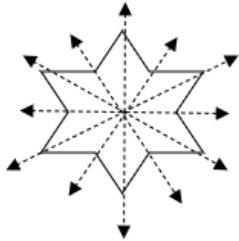
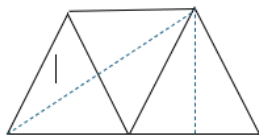
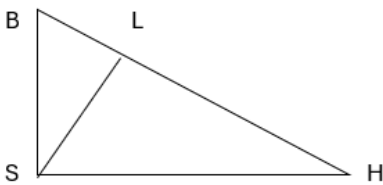
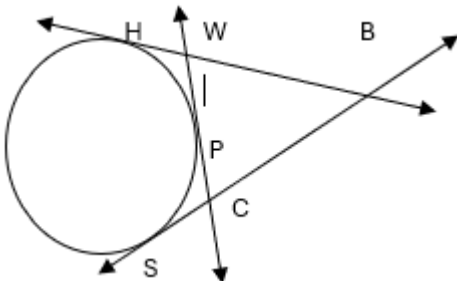
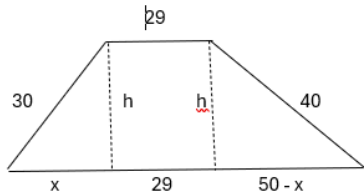
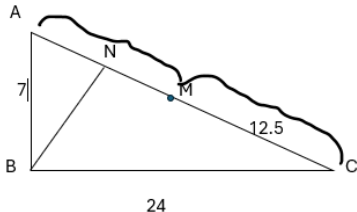


Question	Answer	Solution
1	22	Diagonals of rhombus are perpendicular bisectors of one another and form right triangles. Half of the long diagonal is 60. The side of the rhombus is $244/4 = 61$. The other leg of the right triangle formed is 11, using the Pythagorean Theorem. The short diagonal is $2 \times 11 = 22$.
2	$\frac{64}{49}$	Ratio of volumes of similar solids are in a ratio equal to their linear segments cubed so the ratios of the edges is $8 : 7$. Areas are in a ratio equal to the linear segments squared so the SA ratio is $64 : 49$.
3	92.5°	The sum of the angles of a convex pentagon is 540. Adding the 5 angle measures gives $20x + 415 = 540$ which has solution $x = 6.25$. Substituting makes the smallest angle $12x = 87.5$. The exterior angles are supplementary to the interior angles so the largest exterior angle is 92.5° .
4	6	 Lines of reflective symmetry divide the figure into 2 equal symmetric parts. Six lines may be drawn in this figure as lines of reflective symmetry.
5	36	There are three places where the fourth vertex can be and all will give the same area. The easiest point to find is 9 units to the left of (5, 6) which is (-4, 6). The length of the base of the parallelogram is 9 and the height (since the sides are horizontal lines) is 4. Area = $bh = 9(4) = 36$.
6	122.5	Each arc of the clock face between two of the numbers is 30° . At 6:55 pm, the minute hand will be on 11 and the hour hand will be between 6 and 7. Since 6:00, the minute hand will have covered $11/12$ of the circumference of the clock face, so the hour hand will have covered $11/12$ of the angle between 6 and 7, or $11/12$ of 30° . $(11/12)(30) = 55/2 = 27.5^\circ$. $150 - 27.5 = 122.5$.
7	8	Diagonals = $\frac{n(n-3)}{2}$ so $\frac{n(n-3)}{2} < 44$ $n^2 - 3n - 88 < 0$ and $(n - 11)(n + 8) < 0$ which tells us $-8 < n < 11$ A polygon must have at least 3 sides so there are 8 possibilities for the polygon.
8	$\overline{AD}$	In $\triangle ABC$, $m\angle ACB = 60^\circ$ and the longest side of the triangle is $\overline{AC}$. In $\triangle ADC$, $m\angle DCA = 65^\circ$ and the longest side of that triangle is $\overline{AD}$. Both of these "longest" sides appear in $\triangle ADC$ where $\overline{AD}$ is longer.
9	$2\sqrt{3}$	 Drop an altitude from the end of the upper base to the lower base and draw in the diagonal of the trapezoid. This forms a right triangle with a base of 3 since the equilateral triangle altitude bisects the base. The

		height of the equilateral triangle is $\sqrt{3}$. Using the Pythagorean Theorem to calculate the diagonal, $3^2 + (\sqrt{3})^2 = d^2$, gives $d = 2\sqrt{3}$.
10	$\frac{25}{144}$	 Sides of 5, 12, and 13 make this a right triangle. Use similar triangle proportions to solve for BL and LH. $\frac{BL}{5} = \frac{5}{13} \text{ and } \frac{LH}{12} = \frac{12}{13}$ $BL = \frac{25}{13} \text{ and } LH = \frac{144}{13}$ $\frac{BL}{LH} = \frac{\frac{25}{13}}{\frac{144}{13}} = \frac{25}{144}$
11	31	Use the theorem that if chords intersect within a circle, the product of the two segments of one chord equals the product of the two segments of the other chord. $6(x + 2) = 4(2x - 5)$ which gives $x = 16$. $BD = DE + EB = 27 + 4 = 31$
12	$45\sqrt{2}$	Use $A = \frac{1}{2}ab \sin C = \frac{1}{2}(10)(18)\left(\frac{\sqrt{2}}{2}\right) = 45\sqrt{2}$
13	10	Using Pick's formula: $i = A + 1 - \frac{b}{2}$ where i is the number of interior lattice points, A is the area of the polygon, and b is the number of lattice points on the polygon. The area of the triangle can be found, using a matrix, to be 11. The number of lattice points on the polygon is the 3 vertices plus another point on $\overline{NP}$ which makes $b = 4$. Substituting gives: $i = 11 + 1 - \frac{4}{2} = 10$
14	130	Given 2 sides of a triangle, the third side must be more than their difference and less than their sum. The third side must then be between 3 and 17. Summing the integers from 4 to 16 gives $S_{13} = \frac{13}{2}(4 + 16) = 130$.
15	2	Using the bisected perpendicular diagonals of the rhombus and the Pythagorean Theorem gives us a side of 10 and a perimeter of 40 for the rhombus. For the rectangle, then, $2L + 2W = 40$ and $L + W = 20$. Using the diagonal of $\sqrt{202}$ and the Pythagorean Theorem, $L^2 + W^2 = 202$. Let $W = 20 - L$. This gives $L^2 + (20 - L)^2 = 202$ which simplifies to $2L^2 - 40L + 198 = 0$ or $L^2 - 20L + 99 = 0$. Factoring $(L - 9)(L - 11) = 0$ so $L = 9$ or $L = 11$. The dimensions of the rectangle are 9×11 and the difference is $11 - 9 = 2$.
16	$\frac{16}{49}$	Solving for the areas of the 3 circles gives $\frac{9\pi}{4}$, $\frac{25\pi}{4}$, and $\frac{49\pi}{4}$. The shaded ring has area equal to $\frac{16\pi}{4}$. The probability is the ratio of the shaded ring area to the total dartboard area: $\frac{16\pi}{4} : \frac{49\pi}{4}$ which simplifies to $\frac{16}{49}$.
17	$\frac{\sqrt{6\pi}}{6}$	Setting the surface areas equal: $6e^2 = 4\pi r^2$ or since $r = d/2$, $6e^2 = \pi d^2$ $\frac{e^2}{d^2} = \frac{\pi}{6}$ so $\frac{e}{d} = \sqrt{\frac{\pi}{6}} = \frac{\sqrt{6\pi}}{6}$

18	37.5	AC = CB = 30, AD = DC = 15, DB = 45 so DE=EB = 22.5. That makes CE = 30 – 22.5 = 7.5 and AE = AC + CE = 30 + 7.5 = 37.5
19	$\frac{45}{\pi}$	Using the arc length formula: $s = \frac{\theta}{360} \cdot 2\pi r$, $18 = \frac{72}{360} \cdot 2\pi r$ and solving for r gives $r = \frac{45}{\pi}$
20	$36 - 9\sqrt{3}$	The square has side 6 so its area is 36. The equilateral triangle with side 6 has area = $\frac{6^2\sqrt{3}}{4} = 9\sqrt{3}$. So the described region has area $36 - 9\sqrt{3}$.
21	34	 Tangent segments from an external point to a circle are equal, so WH = WP and CP = CS. Since BH = 17, we know that WB + WP = 17. Since BS = 17, we know that BC + CP = 17. Thus $\triangle BWC$ has a perimeter of 34.
22	1296	 Drawing in the altitudes of the trapezoid gives two right triangles which can be solved to find h. $h^2 + (50-x)^2 = 1600$ $h^2 + x^2 = 900$ Subtracting gives $2500 - 100x = 700$ so $x = 18$. Plugging into the second equation gives $h = 24$. Trapezoid area = $(1/2)(24)(29 + 79) = 1296$.
23	5.75 or $23/4$	The line containing the intersection points of the circles can be obtained by subtracting the circle equations. $x^2 + y^2 - 4x - 4y = 8 \quad \text{so} \quad -8x + 8y = 23 \quad \text{which has intercepts } (0, 23/8) \text{ and } (-23/8, 0)$ $x^2 + y^2 + 4x - 12y = -15$ The sum of the absolute values is $23/4$.
24	20°	$3x + 4x + 5x + 6x = 360$ so $x = 20$ The secant angle $\angle CED = (1/2)(5x - 3x) = x$ Thus $\angle CED = 20$
25	477	 Sides of 7, 24, and 25 form a right triangle and the altitude to the hypotenuse makes 3 similar right triangles. Let AN = x so $\frac{x}{7} = \frac{7}{25}$ and $AN = \frac{49}{25}$. $MA = 12.5$ so $MN = MN - AN = \frac{25}{2} - \frac{49}{25} = \frac{625-98}{50}$

		$\frac{p}{q} = \frac{527}{50}$ and $p - q = 477$.
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