 3 \pmod{11} \\ 5^{2027} &\equiv (1)(3) \pmod{11} = 3 \pmod{11} \end{aligned}$$

The remainder is 3.

$$12. A. \log_2(-3\sin x) = 2\log_2(\cos x) + 1 \rightarrow \log_2\left(\frac{-3\sin x}{\cos^2 x}\right) = 1 \rightarrow -3\sin x = 2\cos^2 x \rightarrow -3\sin x = 2(1 - \sin^2 x)$$

$$2\sin^2 x - 3\sin x - 2 = 0 \rightarrow (2\sin x + 1)(\sin x - 2) = 0 \rightarrow \sin x = -\frac{1}{2} \rightarrow x = \frac{7}{6}\pi, \frac{11}{6}\pi \text{ However, } x = \frac{7}{6}\pi \text{ fails}$$

$$\text{for } \log_2(\cos x), \text{ so the only solution is } x = \frac{11}{6}\pi$$

$$B. \tan\left(\frac{5}{4}\pi + x\right) + \tan\left(\frac{5}{4}\pi - x\right) = 4 \rightarrow \frac{\tan\frac{5}{4}\pi + \tan x}{1 - \tan\frac{5}{4}\pi \tan x} + \frac{\tan\frac{5}{4}\pi - \tan x}{1 + \tan\frac{5}{4}\pi \tan x} = 4 \rightarrow \frac{1 + \tan x}{1 - \tan x} + \frac{1 - \tan x}{1 + \tan x} = 4$$

$$(1 + \tan x)^2 + (1 - \tan x)^2 = 4(1 + \tan x)(1 - \tan x) \rightarrow 1 + 2\tan x + \tan^2 x + 1 - 2\tan x + \tan^2 x = 4 - 4\tan^2 x$$

$$6\tan^2 x = 2 \rightarrow \tan x = \pm \frac{1}{\sqrt{3}} \text{ For the given interval, we have } x = \frac{7}{6}\pi \text{ and } \frac{11}{6}\pi, \text{ whose sum is } 3\pi.$$

$$C. \left| \sin \frac{1}{2}x \right| = |\cos x| \rightarrow \sqrt{\frac{1 - \cos x}{2}} = |\cos x| \rightarrow \frac{1 - \cos x}{2} = \cos^2 x \rightarrow 2\cos^2 x + \cos x - 1 = 0 \rightarrow$$

$$(2\cos x - 1)(\cos x + 1) = 0 \rightarrow x = \frac{5}{3}\pi, \frac{7}{3}\pi, \pi, 3\pi, \text{ which sum to } 8\pi.$$

$$D. \sin 2x = 3\cos x \rightarrow 2\sin x \cos x - 3\cos x = 0 \rightarrow \cos x(2\sin x - 3) = 0 \text{ In the given interval, } x = -\frac{3}{2}\pi, -\frac{\pi}{2}$$

$$13. A. f(x) = (x-1)(x+1)(x-3) \rightarrow f(2) = (1)(3)(-1) = -3$$

B. $\overrightarrow{MN} = i + 0j + 2k$, $\overrightarrow{PQ} = i - 2j + 4k$. $\overrightarrow{MN} \cdot \overrightarrow{PQ} = 1 + 0 + 8 = 9$. $|\overrightarrow{PQ}| = \sqrt{21}$ $\left\| \text{proj}_{\overrightarrow{PQ}} \overrightarrow{MN} \right\| = \frac{9}{\sqrt{21}} = \frac{3\sqrt{21}}{7}$

C. The hour hand turns 30° in 60 minutes. $4(30^\circ) + 48\left(\frac{30^\circ}{60^\circ}\right) = 144^\circ \rightarrow \frac{144\pi}{180} = \frac{4}{5}\pi$

D. If $x > 0$: $3x^2 - 19x - 14 = 0 \rightarrow (3x + 2)(x - 7) = 0 \rightarrow x = -\frac{2}{3}, x = 7$

If $x < 0$: $3x^2 + 19x - 14 = 0 \rightarrow (3x - 2)(x + 7) = 0 \rightarrow x = \frac{2}{3}, -7$

Of these four values, the only true solutions are $x = \pm 7$, whose product is -49 .

14. A. The remainder is $x + 3$.

B. $3x(-1 + 5x) = ax^2 + bx + c$

$15x^2 - 3x + 0 = ax^2 + bx + c \rightarrow a = 15, b = -3, c = 0$

$15 - 6 + 0 = 9$

C. $P(t) = \frac{40,000}{5 + 8e^{-0.03t}} = \frac{8,000}{1 + \frac{8}{5}e^{-0.03t}}$ The carrying capacity is 8,000.

D. $\sqrt{16 + 9 - 2(4)(3)\cos 120^\circ} = \sqrt{25 - 24\left(-\frac{1}{2}\right)} = \sqrt{37}$