

MAΘ National Convention 2026

Probability and Statistics – Hustle – Answers and Solutions

Answers:

1. 4
2. YES
3. 11
4. 0.3
5. $\frac{\sqrt{3}}{4}$
6. $-\frac{15}{16}$
7. $\frac{\sqrt{2}}{25}$
8. 48
9. 0
10. 4
11. 210
12. 12
13. 3
14. $\frac{72}{125}$
15. $\frac{2}{5}$
16. $\frac{1}{4}$
17. $\frac{39}{4}$ or 9.75
18. 0.08 or $\frac{2}{25}$
19. 36
20. $\frac{\sqrt{7}}{40}$
21. $20\sqrt{2}$
22. NO
23. $\frac{75}{16}$
24. 1.45
25. $\frac{9}{35}$

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Solutions:

$$1. \bar{X} = \frac{0+3+5+4+8+3+8+7+8+x}{10} = \frac{46+x}{10} = 5 \rightarrow x = 50 - 46 = \boxed{4}$$

2. The given Normal probability plot shows a fairly strong linear pattern between the observed data values plotted along the x-axis and the expected Z-scores associated with an SRS of 10 independent data values drawn from an approximately Normally distributed population. Thus, YES, it is reasonable to assume that the data was drawn from an approximately Normally distributed population.

$$3. \sigma = \frac{x_1 - x_2}{z_1 - z_2} = \frac{13-6}{2-(-1.5)} = \frac{7}{3.5} = 2 \text{ and } \mu = 13 - 2(2) = 9 \text{ or } \mu = 6 + 1.5(2) = 9 \rightarrow \mu + \sigma = 9 + 2 = \boxed{11}.$$

4. Since $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, $P(A \text{ and } B)$ is a minimum when $P(A \text{ or } B)$ is a maximum, which would, of course, be 1. Therefore, $1 = 0.7 + 0.6 - P(A \text{ and } B)$, implying that $P(A \text{ and } B) = \boxed{0.3}$.

$$5. \text{ The slope of the population LSRL is given by } \beta = \rho \frac{\sigma_Y}{\sigma_X} = \frac{\sqrt{3}}{4} \left(\frac{\sigma}{\sigma} \right) = \boxed{\frac{\sqrt{3}}{4}}.$$

$$6. a = \bar{Y} - b\bar{X} \rightarrow b = \frac{\bar{Y}-a}{\bar{X}} = \frac{15-30}{20} = -\frac{3}{4} \text{ and } b = r \frac{S_y}{S_x} \rightarrow r = b \frac{S_x}{S_y} = -\frac{3}{4} \left(\frac{5}{4} \right) = \boxed{-\frac{15}{16}}$$

$$7. SE(b) = \frac{S_e}{S_x \sqrt{n-1}} = \frac{\sqrt{2}}{5\sqrt{26-1}} = \boxed{\frac{\sqrt{2}}{25}}$$

$$8. S_e = \sqrt{\frac{\sum(y-\hat{y})^2}{n-2}} \rightarrow \sum(y-\hat{y})^2 = S_e^2(n-2) = (\sqrt{2})^2(26-2) = \boxed{48}$$

9. Since random variables X and Y are truly independent, then $E(b) = \boxed{0}$.

10. $P(A | B) + P(A^C | B) = 1$, $P(B | A) + P(B^C | A) = 1$, $P(A | B^C) + P(A^C | B^C) = 1$, and $P(B | A^C) + P(B^C | A^C) = 1$, so $P(A | B) + P(B | A) + P(A | B^C) + P(B | A^C) + P(A^C | B) + P(B^C | A) + P(A^C | B^C) + P(B^C | A^C) = \boxed{4}$.

$$11. \binom{10}{4} = \boxed{210}$$

12. The number of treatments in the experiment is the number of levels of one factor times the number of levels of the other factor, thus: $3(4) = \boxed{12}$.

13. Since blocks do not constitute a treatment, there are only $\boxed{3}$ treatments in the experiment.

$$14. P(2 \leq X \leq 3) = \binom{5}{2} \left(\frac{2}{5} \right)^2 \left(\frac{3}{5} \right)^3 + \binom{5}{3} \left(\frac{2}{5} \right)^3 \left(\frac{3}{5} \right)^2 = 10 \left(\frac{4}{25} \right) \left(\frac{27}{125} \right) + 10 \left(\frac{8}{125} \right) \left(\frac{9}{25} \right) = \frac{360}{625} = \boxed{\frac{72}{125}}$$

$$15. P(2 \leq X \leq 3) = \boxed{\frac{2}{5}}$$

$$16. E(X) = E(Y) \rightarrow np = \frac{1}{p} \rightarrow 16p = \frac{1}{p} \rightarrow 16p^2 = 1 \rightarrow p^2 = \frac{1}{16} \rightarrow p = \boxed{\frac{1}{4}}$$

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$$17. \text{Var}(X - Y) = \text{Var}(X) + \text{Var}(Y) = np(1-p) + \frac{1-p}{p^2} = 25 \left(\frac{2}{5}\right) \left(\frac{3}{5}\right) + \frac{\frac{3}{5}}{\left(\frac{2}{5}\right)^2} = 6 + \frac{15}{4} = \boxed{\frac{39}{4} \text{ or } 9.75}$$

$$18. \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.2(0.8)}{25}} = \boxed{0.08 \text{ or } \frac{2}{25}}$$

$$19. n = \left(\frac{Z^* \sigma}{ME}\right)^2 = \left(\frac{2 \cdot 15}{5}\right)^2 = \boxed{36}$$

$$20. \text{The width of the interval is twice the margin of error: } 2(ME) = 2(Z^*) \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 2(2) \sqrt{\frac{50}{400} \left(\frac{350}{400}\right) \left(\frac{1}{400}\right)} \\ = 4 \sqrt{\frac{1}{8} \left(\frac{7}{8}\right) \left(\frac{1}{400}\right)} = \boxed{\frac{\sqrt{7}}{40}}.$$

$$21. t = \frac{\bar{X}_1 - \bar{X}_2 - 0}{\sqrt{\frac{s_1^2}{n_2} + \frac{s_2^2}{n_1}}} \rightarrow \frac{5\sqrt{2}}{4} = \frac{100 - 90}{\sqrt{\frac{s^2}{50} + \frac{s^2}{50}}} \rightarrow \frac{5\sqrt{2}}{4} = \frac{10}{\sqrt{\frac{2s^2}{50}}} \rightarrow \frac{\sqrt{2}}{4} \cdot \frac{s}{5} = 2 \rightarrow s = \frac{40}{\sqrt{2}} = \boxed{20\sqrt{2}}$$

$$22. t = \frac{5\sqrt{2}}{4} < 1.25(1.5) = 1.875 < t^* = 2.01; \text{ so } \boxed{NO}, \text{ the null hypothesis is not rejected in this scenario.}$$

$$23. \chi^2 = Z^2 = \left(-\frac{5\sqrt{3}}{4}\right)^2 = \boxed{\frac{75}{16}}$$

$$24. \text{The upper bound of the confidence interval is } b + t^* \cdot SE(b) = 0.75 + 2(0.35) = \boxed{1.45}.$$

$$25. \frac{\binom{3}{2} \binom{4}{2} \binom{3}{2}}{\binom{10}{6}} = \frac{3 \cdot 6 \cdot 3}{210} = \boxed{\frac{9}{35}}$$